

# HOMEWORK - 1

Lagrange's ~~Form~~ Interpolation

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1)

|   |   |   |   |    |
|---|---|---|---|----|
|   | 0 | 1 | 2 | 3  |
| x | 7 | 8 | 9 | 10 |
| y | 3 | 1 | 1 | 9  |

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} (3) + \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} (1) \\ + \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} (1) + \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} (9) \\ = 0.1875 + (-0.3125) + 0.9375 + 2.8125$$

$$f(9.5) = \underline{\underline{3.625}}$$

2)

|      |     |     |    |    |
|------|-----|-----|----|----|
|      | 0   | 1   | 2  | 3  |
| x    | 3   | 7   | 9  | 10 |
| f(x) | 168 | 120 | 72 | 63 |

$$f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)} (168) + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} (120) + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} (72) \\ + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} (63)$$

$$= \frac{(-1)(-3)(-4)}{(-4)(-6)(-7)} (168) + \frac{(3)(-3)(-4)}{(4)(-2)(-3)} (120) + \frac{(3)(-1)(-4)}{(6)(2)(-1)} (72) + \frac{(3)(-4)(-2)}{(7)(3)(1)} (63)$$

$$= 12 + 180 - 72 + 18$$

$$f(6) = \underline{\underline{138}}$$

(2)

3)

|        |   |   |   |   |
|--------|---|---|---|---|
| $x$    | 1 | 2 | 7 | 8 |
| $f(x)$ | 4 | 5 | 5 | 4 |

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(4-2)(4-7)(4-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(5-1)(5-7)(5-8)} + \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} \\ + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)}$$

$$= \frac{(4)(-1)(-2)(4)}{(1)(-3)(-4)} + \frac{(5)(-1)(-2)(5)}{(1)(-4)(-3)} + \frac{(5)(4)(-2)(5)}{(6)(5)(-1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$= (-0.7619) + 7.6667 + 6.6667 + 1.9048$$

$$f(6) = \underline{\underline{5.6667}}$$

4)

|        |   |   |   |    |     |
|--------|---|---|---|----|-----|
| $x$    | 1 | 2 | 3 | 4  | 7   |
| $f(x)$ | 2 | 4 | 8 | 16 | 128 |

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)}$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)}$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}$$

$$= \frac{(8)(2)(1)(-2)(2)}{(1)(-2)(-3)(-6)} + \frac{(4)(2)(1)(-2)(4)}{(1)(-4)(-3)(-5)} + \frac{(4)(3)(4)(-2)(8)}{(2)(1)(-1)(-4)}$$

$$+ \frac{(4)(3)(2)(-2)(16)}{(3)(2)(1)(-3)} + \frac{(4)(3)(2)(1)(128)}{(6)(5)(4)(3)}$$

$$= -0.6667 + 6.4 - 8 + 42.6667 + 8.5334$$

$$f(5) = \underline{\underline{32.9334}}$$

(3)

|    |        |      |     |      |      |               |        |      |      |      |     |
|----|--------|------|-----|------|------|---------------|--------|------|------|------|-----|
| 5) | $x$    | 10   | 20  | 14   | 16   | $\Rightarrow$ | $x$    | 10   | 14   | 16   | 20  |
|    | $f(x)$ | 0.25 | 0.2 | 0.15 | 0.10 |               | $f(x)$ | 0.25 | 0.15 | 0.10 | 0.2 |

$$\begin{aligned}
 f(15) &= \frac{(15-14)(15-16)(15-20)(0.25)}{(10-14)(10-16)(10-20)} + \frac{(15-10)(15-16)(15-20)(0.15)}{(14-10)(14-16)(14-20)} \\
 &\quad + \frac{(15-10)(15-14)(15-20)(0.10)}{(16-10)(16-14)(16-20)} + \frac{(15-10)(15-14)(15-16)(0.2)}{(20-10)(20-14)(20-16)} \\
 &= \frac{(1)(-1)(-5)(0.25)}{(-4)(-6)(-10)} + \frac{(5)(-1)(-5)(0.15)}{(4)(-2)(-6)} + \frac{(5)(1)(-5)(0.10)}{(6)(2)(-4)} + \frac{(5)(1)(-1)(0.2)}{(10)(6)(4)} \\
 &= -0.0052 + 0.078125 + 0.052 - 0.0042 \\
 f(15) &\approx \underline{\underline{0.1204}}
 \end{aligned}$$

## HOMEWORK-2

### Regular Falsi Method

1)  $f(x) = x^3 + x - 1 = 0 = y$

$x_1 = 0.5$  ,  $y_1 = -0.375$

$x_2 = 1$  ,  $y_2 = 1$

$x_3 = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{(0.5)(1) - (1)(-0.375)}{1 - (-0.375)} = 0.6364$

$y_3 = -0.1059$

$x_4 = \frac{x_3 y_2 - x_2 y_3}{y_2 - y_3} = \frac{(0.6364)(1) - (1)(-0.1059)}{1 - (-0.1059)} = 0.6712$

$y_4 = -0.0264$

$x_5 = \frac{x_4 y_2 - x_2 y_4}{y_2 - y_4} = \frac{(0.6712)(1) - (1)(-0.0264)}{1 - (-0.0264)} = 0.6797$

$y_5 = -0.0063$

$x_6 = \frac{x_5 y_2 - x_2 y_5}{y_2 - y_5} = \frac{(0.6797)(1) - (1)(-0.0063)}{1 - (-0.0063)} = 0.6817$

$y_6 = -0.0015$

$x_7 = \frac{x_6 y_2 - x_2 y_6}{y_2 - y_6} = \frac{(0.6817)(1) - (1)(-0.0015)}{1 - (-0.0015)} = 0.6822$

(4)

$$y_7 = -0.0003$$

$$\text{Root of } f(x) = \underline{\underline{0.6822}}$$

 $x =$ 

$$2) f(x) = x^2 - 2x + 0.5 = 0$$

$$x_1 = 0, y_1 = 0.5$$

$$x_2 = 1, y_2 = -0.5$$

$$x_3 = \frac{0 - 0.5}{-0.5 - 0.5} = 0.5, y_3 = -0.375$$

$$x_4 = \frac{-0.25 + 0.375}{-0.5 + 0.375} = -1, y_4 = 1.5$$

$$x_5 = x_2 = 1, y_2 =$$

$$x_4 = -1$$

$$x_1 = x_1 = 0, y_1 = 0.5$$

$$x_3 = 0.5, y_3 = -0.375$$

$$x_4 = \frac{0 - 0.25}{-0.375 - 0.5} = 0.2857, y_4 = -0.0481$$

$$x_5 = \frac{0 - 0.14285}{-0.0481 - 0.5} = 0.2606, y_5 = -0.0035$$

$$x_6 = \frac{0 - 0.1303}{-0.0035 - 0.5} = 0.2589, y_6 = -0.0004$$

$$\text{Root of the equation} = \underline{\underline{0.2589}}$$

(5)

$$3) f(x) = x^3 + 2x - 20 = 0 = y$$

$$x_1 = 2, y_1 = -16 - 8$$

$$x_2 = 2.5, y_2 = -9.375 + 0.625$$

$$x_3 = \frac{1.25 + 20}{8.625} = 2.4695, y_3 = -0.1164$$

$$x_4 = \frac{1.5399 + 0.291}{0.625 + 0.1164} = 2.4695, y_4 = -0.0009$$

$$\text{Root of } f(x) = \underline{2.4695}$$

$$4) f(x) = y = 3x - \cos x - 1 = 0$$

$$x_1 = 0, y_1 = -2$$

$$x_2 = 1, y_2 = 2$$

$$x_3 = \frac{0 + 2}{1 + 2} = 0.6667, y_3 = 0.0016$$

$$\text{Root of } f(x) = \underline{0.6667}$$

$$5) f(x) = y = x^3 - 3x - 5$$

$$x_1 = 2, y_1 = -3$$

$$x_2 = 2.5, y_2 = 3.125$$

$$x_3 = \frac{(2 \times 3.125) - (2.5 \times -3)}{3.125 + 3} = 2.2449, y_3 = -0.4214$$

$$x_4 = \frac{(2.2449 \times 3.125) - (2.5 \times -0.4214)}{-3.125 + 0.4214} = 2.2752, y_4 = -0.0479$$

$$x_5 = \frac{(2.2752 \times 3.125) - (2.5 \times -0.0479)}{3.125 + 0.0479} = 2.2786, y_5 = -0.0053$$

$$x_6 = \frac{(2.2786 \times 3.125) + (2.5 \times 0.0053)}{3.125 + 0.0053} = 2.2799 = y_6 = -0.0002$$

$$\text{Root of } f(x) = \underline{2.2799}$$

(6)

$$6) f(x) = y = x \log_{10} x - 1.2 = 0$$

$$x_1 = 2, y_1 = -0.5979$$

$$x_2 = 3, y_2 = 0.2314$$

$$x_3 = \frac{(2 \times 0.2314) - (3)(-0.5979)}{0.2314 + 0.5979} = 2.7201, y_3 = -0.0171$$

$$x_4 = \frac{(2.721 \times 0.2314) + (3 \times 0.0171)}{0.2314 + 0.0171} = 2.7402, y_4 = -0.00038$$

$$\text{Root of } f(x) = \underline{2.7402}$$

$$7) f(x) = y = x^3 - 4x + 1 = 0$$

|   |   |    |
|---|---|----|
| x | 0 | 1  |
| y | 1 | -2 |

$$x_1 = 0, y_1 = 1$$

$$x_2 = 1, y_2 = -2$$

$$x_3 = \frac{0 - 1}{-3} = 0.3334, y_3 = -0.2965$$

$$x_4 = \frac{0 - 0.3334}{-1.2965} = 0.2572, y_4 = -0.0118$$

$$x_5 = \frac{0 - 0.2572}{-1.0118} = 0.2542, y_5 = -0.0003$$

$$\text{Root of } f(x) = \underline{0.2542}$$

$$8) f(x) = y = x^3 + 2x - 8 = 0$$

|   |    |    |   |
|---|----|----|---|
| x | 0  | 1  | 2 |
| y | -8 | -5 | 8 |

$$x_1 = 1, y_1 = -5$$

$$x_2 = 2, y_2 = 8$$

$$x_3 = \frac{8 + 10}{8 + 5} = 1.3846, y_3 = -1.1244 - 1.5113$$

$$x_4 = \frac{(1.5556 \times 4) + (2 \times 1.1244)}{4 + 1.1244} = 1.6531, y_4 =$$

$$x_4 = \frac{(1.3846 \times 8) + (2 \times 1.5113)}{8 + 1.5113} = 1.4824, y_4 = -0.3474$$

$$x_5 = \frac{(1.4824 \times 8) + (2 \times 0.3474)}{8 + 0.3474} = 1.5039, y_5 = -0.0752$$

$$x_6 = \frac{(1.5039 \times 8) + (2 \times 0.0752)}{8 + 0.0752} = 1.5085, y_6 = -0.0162$$

$$x_7 = \frac{(1.5085 \times 8) + (2 \times 0.0162)}{8 + 0.0162} = 1.757, y_7 = 3.598$$

$$x_8 = \frac{(1.5085 \times 3.598) + (1.757 \times 0.0162)}{3.598 + 0.0162} = 1.5017, y_8 = -0.1033$$

$$x_9 = \frac{(1.5017 \times 3.598) + (1.757 \times 0.1033)}{3.598 + 0.1033} = 1.5088, y_9 = -0.0123$$

$$x_{10} = \frac{(1.5088 \times 3.598) + (1.757 \times 0.0123)}{3.598 + 0.0123} = 1.5096, y_{10} = -0.002$$

$$x_{11} = \frac{(1.5096 \times 3.598) + (1.757 \times 0.002)}{3.598 + 0.002} = 1.5097, y_{11} =$$

$$\text{Root of } f(x) = \underline{\underline{1.5096}}$$

9)  $f(x) = y = x^3 - 9x + 1 = 0$

| x | y  |
|---|----|
| 0 | 1  |
| 1 | -7 |

$$x_3 = \frac{0 - 1}{-7 - 1} = 0.125, y_3 = -0.123$$

$$x_4 = \frac{0 - 0.125}{-0.123 - 1} = 0.1113, y_4 = -0.0032$$

$$\text{Root of } f(x) = \underline{\underline{0.1113}}$$

10)  $f(x) = y = x^3 - x - 4 = 0$

| $x$   | $y$   | $x$ | 0  | 1  | 2 |
|-------|-------|-----|----|----|---|
| $x_0$ | 0     | $y$ | -4 | -4 | 2 |
| $x_1$ | $y_1$ |     |    |    |   |

$$x_1 = 1, y_1 = -4$$

$$x_2 = 2, y_2 = 2$$

$$x_3 = \frac{2 + 8}{2 + 4} = 1.6667, y_3 = -1.0368$$

$$x_4 = \frac{(1.6667 \times 2) + (2 \times 1.0368)}{2 + 1.0368} = 1.7805, y_4 = -0.135$$

$$x_5 = \frac{(1.7805 \times 2) + (2 \times 0.136)}{2 + 0.136} = 1.7945, y_5 = -0.0158$$

$$x_6 = \frac{(1.7945 \times 2) + (2 \times 0.0158)}{2 + 0.0158} = 1.7961, y_6 = -0.00192$$

$$x_7 = \frac{(1.7961 \times 2) + (2 \times 0.0019)}{2 + 0.0019} = 1.7963, y_7 = -0.00019$$

Root of  $f(x) = \underline{\underline{1.7963}}$