Name of Practical

Date :

Calculus assignment

(1) $$\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$$

$$\rightarrow \lim_{h \to 0} \frac{2(-3+h)^2 - 2(3)^2}{h}$$

$$\lim_{h \to 0} \frac{2[(-3+h-3)(-\cancel{3}+h+\cancel{3})]}{h}$$

$$\lim_{h \to 0} 2\left[\frac{(h-6)(\cancel{h})}{\cancel{h}}\right]$$

$$\lim_{h \to 0} 2[(h-6)]$$

$$2 \times -6 = -12$$

2) $$g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

(a) at $x = 4$

$$g(4) = 2 \times 4 = 8$$

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$\lim_{x \to 4^-} g(x) = 8$

$\lim_{x \to 4^+} g(x) = 8$

$\therefore g(x)$ is continuous at $x = 4$

(b) $x = 6$

$g(6) = x - 1 = 6 - 1 = 5$

$\lim_{x \to 6^-} g(x) = 2(6) = 12 \rightarrow$ L.H.S

$\lim_{x \to 6^+} g(x) = 6 - 1 = 5 \rightarrow$ R.H.S

(3) $\lim_{x \to 9} \dfrac{x - 9}{3 - \sqrt{x}}$

$\rightarrow \lim_{x \to 9} - \dfrac{\cancel{(\sqrt{x} - 3)}(\sqrt{x} + 3)}{\cancel{(\sqrt{x} - 3)}}$

$\lim_{x \to 9} = \cancel{(\sqrt{x} + 3)} \; -(\sqrt{x} + 3) = -6$

(4) $f(x) = \dfrac{4x+5}{9-3x}$

a] $x = -1$

$f(-1) = \cancel{4(-1)+5} \quad \dfrac{4(-1)+5}{9-3(-1)} = \dfrac{1}{12}$

b] $x = 0$

$f(x) = \dfrac{5}{9}$

c] $x = 3$

$f(3) = \dfrac{4(3)+5}{9-3(8)} = -\dfrac{17}{15}$

5) $\lim\limits_{x \to \infty} \dfrac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$ $\qquad \dfrac{6-0}{8-0+0} = \dfrac{6}{8} = \dfrac{3}{4}$

$\lim\limits_{x \to \infty} \dfrac{\dfrac{6e^{4x} - e^{-2x}}{e^{4x}}}{\dfrac{8e^{4x} - e^{2x} + 3e^{-x}}{e^{4x}}}$

$\lim\limits_{x \to \infty} = \dfrac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{5x}}$

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(6) $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

a] $\lim_{y \to 6} g(y)$

$\lim_{y \to 6} (1-3y) = 1-3(6) = -17$

b] $\lim_{y \to -2} g(y) = \lim_{y \to -2} (1-3y) = 1-3(-2)$
$= 1+6$
$= 7$

(7) $f(t) = (4t^2-t)(t^3-8t^2+12)$

By product rule

$f'(t) = (4t^2-t)\frac{d}{dt}(t^3-8t^2+12) + $

$= (t^3-8t^2+12)\frac{d}{dt}(4t^2-t)$

$= (4t^2-t)(3t^2-16t+10) + (t^3-8t^2+12)(8t$

$= 12t^4 - 64t^3 - 3t^3 - 64t^3 + 8t^2 + 96t - 12$

$= 20t^4 - 162t^3 + 24t^2 + 96t - 12$

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(8) $R(\omega) = \dfrac{3\omega + \omega^4}{2\omega^2+1}$

$R'(\omega) = \dfrac{(2\omega^2+1)\dfrac{d}{d\omega}(3\omega+\omega^4) - (3\omega+\omega^4) \times \dfrac{d}{d\omega}(2\omega^2+1)}{(2\omega^2+1)^2}$

$= \dfrac{(2\omega^2+1)(3+4\omega^3) - (3\omega+\omega^4)(4\omega)}{(2\omega^2+1)^2}$

$= \dfrac{6\omega^2+8\omega^5+3+4\omega^3-12\omega^2-4\omega^5}{(2\omega^2+1)^2}$

$= \dfrac{4\omega^5-6\omega^2+4\omega^3+3}{(2\omega^2+1)^2}$

(9) $f(x) = 2e^x - 8^x$

$f'(x) = 2\dfrac{d}{dx}(e^x) - \dfrac{d}{dx}(8^x)$

$= 2e^x - 8^x \ln 8$

(10) $y = z^5 - e^x \ln(z)$

$$\frac{dy}{dz} = 5z^4 - \left[e^x \frac{d}{dx}(\ln x) + \ln z \frac{d}{dx}(z^5) \right]$$

$$= 5z^4 - \left[e^2 \frac{1}{z} + \ln z (e^x) \right]$$

$$= 5z^4 - e^2 \left[\frac{1}{x} + \ln z \right]$$

(11) a] $f(x) = [6x^2 + 7x)^4$

$$f'(x) = 4(6x^2 + 7x)^3 \frac{d}{dx}(6x^2 + 7x)$$

$$= 4(6x^2 + 7x)^3 (12x + 7)$$

$$= (6x^2 + 7x)^3 (48x + 28$$

b] $f(t) = 5 + e^{4t + t^7}$

$$f'(t) = 0 + e^{4t + t^7} \frac{d}{dt}(4t + t^7)$$

$$e^{4t + t^7}(4 + 7t^6)$$

12 a) $z = \ln(7 - x^3)$

$$\frac{dz}{dx} = \frac{1}{7-x^3} \cdot \frac{d}{dx}(7-x^3)$$

$$= \frac{1}{7-x^3} \cdot (0 - 3x^2)$$

$$= \frac{dz}{dx} = \frac{-3x^2}{7-x^3}$$

$$= \frac{dz}{dx} = \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dx} = \frac{d}{dx}\left[\frac{-3x^2}{7-x^3}\right]$$

$$= \frac{(7-x^3)[-6x] - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 + -9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

$$= \frac{-3x(x^3+14)}{(7-x^3)^2}$$

6] $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2} \cdot 2t$

$f''(t) =$ By u/v rule.

$= \frac{(1+t^2)(2) - (2t)(2t}{(1+t^2)^2}$

$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$

$= 2-2t^2$

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(12) $f(x) = x^3 - 3x^2 - 4x + 12$

$\rightarrow f'(x) = 3x^2 - 6x - 9$

Put $f'(x) = 0$

$\therefore 3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x = 3, \ x = -1$

$f''(x) = 6x - 6$

$f''(3) = 18 - 6 = 12 > 0$

$f''(-1) = -6 - 6 = -12 < 0$

$= 3 \longrightarrow$ pt of minima

min value $= (3)^3 - 3(3)^2 - 9(3) + 12$

$= -15$

$-1 \longrightarrow$ pt of maxima

max value $= (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= 21 - 3 - 1$

$= 21 - 4$

$= 17$

14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$= 12(x-2)(x-1)$

$f'(0) = x = 2, 1$

$f''(x) = 24x - 36$

$f''(1) = 24 - 36 = -12 \longrightarrow$ pt of max

$f''(2) = 48 - 36 = 12 \longrightarrow$ pt of min

$f(1) = \cancel{4(1)}\ 4(1)^3 - 18(1)^2 + 24(1)$

$= 4 - 18 + 24 - 7$

$= 3 \longrightarrow$ Max value

$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 43 - 7$

$= 1 \longrightarrow$ minimum value

(15) $f(x) = x^2(x+3)$

$f'(x) = x^2 \frac{d(x+3)}{dt} + (x+3)\frac{d}{dx}(x^2)$

$= x^2(1) + (x+3)(2x)$

$= 5x^2 + 6x$

Put, $f'(x) = 0$

$3x(x+2) = 0$

$x = 0$,

$x = -2$

$f''(x) = 6x+6$

$f''(0) = 6$

$f''(-2) = -12+6 = -6$

minimum value $= 0$

max value $= 4(3-2) = 4$

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(16) $f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

$f'(x) = 0$

at $x = 1$

$\therefore y = 3(x^2) - 6x$

$= 3 - 6$

$y = -3$

$= 1, -3$ is the lowest point in the graph

Now

at $x = 0$

$y = 0$

$\therefore$ It passes through $(0, 0)$

at $y = 0$, $x = 0$ $(2, 0)$