Financial Mathematics Assignment - 2

1. (i) Annual Percentage Rate $\Rightarrow \dfrac{\text{average annual interest}}{\frac{1}{2} \times \text{original loan}}$

$20000 = X\, a_{\overline{5}|}^{(12)}$

$20000 \Rightarrow (427.90)12\, a_{\overline{5}|}^{(12)}$

$= a_{\overline{5}|}^{(12)} \Rightarrow \boxed{3.89499}$

Flat rate $= \dfrac{(\text{total repayment} - \text{loan})}{\text{Original loan} \times \text{term in years}} \Rightarrow \dfrac{(5 \times 12 \times 427.90 - 20000)}{20000 \times 5}$

$= 0.05674 \Rightarrow \boxed{5.674\%}$

APR $\Rightarrow 2 \times 5.674 \Rightarrow 11.348\%$

(ii)

(i) APR $\Rightarrow 11.348\%$

$\Rightarrow 12 \times a_{\overline{4}|}^{(12)} @ 11.348 \times 427.90 \Rightarrow$

$i \Rightarrow 11\%\ a_{\overline{5}|}^{(12)} \Rightarrow 3.87872$

$i \Rightarrow 10\%\ a_{\overline{5}|}^{(12)} \Rightarrow 3.96154$

By linear interpolation

APR $\Rightarrow \dfrac{i - 10\%}{(3.89499 - 3.96154)} \Rightarrow \dfrac{11\% - 10\%}{(3.89499 - 3.96154)}$

$i \Rightarrow 10.8\%$

At $10.8\% \Rightarrow (12)\ 427.90\ a_{\overline{5}|}^{(12)} \Rightarrow \boxed{20000.2}$

At $10.9\% \Rightarrow (12)\ 427.90\ a_{\overline{5}|}^{(12)} \Rightarrow \boxed{19958.2}$

10.8 is more close $\therefore$ APR $\Rightarrow \boxed{10.8\%}$

(ii) APR $\Rightarrow 10.8\%$

$\Rightarrow 12 \times a_{\overline{4}|}^{(12)} @ 10.8 \times 427.90 \Rightarrow \boxed{16775.98}$

Date : ______ Subject : ______ Topic : ______

$16775.98 = 12 \times 274.949 \cdot a_{t}^{(k)}$

$16775.98 \Rightarrow 3293.88 \dfrac{(1 - 1.108^{-t})}{12 \times (1.108^{\frac{1}{12}} - 1)}$

$1.108^{-t} = 0.4754$

$t \Rightarrow \boxed{7.25 \text{ yrs}}$

iii Total Int for 4 yrs. $\Rightarrow (4 \times 12 \times 427.90) - 16775.68$

$\Rightarrow ₹\boxed{3763.52}$

~~Total Int for 7.25 yrs $\Rightarrow 7.25 \times 12 \times 427.90 - 16775.68$~~

~~$\Rightarrow \boxed{20451.62}$~~

Total Int for 7.25 yrs $\Rightarrow (12 \times 274.49 \times 7.25) - 16775.68$

$\Rightarrow ₹\boxed{7104.95}$

Excess interest paid $\Rightarrow 7104.95 - 3763.52$

$\Rightarrow ₹\boxed{3341.43}$

2.

a) Discounted Payback Period $\longrightarrow$ It is the time for which present value or the accumulated value of the paid up to the time 't' exceeds the present or accumulated value the cost up to time t. It gives the number of years it takes to break even from undertaking the initial expenditure by discounting future cash flows and recognizing the time value of money.

b) Payback Period $\longrightarrow$ It is the same as the discount payback period with an exception that the present value calculation is carried out using an interest rate of 0. In other words it is the ratio time for which the monetary value of the return exceeds the ~~...~~ value of the cost

ii The Payback period and the discount payback period does not give ~~g~~ answer for the profitability of a project. These ignore the cash flows accumulated values ~~of its~~ is zero. Whereas Payback period does not consider the time value of money whereas discount payback period uses discounted cash flows ~~and hence~~ which is more accurate than payback period. Therefore Net Present Value is a superior measure as compared to payback ~~per~~ period and discounted payback period

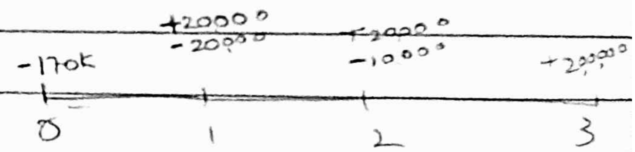
iii Project A

(i) int exp $\Rightarrow$ 170,000

outgo $\Rightarrow$ 20,000 ; 10000

inflow $\Rightarrow$ 20000 at end of 1 & 2.

Income $\Rightarrow$ 200000 at 3.



$$NPV \Rightarrow -170 + 10v^2 + 200v^3 = 0$$

at 7% $\Rightarrow$ NPV $\doteq$ 1.9939

at 8% $\Rightarrow$ NPV = -2.66

at 7.5% $\Rightarrow$ NPV $\Rightarrow$ -0.354

at 7.4% $\Rightarrow$ NPV $\Rightarrow$ 0.0116

By linear interpolation $\Rightarrow \dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

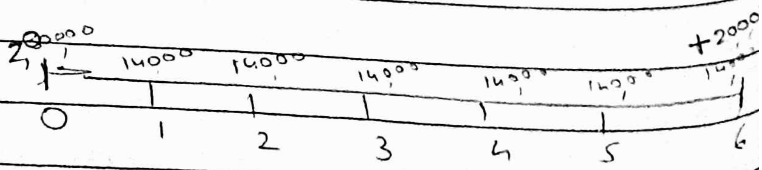
$i \Rightarrow 7.424\% \Rightarrow \boxed{7.4\%} = IRR_A$

Project B

int ~~exp~~ $\Rightarrow$ A + 30,000

income $\Rightarrow$ 14000 p.a. for 6 yrs

after that extra returns



Name : Grade: Div.: Roll No.:

Date : Subject : Topic :

$NPV \Rightarrow -200 + 140\, a_{\overline{6}|} + 200 v^6$

$IRR_B \Rightarrow 7\%$

b) $i = 6\%$

$NPV_A \Rightarrow -170 + 10v^2 + 200v^3 \Rightarrow \boxed{6873.821}$

$NPV_B \Rightarrow -200 + 14 \times a_{\overline{6}|} + 200v^6 \Rightarrow \boxed{9834.64}$

c. $IRR_A = 7.4\%$ and $IRR_B \Rightarrow 7\%$. Maximum rates that the bank could charge are 7.4% and 7% respectively because ~~after~~ any addition after this could make the NPV less than zero.

3. $A_1 \Rightarrow 200\, \ddot{a}_{\overline{22}|}\, v^{\frac{1}{4}} @ i = 8\% \Rightarrow 200 \times (v)^{\frac{1}{4}} \times 10.2007 \times \frac{i}{d}$

$$\frac{200 \times 0.980944 \times 10.2007 \times 0.08}{0.074074} \Rightarrow 2161.366301 \Rightarrow ₹\, \boxed{2161.37}$$

$A_2 \Rightarrow 320\, a^{(4)}_{\overline{16.25}|} \times (1+i)^{\left(\frac{1}{12}\right)} \Rightarrow 320 \times \frac{1-(0.925926)^{16.25}}{0.077706} \times 1.006434$

$\Rightarrow ₹\, 2957.870081 \Rightarrow \boxed{2957.87}$

$A_3 \Rightarrow 180 \times a^{(12)}_{\overline{15.75}|} \Rightarrow 180 \times \frac{1-(0.925926)^{15.75}}{0.077208}$

$\Rightarrow ₹\, 1780.664166 \Rightarrow \boxed{1780.696}$

Total value $\Rightarrow ₹\, \boxed{6899.900548} \Rightarrow$

$(1+i)\left[\cancel{12.4x+13.1x}\right.$

$12.4x + 13.1x^{\frac{3}{4}} + 14.8x^{\frac{1}{2}} + 15.8x^{\frac{1}{4}} = 65.6$

Let the new annuity be ~~A~~ X

$6899.9\cancel{0} = X\, a^{(2)}_{\overline{21.5}|}\,(1+i)^{\frac{1}{4}} \Rightarrow X = \dfrac{6899.90}{1.019427 \times 10.30880}$

$x \Rightarrow \boxed{656.56}$

4. $(I\ddot{a})_{\overline{n}|}$

1 2 3 4 n

0 1 2 3 n-1 n

$(I\ddot{a})_{n} \Rightarrow 1 + 2v + 3v^2 + 4v^3 \ldots nv^{n-1}$

$\Rightarrow (1+i)(v + 2v^2 + v^3 + \ldots nv^n)$

$\Rightarrow (1+i)(Ia)_{\overline{n}|}$

$\Rightarrow \boxed{\dfrac{\ddot{a}_{\overline{n}|} - nv^n}{\left(\frac{i}{1+i}\right)}}$

5.

(i) Time-weighted rate of return for PF $\Rightarrow (1+i) = \dfrac{16.4}{12.4}$

$i \Rightarrow 0.322580645$

$\Rightarrow \cancel{3.225}\ \boxed{32.26\%}$

Time weighted rate of return for EF $\Rightarrow (1+i) = \dfrac{15.5}{12.1}$

$i = 0.280991735$

$\Rightarrow \boxed{28.0991735\ \%}$

(ii)

a) Same no of unit on each date $\Rightarrow 12.4(1+i) + 13.1(1+i)^{\frac{3}{4}} + 14.8(1+i)^{\frac{1}{2}} + 15.8(1+i)^{\frac{1}{4}} \Rightarrow 4 \times 16.4$

$\Rightarrow i = \boxed{29.316\%}$

Grade: _____ Div.: _____ Roll No.: _____

Date : _____ Subject : _____ Topic : _____

5. loan ⇒ 160,000

$n = 10$ yrs. $i = 8\%$

(i) $160000 = C\, a_{\overline{10}|}$ @ 8%

$\frac{160000}{6.7101} = C \Rightarrow \boxed{₹ 23844.65}$

(ii) Capital outstanding after 4 payments

$\Rightarrow 23844.65 \times a_{\overline{6}|} \Rightarrow 23844.65 \times 4.6229$

$\Rightarrow ₹ \boxed{110231.4325}$

New payment be X $\Rightarrow 110231.4325 = X\, a_{\overline{6}|}$ @ 10%

$\frac{110231.4325}{4.3553} = X \Rightarrow \boxed{₹ 25304.722}$

(iii) Capital outstanding after 7^{th} repayment.

$\Rightarrow 25304.722 \times a_{\overline{3}|} \Rightarrow 25304.722 \times 2.4869$

$\Rightarrow \boxed{62942.75}$

New repayment be Y $\Rightarrow 62942.75 = Y\, a_{\overline{3}|}$ @ 9%

$\frac{62942.75}{2.5313} = Y \Rightarrow ₹ \boxed{24865.77}$

7.

(i) Reasons for opting government securities over property are:-

a) The risk of uncertainty plays a very important role. Government securities are much more trusted over property. Property rent may fluctuate a lot.

b) Properties are much more expensive as compared to a government security.

c) Tax levied on property is more as compared to a government security.

Date : Subject : Grade: Div.: Roll No.: Topic :

iii) Tenders of various are submitted to the office which consists of the price they are willing to pay as interest notes. All the tenders are ~~sends~~ scrutinized and then the highest amount which is submitted is the one to whom loan is issued.

b) Advantages → Government will be well known in advance of the issue price at outset. ~~and~~.

Disadvantages → For the sake of winning the bid he might have to pay more than the value.

c 3 securities and names are :-

(i) Shares → The dividend is not known, it might change & the company is not liable to pay the dividend ~~compulso~~ compulsorily.

(ii) Index Linked Bond → The ~~pr~~ interest may fluctuate due to change in the index and inflation it may increase or decrease hence not fixed.

(iii) Property → The rents of the property may vary from time to time and place to place.

8. P.V. of investment $\Rightarrow 2.7 + 0.2a_{\overline{3|}}@i$

expected P.V $\Rightarrow 0.1v^3 + \bar{a}_{\overline{10|}}(0.25v + 0.2v^2 + 0.1v^3)@i$

$\Rightarrow 2.7 + 0.2a_{\overline{3|}} = 0.1v^3 + \bar{a}_{\overline{10|}}(0.25v + 0.2v^2 + 0.1v^3)$

$2.7 = 0.1v^3 + \bar{a}_{\overline{10|}}(0.25v + 0.2v^2 + 0.1v^3) - 0.2a_{\overline{3|}}$

At $i \Rightarrow 9\%$ $\Rightarrow 2.75396$

At $i = 9.2\%$ $\Rightarrow 2.7207$

At $i = 9.3$ $\Rightarrow 2.704$

9.3% is the closest hence $\boxed{9.3\%}$

ii $NPV \Rightarrow 0.1v^3 + 0.55v^2 \bar{a}_{\overline{10}|} - 2.7 - 0.2 \ddot{a}_{\overline{3}|}$ @ 10%

$\Rightarrow 0.1(0.2513) + 0.55(6.4469)(0.2513) - 2.7 - 0.2(2.4868)$

$\Rightarrow \boxed{1.0089}$

Since NPV is ~~a~~ positive integer it is in profit

9

6   4.5   −2.5

1/1/11   1/12/11   30/6/12   1/7/13   31/12/13

30.5   33   41.6

$(1+i)^3 = \frac{33}{30.5} \times \frac{41.05-4.5}{33+6} \times \frac{45.6}{(41.05-4.5+4.5)} \times \frac{47}{(45.6-2.5)}$

$= \frac{33}{30.5} \times \frac{41.05-4.5}{39} + \frac{45.6}{41.05} \times \frac{47}{43.1}$

$= \frac{33}{30.5} \times \frac{36.55}{39} \times \frac{45.6}{41.05} \times \frac{47}{43.1}$

$i \Rightarrow \boxed{7.095128\%}$

ii) Linked rate of return from 1/1/11 − 31/12/13

2011

$30.5(1+i) - 6(1+i)^{\frac{1}{12}} = 38.5$

$\Rightarrow 30.5 + 30.5i + 30.5(x) + 6x^{\frac{1}{12}} = 38.5$

$i \Rightarrow \boxed{6.2566\%}$

2012

$38.5(1+i) + 4.5(1+i)^{\frac{1}{2}} = 45$

$i \Rightarrow \boxed{4.911\%}$

2013

$45(1+i) - 2.5(1+i)^{\frac{1}{2}} = 47$

$i \Rightarrow \boxed{16.2785\%}$

Date : Subject : Topic :

Linked Rate of Return ⇒

$(1+i)^3 \Rightarrow (1.062566)(1.04911)(1.02735)$

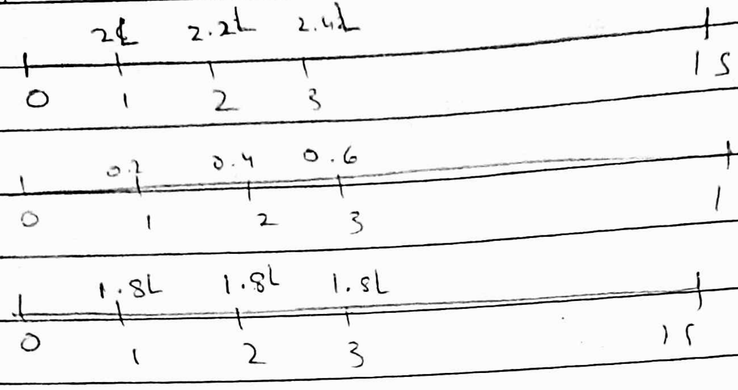
$i \Rightarrow 0.180584385 \quad 0.0712461252$

$\Rightarrow \boxed{7.124621\%}$

iii TWRR is a better measure of investment than MWRR as ~~MWRR~~ TWRR does not consider the amount and timing of the cashflows during calculations whereas MWRR considers the amount and timing of the cashflows.

10.

(i)



$\Rightarrow i = 12\%$

Let the amount be 'x'

$x \Rightarrow 1.8\, a_{\overline{15|}} + 0.2\, Ia_{\overline{15|}}$ @ $i = 12\%$

$\Rightarrow 1.8 \times 6.8109 + 0.2 \times 40.7310$

$\Rightarrow 20.40582$

So $x \Rightarrow \boxed{₹ 2040582}$

Let the cost of house be 'C'

$x = 80\%$ of C

$x = \frac{80}{100} C$

$\frac{100\, x}{80} = C$

$C \Rightarrow \frac{100}{80} \times 2040582$

$\Rightarrow \boxed{₹ 2550727.5}$

(ii) Loan o/s after 8^{th} repayment

$\Rightarrow [1.8 + (0.2 \times 8)]\, a_{\overline{7|}} + 0.2\, Ia_{\overline{7|}}$

$\Rightarrow [1.8 + 1.6]\, a_{\overline{7|}} + (0.2 \times 16.2080)$

$3.4(4.5638) + 3.2416$

$\Rightarrow \boxed{18.75852} \times 100000 \Rightarrow \boxed{₹ 1875852}$

Grade: ______ Div.: ______ Roll No.: ______

Date : ______ Subject : ______ Topic : ______

No.	Loan at start	~~Interest~~ Installment	~~Installment~~ Interest	Capital Rep.	Loan at end
9.	1875852	36,000	$\cancel{2}$ 235102.24	134897.76	1740954.2
10	1740954.27	38000 ~~208914.51~~	208914.51	171085.49	1569868.78

Let the new annual payment be 'N'

$15698\overset{68}{\wedge}.78 = N a_{\overline{5}|} @ i = 10\%$

$\Rightarrow N = \dfrac{1569868.78}{3.790786} \Rightarrow \boxed{₹\,414127.5734}$

Interest for new bank $\Rightarrow (414127.5734)5 - 1569868.75$

$\Rightarrow \boxed{₹\,500769.117}$

Interest from the new bank is less as compared to the prior one.

11. TWRR $\Rightarrow 20\% = i$

(i) $(1+i)^2 = \dfrac{500}{460} \times \left(\dfrac{550+50}{500+40}\right) \times \dfrac{X}{(550+50-50)}$

$(1+0.2)^2 = \dfrac{\cancel{500}^{10}}{46\cancel{0}} \times \dfrac{\cancel{600}}{\cancel{540}_{9}} \times \dfrac{X}{\cancel{550}_{11}} \Rightarrow$

$1.44 = \dfrac{10X}{4554} \Rightarrow X = \boxed{₹\,655.776}$

(ii) MWRR

$460(1+i)^2 + 40(1+i)^{1.75} - 50(1+i) = \cancel{565}\ 655.776$

At $i = 19\% \Rightarrow \boxed{-9.6365}$

$i = 20\% \Rightarrow \boxed{1.6575}$

$i = 19.8\% \Rightarrow -0.609$

$i = 19.9\% \Rightarrow 0.5237$

By interpolation $\Rightarrow \dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$i \Rightarrow \boxed{19.854\%}$

(iii) Linked rate of return using subintervals of 1 year

For year 2015 $\Rightarrow 460(1+i) + 40(1+i)^{\frac{3}{4}} - 50 = 550$

$i \Rightarrow \boxed{14.4644\%}$

For year 2016 $\Rightarrow 550(1+i) = 655.776$

$i = 0.19232 = \boxed{19.232\%}$

~~For year 2~~ $(1+i)^2 = (1.144644)(1.19232)$

$i = \boxed{16.82388\%}$

(iv) It will be ~~sol~~ identical when the subintervals are ~~equal~~ same in each calculation.

(v) TWRR is better than MWRR as it does not consider amount & timing of the cash flow.

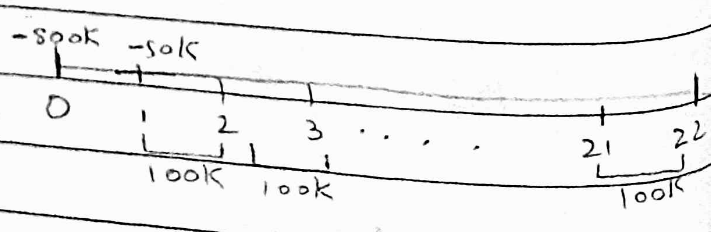
12.

(i) Discounted payback period is the time for which present or accumulated value of the paid up to time 't' exceeds the present or accumulated value of the cost up to time 't'.

(ii)(a) $i = 7\%$

P.V of investment $\Rightarrow -800B + 50V$

$\Rightarrow \boxed{846.7289}$



Grade: ______ Div.: ______ Roll No.: ______

Date : ______ Subject : ______ Topic : ______

PV of income $\Rightarrow 100\, \bar{a}_{\overline{n}|}\, v^{2}$

$$846.7289 = 100 \left(\frac{1-(1.07)^{-n}}{\log(1.07)} \right) (1.07)^{-2}$$

$$\cancel{\frac{846.7289}{100} \times \frac{\log 1.07}{1-(1.07)^{-n}} = (1.07)^{-n}}$$

$$0.3441 = (1.07)^{-n}$$

$\therefore$ Discounted Payback Period $= n =$

b) Lending ratio $= i \Rightarrow 6\%$

Acc ~~profit~~ amount $= 100\, \bar{s}_{\overline{4.33}|}$ @ 6%

$$\Rightarrow \frac{100 \times (1+0.06)^{4.33} - 1}{\ln(1.06)}$$

$$\Rightarrow 492.522874$$

Acc. ~~profit~~ amount $\Rightarrow$ ₹ $\boxed{492522874}$

(iii) DPP and the accumulated amount in the account ~~of~~ after 22 years.

13. Loan ⇒ 988000

$i = 7\%$ p.a.

$+988000$ 10/7/15 ... 25 yrs

(i) Loan repayment

$\Rightarrow 988000 = 12\, C\, a_{\overline{25}|}^{(12)}$ @ 7%.

$$\frac{988000}{12} = C\left[\frac{1-0.18424}{0.067850}\right]$$

$$82333.33 = C\,(12.02299189)$$

$$C \Rightarrow \boxed{6847.990424}$$

(ii) Loan after 10th March 2029 $n=136$

$$\Rightarrow 6847.990424 \times a_{\overline{136}|} @ \frac{i^{(12)}}{12}$$

$6847.990424 \times$

(iii) Loan after repaid on 10th October 2026 $n=165$

$$\Rightarrow 6847.99\, a_{\overline{165}|} @ \frac{i^{(12)}}{12}$$

⇒

Capital repaid ⇒ Installment − Interest

(iv) Loan repaid for the year (10th Apr − 10th March 2034)

$$\Rightarrow 6847.99\, a_{\overline{88}|} - 6847.99\, a_{\overline{76}|}$$

$$\Rightarrow 6847.99\left(a_{\overline{88}|} - a_{\overline{76}|}\right)$$