classmate
Date
Page

Calculus

1. $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

let $t = \frac{2}{w}$

$\frac{dt}{dw} = -2w^{-2}$

$\frac{dt}{-2} = w^{-2} \cdot dw$

when $w = \frac{1}{50}$, $t = 100$

when $w = 2$, $t = 1$

$I = \int_{100}^{1} \frac{e^t}{-2} dt$

$= -\frac{1}{2} \int_{100}^{1} e^t dt.$

$= -\frac{1}{2} \left[e^t \right]_{100}^{1}$

$= -\frac{1}{2} \left[2.718 - 2.68117 \times 10^{43} \right]$

$= 1.30585 \times 10^{43}$

classmate
Date
Page

2. $\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

Let $u = t^4 + 2t$

$\frac{du}{dt} = 4t^3 + 2$

$\frac{du}{2} = 2(2t^3+1)$

Now, $I = \frac{1}{2}\int \frac{1}{u^3}\,du$

$= \frac{1}{2}\int u^{-3}\,du$

$= \frac{1}{2}\,\frac{u^{-2}}{-2} + c$

$= -\frac{1}{4}u^{-2} + c$

from a, $-\frac{1}{4}(t^4+2t)^{-2} + c$

3. $f'(x) = x^4 + 3x - 9$

$f(x) = \int x^4 + 3x - 9\,dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

Date
Page

4, $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

Evaluate
$$\int_{-2}^{3} f(x)\, dx$$

$$I = \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$$
$$= \left[\frac{3(2x^3)}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= \frac{47}{3}$$

5. $\int 3(8y-1)e^{4y^2-y}\, d\cancel{x}y$

let $t = 4y^2 - y$
$$\frac{dt}{dy} = 8y - 1$$
$$dt = (8y-1)\, dy$$

$$I = 3\int e^t . dt$$
$$= 3e^t + C$$

$$I = 3\cancel{x}e^{(4y^2-y)} + C$$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$
$f(1) = -\frac{5}{4}$ $f(4) = 404$

To find $f(x)$

$f'(x) = \int f''x\,dx$

$= \int 15x^{1/2} + 5x^3 + 6\,dx$

$= \left[\frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x\right] + C$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C.$

$f(x) = \int f'(x)\,dx$

$= \int\left[10x^{3/2} + \frac{5}{4}x^4 + 6x + C\right]$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + K$

Now $f(1) = -\frac{5}{4}$ hence,

$f(1) = 4 + \frac{1}{4} + 3 + C + K = -\frac{5}{4}$

$4[C+K] = -34$

$K = -\frac{17}{2} - C$

Also $f(4) = 404$

$404 = 4(4)^{5/2} + \frac{4^5}{4} + 3(4)^2 + 4C + K$

classmate

Date

Page

$4C + K = -28$

$4C - \frac{17}{2} - 2\cancel{K} = -28$

$3C - 17 - 2C = -56$

$6C = -39$

$C = -6.5$

$K = \frac{-17}{2} - (-6.5)$

$K = -2$

So $f(x) =$

$4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2$

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $r(1) = 2$

$\frac{dr}{r^2} = \frac{d\theta}{\theta}$

integrating both sides

$\int \frac{1}{r^2} dr = \int \frac{1}{\theta} d\theta$

$\frac{r^{-2+1}}{-2+1} + C = \log \theta$

$C = \log \theta + r^{-1}$

Page

$v(1) = 2$

$C = \log(1) + \frac{1}{2}$
$= 0 + \frac{1}{2}$

$C = \frac{1}{2}$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\left(\frac{1}{9.8 - 0.196v}\right) dv = dt$

$\int \frac{1}{1.002v} dv = 9.8 \int dt$

$\log \frac{(1 - 0.02v)}{-0.02} = 9.8t + C$

using IVP

$ty + 2y = t^2 - t + 1$ Now $y(1) = \frac{1}{2}$

$\frac{dy}{dt} + 2y = t^2 - t + 1$

11. $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$y(0) = -1$

$\int \frac{1}{y^3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx$

$\frac{-1}{2y^2} = \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$

Now $1+x^2 = t$

$2x\,dx = dt$

$dx = \frac{1}{2}\,dt$

$\frac{-1}{2}\frac{1}{y^2} = \sqrt{1+x^2} + C$

$\frac{-1}{2} = 1 + C$

So $C = \frac{-3}{2}$

$\frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$

Date
Page

12. $f(x) = x^3 - 10x^2 + 6$

Now for $x = 3$

$f'(x) = 3x^2 - 20x + 0$
$f''(x) = 6x - 20$
$f'''(x) = 6$
$f^{4}(x) = 0$

$f(3) = -57$
$f'(3) = -33$
$f''(3) = -2$
$f'''(3) = 6$

Now the taylor series is given as

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a).$$

$$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{2!}(-33) + \frac{(x-3)^3}{3!}(6) + 0$$

So $f(x) = -57 + (x-3)(-33) - (x-3)^2 + (x-3)^3$

classmate Date Page

13. $f(x) = (1+x)^{\mu}$

The Maclaurin series:

$$f(x) = f(0) + x' f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$f(x) = (1+x)^{\mu}$

$f'(x) = \mu(1+x)^{\mu-1}$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$

$f'''(x) = \mu^3 - 3\mu^2 + 2\mu \; (1)^{\mu-3}$

$= \cancel{\mu^3 - 3\mu^2 - 2\mu}$

$f(0) = 1$

$f'(0) = \mu$

$f''(0) \; \mu^2 - \mu$

$f'''(0) = \mu^3 - 3\mu^2 - 2\mu$

The series

$$f(x) = 1 + x(\mu) \; \frac{x^2}{2!}(\mu)(\mu-1) + \frac{x^3}{3!}(\mu)(\mu-1)(\mu-2)$$

14. $f(x) = \sqrt{1+x}$

The maclaurin series

$$f(x) = f(0) = xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$f(x) = \sqrt{1+x}$

$f'(x) = \frac{1}{2\sqrt{1+x}}$

$f''(x) = \frac{-1}{4}(1+x)^{-1/2-1}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$

$f(0) = 1$

$f'(0) = 1/2$

$f''(0) = -1/4$

$f'''(0) = 3/8$

The series

$f(x) = 1 + x\left(\frac{1}{2}\right) + \frac{x^2}{2!}\left(\frac{-1}{4}\right) + \frac{x^3}{3!}\left(\frac{3}{8}\right) \ldots$

15. Given that

$f(x) = e^{-6x}$

~~Also~~ $x = -4$

Taylor series is given as

$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a)$

$f(x) = e^{-6x}$ | $f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ | $f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ | $f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ | $f'''(-4) = -216e^{24}$

$f(x) = e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2!}(36e^{24})$
$+ \frac{(x+4)^3}{3!}(-216e^{24})$

6. ~~$f(x) = e^{6x}$ for~~ $f(x) = \log(3+4x)$ for $x = 0$

Taylor series

$f(x) = f(a) + f'(a)(x-a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a)$

$f(x) = \log(3+4x)$

$f'(x) = \frac{1}{(3+4x)} \cdot 4$

$f''(x) = \frac{-4}{(3+4x)^2}$

$f'''(x) = \frac{8}{(3+4x)^3}$

$f(0) = \log 3$

$f'(0) = \frac{4}{3}$

$f''(0) = \frac{-4}{9}$

$f'''(0) = \frac{8}{27}$

The series.

$f(x) = \log 3 + x\,4/3 + \frac{x^2}{2!}\left(\frac{-4}{9}\right) + \frac{x^3}{3!}\left(\frac{8}{27}\right) \ldots$