

Numerical methods and algebra

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1. $\sqrt{7 + \sqrt{7} + \sqrt{7} \dots}$

$$y = \sqrt{7 + \sqrt{7} + \sqrt{7}}$$

$$y = \sqrt{7 + y}$$

$$y^2 = 7 + y$$

$$y^2 - y - 7 = 0$$

$$y = \frac{1 \pm \sqrt{29}}{2}$$

$$\Rightarrow \frac{1 + \sqrt{29}}{2} \text{ as } y \text{ is always } > 0$$

2. $7x^2 - 6y^2 = 11x^2 - 4y^2$
 $18x^2 = 2y^2$

$$\frac{x^2}{y^2} = \frac{2}{18}$$

$$\boxed{\frac{x}{y} = \sqrt{\frac{2}{18}}}$$

2. ~~$7x^2 - 6y^2 = 112$~~

3. $x + 3y = 4$
 $x^2 + y^2 - 4y - 12 = 0 \rightarrow \textcircled{1}$

$x = 4 - 3y \rightarrow \textcircled{2}$

② in ①

$(4 - 3y)^2 + y^2 - 4y - 12 = 0$

$16 + 9y^2 - 24y + y^2 - 4y - 12 = 0$

$10y^2 - 28y + 4 = 0$

$5y^2 - 14y + 2 = 0$

$y = \frac{14 \pm \sqrt{156}}{10} = \frac{14 \pm 12.49}{10}$

$= -2.649$

$= 2.649, -0.151$

$x_1 = 4 - 3(2.649)$
 $= -3.9470$

$x_2 = 4 - 3(-0.151)$
 $= 8.53$

The pairs are

$(-3.9470, 2.649), (3.53, -0.151)$

$$4 \quad d = 2x^2 - 16x + 34 \Rightarrow \text{density}$$

$$i) \quad x = 3.5$$

$$d = 2.5 \text{ ml particles / M}^3$$

$$ii) \quad d - 2x^2 - 16x + 34 = 0$$

differentiation

$$4x - 16 = 0 \rightarrow \textcircled{1}$$

$$x = 4 \quad (400 \text{ RPM})$$

differentiation of $\textcircled{1}$

minimum

$$d = 2(4)^2 - 16(4) + 34$$

$$d = 2$$

$\therefore$ The no. of revolution for minimum
Smoke = 4

$$iii) \quad d = 100$$

$$100 = 2x^2 - 16x + 34$$

$$30 = x^2 - 8x + 17$$

$$0 = x^2 - 8x + (-33)$$

$$x = (x-11)(x+3)$$

$$\boxed{x = 11}$$

The speed must be 11 revolution per minute

$$5) \quad C_1 = 21500 + 25(\text{km})$$

$$C_2 = 2100 + 22(\text{km}).$$

i) cost to rent car from Co. 1

$$C_1 = 1500 + 25x$$

ii) cost to rent car from Co. 2

$$C_2 = 2100 + 22x$$

iii) $C_2 < C_1$

$$1500 + 25x > 2100 + 22x$$

$$3x > 600$$

$$x > 200$$

It is less expensive to take C_2 when the number of km is more than 200.

$$6) \quad x^3 - 7x^2 + 14x - 6 = 0$$

$$a) \quad a=0, \quad b=1$$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f(a+b/2)$
0	1	0.5	-6	2	-0.65
0.5	1	0.75	-0.625	2	0.2849
0.5	0.75	0.625	-0.628	0.9814	0.2598

$$x = 0.625$$

b) $a=1$ $b=3.2$ $f(x)=x^3-7x^2+14x-6$

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f(\frac{a+b}{2})$
1	3.2	2.1	2	1.701	-0.11211771
2.1	3.2	2.65	1.791	-0.42	0.5521
2.65	3.2	2.925	1.5521	-0.112	0.08522

$x = 2.925$

c) $[3.2, 4]$

a	b	$\frac{a+b}{2}$	$f(a)$	$f(b)$	$f(\frac{a+b}{2})$
3.2	4	3.6	-0.112	2	0.336
3.6	4	3.8	-0.112	0.336	-0.016
3.8	4	3.9	-0.016	0.336	0.125
3.9	4	3.95	-0.016	0.125	0.0461
3.95	4	3.975	-0.016	0.0461	0.0130

$x = 3.975$

7 $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
1	-1	156.1300	-1105	-0.608121
2	-0.6081	43.994	-418.898	-0.27521
3	-0.2354	1.534	-234.246	-0.04754
4	-0.04754	0.0001	-221.704	-0.04066
5	-0.04066	~0	-221.70443	-0.04066
6	-0.04066	~0	-221.70443	-0.04066

$x_c = -0.04066$

b)

i	x_i	$f(x_i)$	$F(x_i)$	$F(x_{i+1}) = \frac{F(x_i) + x_i}{F(x_i)}$
1	0.1	-36.585	507.62	0.97179
2	0.97179	6.37668	691.80	0.96254
3	0.96254	0.11523	666.81	0.9629
	0.9629	0.3267	666.411	0.9624

$x = 0.9624$

8 a) Average = $\frac{70 + 76 + 81 + 83 + x}{5}$

$= \frac{320 + x}{5}$

b) Average = 8

$80 < \frac{320 + x}{5} < 90$

$400 < 320 + x < 450$

$80 < x < 130$

$80 < x < 130$

9. $x_1 = 0.04$, $x_2 = 0.2$ $x = 0$
 $f(x_1) = 3.54$ $f(x_2) = -11.81$ $f(x) = ?$

$$\frac{x - x_1}{x_2 - x_1} = \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)}$$

$$\frac{-0.04}{0.16} = \frac{f(x) - 3.54}{-20.35}$$

$f(x) = 13.6275$

10 $R(x) = 32x - 0.21x^2$
 $C(x) = 195 + 12x$
for profit $R(x) > C(x)$

$$\therefore 32x - 0.21x^2 > 195 + 12x$$

$$20x - 195 - 0.21x^2 > 0$$

$$0.2x^2 - 20x + 195 < 0$$

$$0.21x^2 - 20x + 195 \neq 0$$

$$x = \frac{20 \pm \sqrt{2128}}{0.42}$$

$$x = 82.38, 12.88$$

$$(x - 82.38)(x - 12.88) < 0$$

$$x \in (12.88, 82.38) //$$