Tisha Kothari - 41

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Calculus - Assignment 2.

Q1] $\displaystyle\int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw$

let $t = e^{2/w}$

diff wrt w

$\frac{dt}{dw} = -2e^{2/w} \frac{1}{w^2}$

$= \displaystyle\int_{1/50}^{2} \frac{t w^2}{-2w^2 e^{2/w}}$

$= -\frac{1}{2} \displaystyle\int_{1/50}^{2} dt$

$= -\frac{1}{2} [x]_{1/50}^{2}$

$= -\frac{1}{2}(2 - 0.02)$

$= -0.99$

Q2] $\displaystyle\int \frac{2t^3+1}{(t^4+2t)^3} \, dt$

let $x = (t^4+2t)^3$

diff wrt t.

$\frac{dx}{dt} = 3(t^4+2t)^2(4t^3+2)$

$= \displaystyle\int \frac{2t^3+1}{x(3)(t^4+2t)^2(4t^3+2)}$

Multiply & divide by 2.

$= \displaystyle\int \frac{4t^3+2}{(x)(6)(4t^3+2)(x)^{2/3}}$

$= \frac{1}{6} \displaystyle\int x^{-5/3} \, dx$

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$= \frac{1}{6}\left(\frac{x^{-5/3+1}}{-5/3+1}\right) + c$

$= -\frac{1}{4} x^{-2/3} + c$

$= -\frac{1}{4} (t^4 + 2t)^{3(-2/3)} + c$

$= -\frac{1}{4} (t^4 + 2t)^{-2} + c$

Q3] $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)\, dx$

$f(x) = \int (x^4 + 3x - 9)\, dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

Q4] $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$\int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x)\, dx + \int_{1}^{3} f(x)\, dx$

$= \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$

$= [x^3]_{-2}^{1} + [6x]_{1}^{3}$

$= 1 + 8 + 18 - 6$

$= 4$

Q5] $\int 3(8y - 1) e^{4y^2 - y}\, dy$

Let $x = e^{4y^2 - y}$

diff. wrt y

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$$\frac{dx}{dy} = (e^{4y^2-y})(8y-1)$$

$$\int \frac{3(8y-1)e^{4y^2-y}}{(e^{4y^2-y})(8y-1)} dx$$

$$= 3x + c$$

$$= 3[e^{4y^2-y}] + c$$

Q6] $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$$f'(x) = \int 15\sqrt{x} + 5x^3 + 6 \, dx$$

$$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k$$

$$f(x) = \int \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k \, dx$$

$$= 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + c$$

$$f(1) = \frac{-5}{4}$$

$$4(1)^{5/2} + \frac{(1)^4}{4} + 3(1)^2 + k(1) + c = \frac{-5}{4}$$

$$k + c = -8.5$$

$$f(4) = 404$$

$$4(4)^{5/2} + \frac{(4)^4}{4} + 3(4)^2 + k(4) + c = 404$$

$$4k + c = 164$$

$$4k + c = 164$$

$$(-) \quad k + c = -8.5$$

$$3k = 172.5$$

$$k = 57.5$$

$$k + c = -8.5$$

$$c = -66$$

$$\therefore f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + 57.5x - 66$$

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(9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\int \frac{dv}{9.8-0.196v} = \int dt$

$\log(9.8-0.196v) = -0.196t + C$

(12) Taylor series for $f(x) = x^3 - 10x^2 + 6$ $\boxed{x=3}$

$f(x) = x^3 - 10x^2 + 6$ $f(3) = -57$

$f'(x) = 3x^2 - 20x$ $f'(3) = -33$

$f''(x) = 6x - 20$ $f''(3) = -2$

$f'''(x) = 6$ $f'''(3) = 6$

$f(x) = f(3) + (x-3)f'(x) + \frac{(x-3)^2}{2!} f''(3) + \frac{(x-3)^3}{3!} f'''(3) + \ldots$

$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{6}(6)$

$f(x) = -57 + (x-3)(-33) + (x-3)^2(-1) + (x-3)^3 \ldots$

(13) Maclaurin series for $(1+x)^{\mu}$

$f'(x) = \mu(1+x)^{\mu-1}$

$f''(x) = \mu(\mu-1)(1-x)^{\mu-2}$

$f'''(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$

$f(x) = f(0) + \frac{x^1}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$

$(1-x)^{\mu} = 1 + \mu + (\mu-1)(\mu) + \mu(\mu-1)(\mu-2) \ldots$

(14) $f(x) = \sqrt{1+x}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{-3}{8}(1+x)^{-5/2}$ $f'''(0) = \frac{-3}{8}$

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$$f(x) = \sqrt{1+x} + \frac{x^1}{1!}\left(\frac{1}{2}\right) + \frac{x^2}{2!}\left(\frac{-1}{4}\right) + \frac{x^3}{3!}\left(\frac{-3}{8}\right) \cdots$$

Q15]

$f(x) = e^{-6x}$ $\qquad x = 4$

$f(x) = e^{-6x}$ $\qquad f(4) = e^{24}$

$f'(x) = e^{-6x}(-6)$ $\qquad f'(4) = e^{24}(-6)$

$f''(x) = (-6)(e^{-6x})(-6)$ $\qquad f''(4) = e^{24}(-6)^2$

$f'''(x) = (-6)^3(e^{-6x})$ $\qquad f'''(4) = e^{24}(-6)^3$

$$e^{-6x} = e^{24} + \frac{(x-4)}{1!}(e^{24})(-6) + \frac{(x-4)^2}{2!}(e^{24})(-6)^2 + \frac{(x-4)^3}{3!}(e^{24})(-6)^3$$

Q16]

$f(x) = \ln(3+4x)$ $\qquad x = 0$

$f'(x) = \frac{4}{3+4x}$ $\qquad f'(0) = \frac{4}{3}$

$f''(x) = (-4)(4x+3)^{-2}(4)$ $\qquad f''(0) = (-1)(4)^2(3)^2$

$f'''(x) = (8)(4x+3)^{-3}(4)^2$ $\qquad f'''(0) = (-1)(-2)(3)^{-2}(4)^3$

$$\ln(3+4x) = \ln(3) + \frac{x}{1!}\left(\frac{4}{3}\right) + \frac{x^2}{2!}(-1)(4)^2(3)^2 + \frac{x^3}{3!}(-1)(-2)(3)^{-2}(4)^3$$