NMA ASSIGNMENT

Roll no. 69
Div B

1. Observed radius = 12.65 cm
Actual radius = 12.5 cm

Observed area = πr^2

$A_o = \pi \times (12.65)^2$

$\therefore A_o = 502.725 \text{ cm}^2$

Actual Area = πr^2

$A_A = \pi \times (12.5)^2$

$A_A = 490.874 \text{ cm}^2$

(i) Absolute error = Observed value − Actual value

$= 502.725 - 490.874$

$= 11.857 \text{ cm}^2$

(ii) Relative error = $\dfrac{\text{Absolute error}}{\text{Actual } \cancel{\text{error}} \text{ val.}}$

$= \dfrac{11.857}{490.874} = 0.02414$

(iii) Percentage Error = Relative Error × 100

$= 2.414\%$

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2 (i) $x_1 = 4$ $y_1 = 0.60206$

$x = 5$ $y = ?$

$x_2 = 6$ $y_2 = 0.7781513$

$$\therefore y = \frac{y_2(x-x_1) + y_1(x_2-x_1)}{(x_2-x_1)}$$

$$= \frac{0.7781513(1) + 0.60206(1)}{2}$$

$$\therefore y = 0.6901$$

(ii) $x_1 = 4.5$ $y_1 = 0.6532125$

$x = 5$ $y = ?$

$x_2 = 5.5$ $y_2 = 0.7403627$

$$\therefore y = \frac{y_2(x-x_1) + y(x_2-x)}{x_2-x_1}$$

$$= \frac{0.7403627(0.5) + 0.6532125(0.5)}{1}$$

$$y = 0.6901$$

3. $x^4 - x - 10 = 0$

$a = 1.5$ $b = 2$

$f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$

$= -6.4375$

$f(b) = f(2) = 2^4 - 2 - 10$

$= 4$

1st iteration:

$$x_1 = \frac{2+1.5}{2} = 1.75$$

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$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = -2.3711$

2nd iteration

$x_2 = \frac{1.75 + 2}{2} = 1.875$

$f(x_2) = 1.875^4 - 1.875 - 10 = 0.4846$

3rd iteration

$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

$f(x_3) = f(1.8125) = 1.8125^4 - 1.8125 - 10$
$= -1.0202$

4th iteration

$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$

$f(x_4) = f(1.84375) = 1.84375^4 - 1.84375 - 10$
$= -0.2877$

5th iteration

$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$

$f(x_5) = 1.859375^4 - 1.859375 - 10$
$= 0.093$

$\therefore$ Root of $x^4 - x - 10 = 0$ is 1.859375

which $\approx$ 1.86%

4. $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$\therefore x_0 = \frac{1+2}{2} = 1.5$

$f(x_0) = 0.875 \qquad f'(x_0) = 5.75$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1.5 - \frac{0.875}{5.75} = 1.34783$

$f(x_1) = 0.10068 \qquad f'(x_1) = 4.44991$

$x_2 = 1.34783 - \frac{0.10068}{4.44991}$

$= 1.3252$

$f(x_2) = 0.00205$

$f'(x_2) = 4.2684$

$x_3 = 1.3252 - \frac{0.00205}{4.2684}$

$x_3 = 1.3247$

$f(x_3) = 0.000007545 \qquad f'(x_3) = 4.2645$

$x_4 = 1.3247 - \frac{0}{4.2645}$

$x_4 = 1.3247$

Root of $x^3 - x - 1 = 0$ is 1.3247

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5. $A^2 = A$

$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - I^3 - A^2 A - 3A^2 - 3A$

$= 7A - I - A \cdot A - 3A - 3A$

$= 7A - I - A - 6A$

$= 7A - 7A - I$

$\therefore 7A (I+A)^3 = -I$

6. let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\therefore |A| = 1 \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2 \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix}$

$|A| = 1(-6) + 1(-4) + 2(4) = -2$

$|A| \neq 0 \therefore$ inverse exists.

$\therefore$ co-factor matrix $= \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 1 & 4 \end{bmatrix}$

$\therefore$ Adjoint matrix $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \times$ Adjoint

$-\frac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

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$$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & 2 \end{bmatrix}$$

9. The smallest 3 digit no. that leaves remainder of 2 is 101 and the largest is 998.

$d = 3$

$\therefore t_n = a + (n-1) d$

$998 = 101 + (n-1)3$

$n - 1 = \frac{998 - 101}{3} \Rightarrow 299$

$n = 299 + 1$

$\boxed{n = 300}$

$S_n = \frac{n}{2}(a+l) = \frac{300}{2}(101 + 998)$

$S_n = 164850$ //

$\therefore$ Sum of all nos. that leave a remainder of 2 when divided by 2 is 164850.

10. $t_6 = 32$

$t_8 = 128$

$t_6 = ar^5$

$t_8 = ar^7$

$\therefore \frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$\boxed{r = 2}$

11. Expand $(4+3x)^5 = (3x+4)^5$

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-1)}{3!}a^{n-3}.b^3$$

$$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4 . 4 + \frac{5 \times 4}{2}(3x)^3 4^2$$

$$+ \frac{5\times4\times3}{3\times2}(3x)^2 \times 4^3$$

$$+ \frac{5\times4\times3\times2}{4\times3\times2} \times 3x \times (4)^4$$

$$= 243x^5 + 1625x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\therefore \lambda^3 - 0(\lambda^2) + (-13)\lambda - (-30+18+0) = 0$$

$$\therefore \lambda^3 - 13\lambda + 12 = 0$$

$\therefore \lambda = 1$ satisfies.

$$\therefore \begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & \downarrow & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$$\therefore \lambda^2 + \lambda - 12 = 0$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$

$$(\lambda+4)(\lambda-3) = 0$$

$$\lambda = -4 \text{ or } \lambda = 3$$

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13. $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$\therefore f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = -13$

$f'(x) = 3x^2 - 14x + 8$

$f'(5) = 13$

$\therefore x_1 = 5 - \left(\frac{-13}{13}\right)$

$x_1 = 5 + 1 = 6$

$f(6) = 9$

$f'(6) = 32$

$x_2 = 6 - \left(\frac{9}{32}\right)$

$\therefore \boxed{x_2 = 5.71875}$.

14. $(1 - x + x^2)^4$

$\therefore [x^2 + (1-x)]^4$

let $1 - x = y$

$\therefore [x^2 + y]^4 =$

using binomial theory:

$(a+b)^n = a^n + n a^{n-1} \cdot b + \frac{n(n-1)}{2!} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!}$

$[x^2+y]^4 = (x^2)^4 + 4(x^2)^3 y + \frac{4 \times 3}{2} (x^2)^2 y^2 + \frac{4 \times 3 \times 2 (x^2)}{3 \times 2} + y^4$

$= \cancel{y}\, x^8 + 4x^6 y + 6x^4 y^2 + 4x^2 y^3 + y^4$

$= \therefore [x^2 + (1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2$
$+ 4x^2 (1-x)^3 + (1-x)^4$

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15. $e^{-x} = 3\log(x)$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1$ $\quad b = 2$

$\therefore f(a) = -0.367879$

$f(b) = 1.944106$

$\therefore \frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$

$f(x_1) = 0.993265$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = \cancel{\frac{1+1.5}{2}}\ 3\log(1.25) - e^{-1.25}$

$f(x_2) = \underline{\underline{0.382925}}$

$x_3 = \frac{1+1.25}{2} = 1.125$

$f(x_3) = 3\log(1.125) - e^{-1.125}$

$= 0.02869664$

$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = 3\log(1.0625) - e^{-1.0625}$

$= -0.16372$