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Q1] $X \sim Poi(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \div N(0,1)$$

$$P(X=200) = P[199.5 < Z < 200.5)$$

$$= P\left[\frac{199.5-\theta}{\sqrt{\theta}} < Z \frac{200.5-\theta}{\sqrt{\theta}}\right]$$

$$95\% \ CI = \theta \pm 1.96\sqrt{\theta}$$

Q2]

Q2] $x_i = 1, 2, 3 \dots n$

$$\bar{x} = \frac{\Sigma x_i}{n} \qquad s^2 = \frac{1}{n-1}\sum_1^n (x_i - \bar{x})^2$$

i] $\bar{x} = \frac{\Sigma x_i}{n}$

$$E(\bar{x}) = \frac{\Sigma E(x_i)}{n} = \frac{\Sigma \mu}{n} = \frac{n\mu}{n} = \mu$$

$$var(\bar{x}) = V\left(\frac{\Sigma x_i}{n}\right] = \frac{1}{n^2}\Sigma V(x_i)$$

$$= \frac{\Sigma \sigma^2}{n^2} = \frac{n\sigma^2}{n^2}$$

$$= \sigma^2/n$$

ii] $s^2 = \frac{1}{n-1}\Sigma(x_i - \bar{x})^2$

$$E(s^2) = \frac{1}{n-1}\left[\Sigma E(x_i)^2 - nE(\bar{x})^2\right]$$

$$= \frac{1}{n-1} \Sigma (\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)$$

$$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \frac{\sigma^2 (n-1)}{n-1} = \sigma^2$$

iii] $V\left(\frac{(n-1)s^2}{\sigma^2}\right) = V(\chi^2_{n-1}) = 2(n-1)$

$$\frac{(n-1)^2}{\sigma^4} V(s^2) = 2(n-1)$$

$$V(s^2) = \frac{2(n-1)\sigma^4}{(n-1)^2}$$

$$= \frac{2\sigma^4}{n-1}$$

iv] $n = 10, \sigma^2 = 100$

Q3] $x \sim Bin(n, p)$ $n = 180$

i] $L(p) = \prod_{i=0}^{4} {}^{180}C_0 p^0 q^{180} \times {}^{180}C_1 p^1 q^{179} \times {}^{180}C_2 p^2 q^{178} \times {}^{180}C_3 p^3 q^{177} \times {}^{180}C_4 p^4 q^{176}$

$= \text{constant} \times p^0 q^{180} \times p^1 q^{179} \times p^2 q^{178} \times p^3 q^{177} \times p^4 q^{176}$

$\log L(p) = \text{constant} + 0 + 180 \log p + 1 \log p + 179 \log q + 2 \log p + 178 \log q + 3 \log p + 177 \log q + 4 \log p + 176 \log q.$

$= \cancel{100}$ constant $+ 10 \log p + 890 (1-p)$

$\frac{d}{dp} \log L(p) = \frac{-890}{\log p}$

Q 4) $f(x) = \dfrac{c\beta^3}{(x+\beta)^4}$; $x>0$, $\beta<0$

i.) $E(x) = \int x \cdot f(x)\, dx$

$= \int_0^\infty x \cdot \dfrac{c\beta^3}{(x+\beta)^4}\, dx$

Q5) $X \sim U(a,b)$

$E(X) = \bar{X}$ $\qquad f(x) = \frac{1}{b-a}$

$\cancel{\frac{b-a}{2} = \bar{X}}$ $\qquad E(X) = \int_a^b x \cdot f(x)\,dx$

$\cancel{a = 2\bar{X} - b}$ $\qquad = \int_a^b x \cdot \frac{1}{b-a}\,dx$

$\cancel{\frac{(b-a)^2}{12} =}$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{b^2-a^2}{2(b-a)} =$$

$$\frac{(b-a)(b+a)}{2(b-a)}$$

$$E(X) = \frac{a+b}{2} \quad -(1)$$

$$E(x^2) = \int_a^b x^2 f(x)\,dx$$

$$= \int_a^b x^2 \frac{1}{b-a}\,dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{b^3-a^3}{3(b-a)}$$

$$b^3 - a^3 = (b-a)(b^2+a^2+ab)$$

$$E(x^2) = \frac{a^2+b^2+ab}{3} \quad -(2)$$

$$\bar{X} = \frac{1}{n}\sum X_i \quad -(3)$$

$$\bar{x}_2 = \frac{1}{n}\sum x_i^2 \quad -(4)$$

$E(X) = \bar{X}$ $\qquad \frac{a+b}{2} = \bar{X}$

$E(X^2) = \bar{X}_2$ $\qquad \frac{a^2+b^2+ab}{3} = \bar{X}_2$

$a+b = 2\bar{X} \Rightarrow a = 2\bar{X} - b$

$$\frac{(2\bar{X}-b)^2 + b^2 + (2\bar{X}-b)b}{} = 3\bar{X}_2$$

$$b^2 + 4\bar{X}^2 - 4b\bar{X} + \cancel{b^2} - \cancel{b^2} + 2b\bar{X} = 3\bar{X}_2$$

$$b^2 - 2b\bar{X} + 4\bar{X}^2 - 3\bar{X}_2 = 0$$

$$\hat{b} = \frac{-2\bar{X} \pm \sqrt{2(\bar{X})^2 - 4(4\bar{X}^2 - 3\bar{X}_2)}}{}$$

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$\therefore \hat{b} = \bar{x} + \frac{1}{2}\sqrt{4\bar{x}^2 - 4\cdot 4\bar{x}^2 + 4\; 3\bar{x}_2}$

$\hat{b} = \bar{x} + \sqrt{3(\bar{x}_2 - \bar{x}^2)}$

iii.) $n = 2, 4, 6, 8, 10 \rightarrow U(2, 10)$

$\bar{x} = \frac{a+b}{2} = \frac{2+10}{2} = 6$

$s^2 = \frac{(10-2)^2}{12} = 5.333$

iv.) $f(x) = \frac{1}{b-a}$

$L(b) = \prod_{i=1}^{n} \frac{1}{b-a}$

$= \frac{1}{b^n - a^n} = \frac{1}{(b-a)^n}$

$= (b-a)^n$

$\log L(b) = n \log(b-a)$

$\frac{d}{db} \log L(b) = \frac{n}{b-a} (-a)$

$= -\frac{na}{b-a}$

~~Q6.)~~

Q7.) i.) $\bar{x}_1 = 30$ $\quad \sigma_1^2 = 57.1667$

$\bar{x}_2 = 35.0556$ $\quad \sigma_2^2 = 59.91358$

iii.) $\bar{x} = 35.0556$ $\quad \sigma_2^2 = 59.91358$

95% of CI $= 35.0556 \pm 1.96 \sqrt{\frac{59.91358}{9}}$

$= (29.9985, 40.11265)$

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Q8.] i.] If we have a random sample $X = (X_1, X_2, \cdots, X_n)$ from a distribution with an unknown parameter θ and $g(\underline{X})$ is an estimator of θ, it seems desirable that $E[g(\underline{X})] = \theta$

ii.] When the expected value of the true value is not equal it is called bias.

iii.] The mean square error of the estimator $\hat{\theta}$ is $MSE(\hat{\theta}) = E[\hat{\theta} - \theta]^2$

~~iv.] The MSE with lower~~

iv.] An estimator with lower MSE is more efficient. Therefore, $\tilde{\theta}$ is more efficient the $\hat{\theta}$.

v.] They both be considered ~~lower~~ consistent when both MSE are lower

vi.] We can estimate θ by using methods of moments & Maximum likelihood,

Q9.] i.] The sampling distribution for $\bar{X}$ i.e.
$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \text{ or } \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$
The t_K variable is defined by
$$t_K = \frac{N(0,1)}{\sqrt{\frac{\chi^2_K}{K}}}$$ where the $N(0,1)$ & χ^2_K random variables are independent

$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is the $N(0,1)$ & $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ is the χ^2_K

ii] t_k distribution has mean 0 & variance $\frac{k}{k-2}$

iii] $n = 10$

$\bar{x} = 50$

$\sigma^2 = 48.667$

$\bar{x} \sim N(0,1)$

$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

99% of CI $= \bar{x} \pm t_{\alpha/2, n-1} \sqrt{\frac{\sigma^2}{n}}$

$= 50 \pm 3.250 \sqrt{\frac{48.667}{10}}$

$= (142.83, 57.169)$

Q) $n = 12$

$\Sigma x_i = 21320$

$\Sigma x_i^2 = 4919860000$

$\bar{x} = 17767 \qquad s^2 = 10144$

i] $\bar{x} = \frac{\Sigma x_i}{n} \qquad s^2 = \frac{1}{n-1} [(\Sigma x_i - x)^2]$

$\bar{x} = \frac{\Sigma x_i}{12} = \Sigma x_i = 213204$

$\Sigma x_N = 213204 - 11000 - 48000 + 27300 + 31500$

$= 213204$

$\bar{x}_N = \frac{213204}{12} = 17767$

$SD^2 = \frac{1}{11} (4919860000 - 12 \times 17767)$

$SD^2 = 447240617.8$

$SD = 21148.06416$