Probability & Statistics

Assignment 1

Q1 i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25.

Mean = $\frac{4+7+9+9+11+15+18+18+18+25}{10}$

$= \frac{134}{10}$

$= 13.4$.

Median = $\frac{\frac{n}{2}\text{ th term} + \frac{n+1}{2}\text{ th term}}{2}$.

$= \frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2} \Rightarrow \frac{11+15}{2}$

$= \frac{26}{2} = 13$

Mode = 18.

ii)

| no. of households ($x_i$) | no. of people ($f$) | c.f | $f_i x_i$ |
|---|---|---|---|
| 0 | 5 | 5 | 0 |
| 1 | 11 | 16 | 11 |
| 2 | 26 | 42 | ~~4~~52 |
| 3 | 15 | 57 | 45 |
| 4 | 3 | 60 | 12 |

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow \frac{120}{60} = 2$.

Median = $l + \frac{(N/2 - cf)}{f} \times h$.

$= 1.5 + \frac{30-16}{26} \times 1$.

$= 1.5 + \frac{16}{26} \times 1$ ~~$\Rightarrow 1.5$~~ +

$= 2.03846.$

Mode. $= 3$ Median $- 2$ mean

$= 3(2.038466) - 2(2)$

$= 2.11538$

For IQR, $Q_1 = l_1 + \dfrac{N/4 - cf}{f}$

$Q_1 = \dfrac{12 - 5}{}$

$Q_1 = \left(\dfrac{N+1}{4}\right)^{th}$ item $= \left(\dfrac{61}{4}\right)^{th}$ item

$=$ 12.25 th item

$=$ 2.1

$Q_3 = 3\left(\dfrac{N+1}{4}\right)^{th}$ item $=$ 36.75th item

$= 2.$

$\therefore$ $IQR = Q_3 - Q_1 = 2-1 = \underline{1}$

Standard deviation or $\sigma = \sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$

| $x_i$ | $f_i$ | $x-\bar{x}$ | $(x-\bar{x})^2$ | $f(x-\bar{x})^2$ |
|---|---|---|---|---|
| 0 | 5 | -2 | 4 | 20 |
| 1 | 11 | -1 | 1 | 11 |
| 2 | 26 | 0 | 0 | 0 |
| 3 | 15 | 1 | 1 | 15 |
| 4 | 3 | 2 | 4 | 12 |
| | 60 | | 10. | 58. |

$\sigma = \sqrt{\dfrac{58}{60}} = 0.98319$

Q2 i) $X \sim P(0.5)$

0 claims in 1 year. $P(X=0)$.

$$P(X=0) = \frac{e^{-x} x^{x}}{0!} = \frac{e^{-0.5}(0.5)^0}{1}$$

$$= e^{-0.5} = 0.60653$$

ii) $P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.0'60635$

$= 0.439347$

Binomial Distribution =

$P(X<1) = {}^nC_x \, p^x (1-p)^{n-x}$

$= {}^3C_1 \, p^1 (1-p)^{3-1}$

$= 6(0.439347) \times (0.60653)^2$

$= 0.86849$

iii) As one claim has occurred and we have to find the time till the next one, we will use an exponential distribution

$X \sim Exp(\lambda = 0.5)$

$P(X \geq 2) = 1 - P(X \leq 2)$

$= 1 - [1 - e^{-\lambda x}]$

$= e^{-\lambda x}$

$= e^{-0.5 \times 2} = 0.367879$

Q3) $X \sim Gamma(2, 0.5)$ $\quad 0 < x < \infty$

i) $E(X) = \frac{\alpha}{\lambda} = \frac{2}{0.5} = 4.$

$V(X) = \frac{\alpha}{\lambda^2} = \frac{2}{(0.5)^2} = 8.$

$S.D(X) = 6 = \sqrt{8} = 2.828427$

$\therefore$ Mean $= 4$ & S.D $= 2.8284$.

ii) The distribution is probably skewed due to the high variance of 8. As the mean $= 4$ is close to the range $0 < x < \infty$, it can be assumed that it is a positively skewed distribution.

iii) $$F_X(x) = \int_0^x f_X(x)\, dx.$$

$$= \int_0^x \frac{(0.5)^{(2)}}{\sqrt{2}} \cdot e^{-\cancel{2}x/2} \cdot x^{(2-1)}$$

$$= (0.5)^2 \int_0^x x \cdot e^{-x/2}\, dx$$

$$= (0.5)^2 \left[ x \int e^{-x/2} dx - \int \frac{d}{dx} x \cdot \left( \int e^{-x/2} dx \right) \right]_0^x$$

$$= (0.5)^2 \left[ x \frac{e^{-x/2}}{-1/2} - \int (1) \frac{e^{-x/2}}{-1/2} \right]_0^x$$

$$= (0.5)^2 \left[ -2 . x . e^{-x/2} - \frac{e^{-x/2}}{-1/2 . -1/2} \right]_0^x$$

$$= (0.5)^2 \left[ -2xe^{-x/2} - 4e^{-x/2} \right]$$

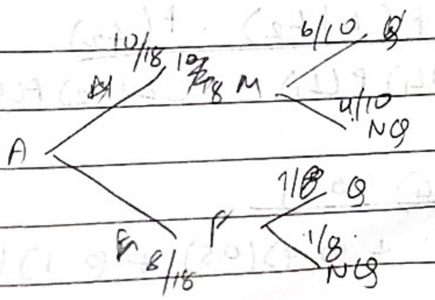
$$= (0.5)^2 \left[ \left( -2xe^{-x/2} - 4e^{-x/2} \right) - (-4) \right]$$

$$= (0.5)^2 \left[ -2xe^{-x/2} - 4e^{-x/2} + 4 \right]$$

$$= \cancel{(0.5)^2}\ \frac{1 - 0.5x}{\sqrt{e^x}} - \frac{1}{\sqrt{e^x}}$$

Q4) Total Males (M) $= 10$  Total Females (F) $= 8$

$P(M) = 10/18$  $P(F) = 8/18$



$P(Q|F) = \frac{7}{8}$  $P(Q|M) = \frac{6}{10}$

~~*~~ Probability of picking atleast 1 female = 1 − P(picking 2 males)

$1 - \left(\frac{10}{18} \times \frac{9}{17}\right) = \left(1 - \frac{5}{17}\right) = \frac{12}{17}$

P( one unqualified female & one unqualified male) = $\left(\frac{12}{17} \times \frac{1}{8}\right) +$

$\left(\frac{125}{153} \times \frac{4}{10}\right)$

$= \frac{3}{34} + \frac{50}{153} = \frac{127}{306}$

P(one qualified female & one unqualified male) =

$\left(\frac{12}{17} \times \frac{7}{8}\right) + \left(\frac{125}{153} \times \frac{4}{10}\right) = \frac{21}{34} + \frac{50}{153} = \frac{17}{18}.$

P( ~~two~~ qualified females) ~~and one~~ $= \frac{147}{272}$

$= 0.54044$ (ans)

Q5) $P(L_1) = 0.4$  $P(L_2) = 0.5$  $P(L_3) = 0.1$

~~$P(L|L_1)$~~ ~~*~~ ~~$P(L_1|L) = 0.2$~~
~~$P(L_2|L) = 0.4$~~
~~$P(L_3|L) = 0.1$~~

$P(L|L_1) = 0.2$  $P(L|L_2) = 0.4$  $P(L|L_3) = 0.1$

$$P(L_2|L) = \frac{P(L|L_2) \cdot P(L_2)}{P(L|L_1)P(L_1) + P(L|L_2)P(L_2) + P(L|L_3)P(L_3)}$$

$$= \frac{(0.4)(0.5)}{(0.2)(0.4) + (0.4)(0.5) + (0.1)(0.1)}$$

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$$= \frac{0.2}{0.08 + 0.2 + 0.01} = \frac{0.2}{0.29} \Rightarrow 0.689655.$$

$$\text{or } \frac{20}{29} \quad (\text{ans})$$

96) $X \sim U(1, 2)$ $\quad Y = \frac{3}{6 - X}$

$a = 1, b = 2$

$$f(x) = \frac{1}{2-1} = 1$$

$$Y = \frac{3}{6-X} \Rightarrow 6 - X = \frac{3}{Y} \Rightarrow$$

$$\Rightarrow X = 6 - \frac{3}{Y} = w^{-1}(Y)$$

$$x = 6 - 3\left(\frac{1}{y}\right) \Rightarrow \frac{dx}{dy} = \frac{3}{y^2}$$

$$f_Y(y) = f_X(w^{-1}(y)) \left| \frac{d}{dy} w^{-1}(y) \right|$$

$$= f_X\left(6 - \frac{3}{Y}\right) \cdot \frac{3}{y^2}$$

$$= 1 \cdot \frac{3}{y^2} = \frac{3}{y^2} \quad (\text{ans}).$$

97)

| $X$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.05 | 0.15 | 0.35 | 0.4 | 0.05 |

i) $F(3) = 0.35$

ii) $E(X) = \sum P(X = x) \cdot x \, dx$

$= 1 \times 0.05 + 2 \times 0.15 + 3 \times 0.35 + 4 \times 0.4 + 5 \times 0.5$

$= 3.25$

iii) $V(x) = E(x)^2 - [E(x)]^2$

$= 1\times 0.05 + 4\times 0.15 + 9\times 0.35 + 16\times 0.4 + 25\times 0.05 - (3.25)^2$

$= 11.45 - (3.25)^2$

$= 0.8875$.

Q8. $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$= {}^{15}C_0 \times (0.02)^0 \times (0.8)^{15} + {}^{15}C_1 \times (0.2)^1 \times (0.8)^{14} + {}^{15}C_2 \times (0.2)^2 \times (0.8)^{13}$

$= 0.035184372 + 0.2308917442 + 0.1319418953$

$= 0.3980232093$

Q9) $r = 7$ $\quad p = 0.01$

$p + q = 1$ $\quad q = 1 - 0.01 = 0.99$

$X \sim$ -ve Bin $(7, 0.01)$

$x = 3$

$P(X = 3) = {}^6C_2 \, (0.01)^3 \, (0.99)^4$

$= 0.0000144089$

Q10) $\lambda = 2$ per working week (5 days)

$\lambda = 2/5$ for one day.

$X \sim$ Poi $(\lambda = 2/5)$

$P(X > 2) = 1 - P(X \le 2)$

$= 1 - 0.99207$

$= 0.00793$

Q11) P(Having a boy) $= \frac{1}{2}$

P(Having a girl $= \frac{1}{2}$

Since probability of having a boy in the first child and a boy in the second child are independent events,

P(Having 3 boys in a row) $= \left(\frac{1}{2}\right)^3$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8} = 0.125$

Q12) $X \sim Poi$ (10 per day)

$X \sim Exp(\lambda = 10)$

$P(X < 0.1) = 1 - e^{-\lambda x}$

$= 1 - e^{-10 \times 0.1}$

$= 1 - e^{-1}$

$= 0.63212$

Q13) $X \sim U(75, 120)$ if we find individual f x it is constant due to equal probability density in uniform dist.

$P(80 < x < 100) = F(100) - F(80)$

$= \frac{100-75}{120-75} - \frac{80-75}{120-75}$

$= \frac{25-5}{45} = \frac{20}{45} = 0.44\bar{4}$

Q14) $X \sim$ Gamma $(5, 0.4)$

$P(x < 22.89) = ?$

$2\lambda x \sim \chi^2_{10}$

$0.8x \sim \chi^2_{10}$

$P(x < 22.89) = P(0.8x < 18.312)$

$\Rightarrow 0.9489296$

Q15) $\dfrac{K}{(x+5)^4}, \quad x > 0$

i) $\displaystyle\int_0^\infty \frac{K}{(x+5)^4}\,dx = 1.$

$\displaystyle K\int_0^\infty (x+5)^{-4}\,dx = 1$

$\Rightarrow K\left[\dfrac{(x+5)^{-3}}{-3}\right]_0^\infty = 1$

$\Rightarrow K\left[0 + \dfrac{(5)^{-3}}{3}\right] = 1$

$\dfrac{K}{375} = 1 \Rightarrow K = 375$ (ans)

ii) $\displaystyle\int_0^{10} \frac{375}{(x+5)^4}\,dx \Rightarrow \int_0^{10} 375\cdot(x+5)^{-4}\,dx$

$\Rightarrow 375\left[\dfrac{(x+5)^{-3}}{-3}\right]_0^{10}$

$\Rightarrow 375\left[\dfrac{(15)^{-3}}{-3} - \dfrac{(5)^{-3}}{-3}\right]$

$\Rightarrow 375\left[-\dfrac{1}{10125} + \dfrac{1}{375}\right]$

$= \cancel{375}\left[\dfrac{-1+27}{10\cancel{125}\,27}\right] \Rightarrow \dfrac{26}{27} = 0.96296$

HT22) $E(x)$

———— x ————