

$$1.] \lim_{h \rightarrow 0} \frac{2(-3+h)^2 - 18}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{2(-9 + 6h + h^2) - 18}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{18 + 2h^2 - 12h - 18}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{2h^2 - 12h}{h}$$

$$\Rightarrow \lim_{h \rightarrow 0} (2h - 12)$$

$$\Rightarrow -12$$

$$2.] \quad g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

$$a.] \quad x = 4$$

$$\therefore g(4) = 8$$

$$\therefore \lim_{x \rightarrow 4^-} g(x) = \lim_{x \rightarrow 4^-} 2x = 2(4) = 8$$

$$\lim_{x \rightarrow 4^+} g(x) = \lim_{x \rightarrow 4^+} 2x = 8$$

$$\therefore \text{LHL} = \text{RHL}$$

$$\text{Hence } \lim_{x \rightarrow 4} g(x) \text{ exists}$$

$\therefore$ The given function is continuous at $x = 4$

$$b.] \quad x = 6$$

$$\therefore g(6) = 6-1 = 5$$

$$\therefore \lim_{x \rightarrow 6^-} g(x) = \lim_{x \rightarrow 6^-} 2x = 12$$

$$\lim_{x \rightarrow 6^+} g(x) = \lim_{x \rightarrow 6^+} x-1 = 5$$

$$\therefore \lim_{x \rightarrow 6^-} g(x) \neq \lim_{x \rightarrow 6^+} g(x)$$

$$\therefore \lim_{x \rightarrow 6} g(x) \text{ does not exist}$$

$\therefore$ The given function $g(x)$ is discontinuous at $x = 6$

$$3.7 \quad \lim_{x \rightarrow 9} \frac{x-9}{3-\sqrt{x}}$$

$$\Rightarrow \lim_{x \rightarrow 9} \frac{(\sqrt{x})^2 - (3)^2}{3-\sqrt{x}}$$

$$\Rightarrow \lim_{x \rightarrow 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(3-\sqrt{x})}$$

$$\Rightarrow \lim_{x \rightarrow 9} \frac{(\cancel{\sqrt{x}-3})(\sqrt{x}+3)}{-(-\cancel{3}+\sqrt{x})}$$

$$= \lim_{x \rightarrow 9} -(\sqrt{x}+3)$$

$$\Rightarrow -(\sqrt{9}+3)$$

$$\Rightarrow -6.$$

4.] $f(x) = \frac{4x+5}{9-3x}$

At $x = -1$, $f(-1) = \frac{4(-1)+5}{9-3(-1)} = \frac{-4+5}{9+3} = \frac{1}{12}$.

$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \left(\frac{4x+5}{9-3x} \right) = \frac{4(-1)+5}{9-3(-1)} = \frac{1}{12}$.

Therefore the function is continuous at $x = -1$
 $\therefore f(-1) = \lim_{x \rightarrow -1}$

At $x = 0$, $f(0) = \left(\frac{4(0)+5}{9-3(0)} \right) = \frac{5}{9}$.

$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{4x+5}{9-3x} \right) = \frac{4(0)+5}{9-3(0)} = \frac{5}{9}$.

Therefore, $f(0) = \lim_{x \rightarrow 0}$

The function is continuous at $x = 0$

At $x = 3$, $f(3) = \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \infty$

$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{4(3)+5}{9-3(3)} = \frac{17}{0} = \infty$

$\therefore \lim_{x \rightarrow 3} f(x) = f(3)$

The function is continuous at $x = 3$

5.] $\lim_{x \rightarrow \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}} \quad \left[\because \frac{e^a}{e^b} = e^{a-b} \right]$

$\lim_{x \rightarrow \infty} \frac{e^{4x} [6 - e^{-6x}]}{e^{4x} [8 - \frac{e^{2x}}{e^{6x}} + 3e^{-5x}]}$

$\lim_{x \rightarrow \infty} \left[\frac{6 - \frac{1}{e^{6x}}}{8 - \frac{1}{e^{2x}} + \frac{3}{e^{5x}}} \right] = \frac{6 - \frac{1}{\infty}}{8 - \frac{1}{\infty} + \frac{3}{\infty}}$

$= \frac{6}{8} = \frac{3}{4}$.

6.] $g(y) = \begin{cases} y^2 + 5, & \text{if } y < -2 \\ 1 - 8y, & \text{if } y \geq -2 \end{cases}$

a.] $\lim_{y \rightarrow 6} g(y)$

$\lim_{y \rightarrow 6} g(y) = \lim_{y \rightarrow 6} 1 - 3y$

$= 1 - 3(6)$

$= 1 - 18 = -17$

$\lim_{y \rightarrow 6} g(y) = -17$

b.] $\lim_{y \rightarrow -2} g(y) = \lim_{y \rightarrow -2} 1 + 3y$

$= 1 + 3(-2) = 7$

$\lim_{y \rightarrow -2} g(y) = 7$

7.] $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

differentiate w.r.t. t

$f'(t) = (4t^2 - t) \frac{d}{dt} (t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12) \frac{d}{dt} (4t^2 - t)$

$f'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$f'(t) = 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$

$f'(t) = 20t^4 - 132t^3 + 24t^2 + 96t - 12$

8.] $R(w) = \frac{3w + w^4}{2w^2 + 1}$

Differentiate w.r.t. w

$f'(w) = (2w^2 + 1) \frac{d}{dw} (3w + w^4) - (3w + w^4) \frac{d}{dw} (2w^2 + 1)$

$(2w^2 + 1)^2$

$$R'(w) = \frac{2w^2 + (3 + 4w^3) - (3w + w^4)(4w)}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{6w^2 + 8w^5 + 3 + 4w^3 - [12w^2 + 4w^5]}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{6w^2 + 8w^5 + 3 + 4w^3 - 12w^2 - 4w^5}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{4w^5 - 6w^2 + 3 + 4w^3}{(2w^2 + 1)^2}$$

$$R'(w) = \frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2 + 1)^2}$$

9.] $f(x) = 2e^x - 8^x$

Differentiate w.r.t. x

$$f'(x) = 2 \frac{d}{dx} e^x - \frac{d}{dx} 8^x$$

$$f'(x) = 2e^x - 8^x \log 8$$

10.] $y = z^5 - e^z \ln(z)$

Differentiate w.r.t. z

$$\frac{dy}{dz} = \frac{d}{dz} z^5 - \left[e^z \frac{d}{dz} \ln(z) + \ln(z) \frac{d}{dz} e^z \right]$$

$$\frac{dy}{dz} = 5z^4 - \left[\frac{e^z}{z} + \ln(z) e^z \right]$$

$$\frac{dy}{dz} = 5z^4 - \frac{e^z}{z} - \ln(z) e^z$$

11.] a.] $f(x) = (6x^2 + 7x)^4$

$$\therefore \text{Let } y = (6x^2 + 7x)^4$$

$$\text{Let } u = 6x^2 + 7x$$

$$y = u^4$$

Differentiate w.r.t. u

$$\frac{dy}{du} = 4u^3$$

$u = 6x^2 + 7x$
 $\frac{dy}{dx}$ - Differentiate w.r.t. x

$$\frac{du}{dx} = 12x + 7$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = 4u^3 \times (12x + 7)$$

$$\frac{dy}{dx} = 4(6x^2 + 7x)(12x + 7)$$

b.] $f(t) = 5 + e^{4t+t^7}$

Let $y = 5 + e^{4t+t^7}$

Let $u = 4t + t^7$

Differentiate w.r.t. t

$$\frac{du}{dt} = 4 + 7t^6$$

$$\therefore y = 5 + e^u$$

differentiate w.r.t. u

$$\frac{dy}{du} = 0 + e^u$$

$$\frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dt} = e^{4t+t^7} (4 + 7t^6)$$

12.] a.] $z = \ln(7 - x^3)$

Differentiate w.r.t. x

$$\frac{dz}{dx} = \frac{1}{7 - x^3} \times \frac{d}{dx} (7 - x^3)$$

$$\frac{dz}{dx} = \frac{-3x^2}{7 - x^3}$$

Again differentiate w.r.t. x

$$\frac{d^2z}{dx^2} = \frac{7-x^3}{(7-x^3)^2} \frac{d}{dx} (-3x^2) - (-3x^2) \frac{d}{dx} \frac{7-x^3}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{7-x^3}{(7-x^3)^2} (-6x) - (-3x^2) \frac{(-3x^2)}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42x + 9x^4}{(7-x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42x - 3x^4}{(7-x^3)^2}$$

b.] $Q(v) = \frac{(6+2v-v^2)^4}{(6+2v-v^2)^4}$

Differentiate w.r.t. v

$$Q'(v) = \frac{(6+2v-v^2)^4}{(6+2v-v^2)^4} \frac{d}{dv} \frac{2-2}{(6+2v-v^2)^4}$$

$$Q'(v) = 0 - 2 \cdot 4(6+2v-v^2)^3 \frac{d}{dv} (6+2v-v^2)$$

$$Q'(v) = -8(6+2v-v^2)^3(2-2v)$$

$$Q'(v) = -8(2+2v)(6+2v-v^2)$$

$$Q'(v) = (-16+16v)(6+2v-v^2)$$

Again differentiate w.r.t. v

$$Q''(v) = (-16+16v)(0+2-2v) + (6+2v-v^2)(16)$$

$$Q''(v) = (-16+16v)(2-2v) + 96 + 32v - 16v^2$$

$$Q''(v) = -32 + 32v - 16v^2 + 32v + 96$$

c] $f(t) = \ln(1+t^2)$

Differentiate w.r.t. t

$$f'(t) = \frac{1}{1+t^2} \frac{d}{dt} (1+t^2)$$

$$f'(t) = \frac{2t}{1+t^2}$$

Again differentiate w.r.t. t

$$f''(t) = \frac{1+t^2 \frac{d}{dt} (2t) - 2t \frac{d}{dt} (1+t^2)}{(1+t^2)^2}$$

$$f''(t) = \frac{1+t^2 (2) - 2t (2t)}{(1+t^2)^2}$$

$$f''(t) = \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$f''(t) = \frac{-2t^2+2}{(1+t^2)^2}$$

(13)

$$f(x) = x^3 - 3x^2 - 9x + 12$$

$$f'(x) = 3x^2 - 6x - 9 \quad \& \quad f''(x) = 6x - 6$$

$$\text{Consider } f'(x) = 0$$

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 + x - 3x - 3 = 0$$

$$x(x+1) - 3(x+1) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1, \text{ or } x = 3$$

$$\text{for } x = -1$$

$$f''(-1) = -6 - 6 = -12 < 0$$

$f(x)$ attains maximum value at $x = -1$

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$$

$$= -1 - 3 + 9 + 12$$

$$= 17$$

The function $f(x)$ has max value 17 at $x = -1$

For $x = 3$

$$f''(3) = 6(3) - 6 \\ = 12 > 0$$

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$f(x)$ attains min value at $x = 3$

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$$

$$= 27 - 27 - 27 + 12 \\ = 12$$

The function $f(x)$ has min value 12 at $x = 3$.

(14)

$$f(x) = 4x^3 - 18x^2 + 24x - 7$$

$$f'(x) = 12x^2 - 36x + 24 \quad \& \quad f''(x) = 24x - 36$$

consider $f'(x) = 0$

$$12x^2 - 36x + 24 = 0$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - x - 2x + 2 = 0$$

$$x(x-1) - 2(x-1) = 0$$

$$(x-1)(x-2) = 0$$

$$x-1 = 0 \quad \text{or} \quad x-2 = 0$$

$$x = 1 \quad \text{or} \quad x = 2$$

For $x = 1$

$$f''(1) = 24(1) - 36 = -12 < 0$$

$$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7 = 3$$

$f(x)$ attains max value at $x = 1$

$$f(1) = 4(1)^3 - 18(1)^2 + 24(1) - 7$$

$$= 4 - 18 + 24 - 7 = 3$$

$$= 3$$

The function $f(x)$ has max value 3 at $x = 1$

For $x = 2$

$$f''(2) = 24(2) - 36 = 12 > 0$$

$$= 12 > 0$$

$f(x)$ attains min value at $x = 2$

$$f(2) = 4(2)^3 - 18(2)^2 + 24(2) - 7$$

$$= 36 - 72 + 48 - 7 = 5$$

The function $f(x)$ has min value 5

at $x = 2$

15.] $f(x) = x^2(x+3)$

Differentiate w.r.t. x on both sides

$$\therefore f'(x) = 2x(x+3) + x^2$$

$$= 2x^2 + 6x + x^2$$

$$= 3x^2 + 6x$$

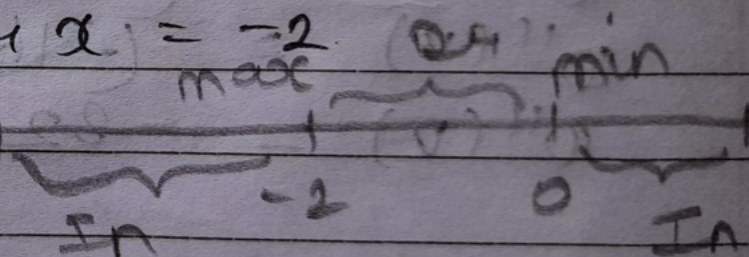
Put $f'(x) = 0$

$$3x^2 + 6x = 0 \quad \therefore 3(x^2 + 2x) = 0$$

$$x(x+2) = 0$$

$$x = 0 \text{ or } (x+2) = 0$$

$$x = 0 \text{ or } x = -2$$



Diff. w.r.t. x again

$$f''(x) = 6x + 6$$

$$\text{when } f''(0) = 6(0) + 6 = 6 > 0$$

$$\therefore f''(0) > 0$$

$f(x)$ attains minimum at $x=0$

$$f(0) = 0^2(0+3) = 0$$

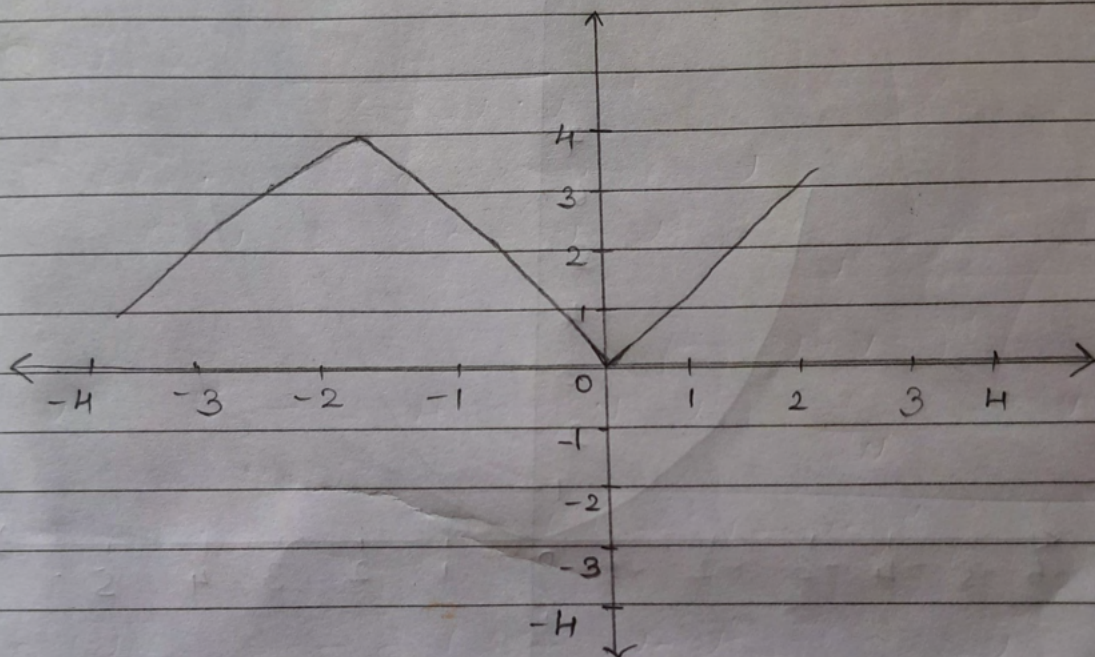
$\therefore$ The function $f(x)$ ~~is~~ minimum at $x=0$ has minimum value 0 at $x=0$

$$f''(-2) = 6(-2) + 6 = -6 < 0$$

$\therefore f(x)$ attains maximum at $x=-2$

$$f(-2) = (-2)^2(-2+3) = 4$$

$\therefore$ The function $f(x)$ has maximum value 4 at $x=-2$.



16] $f(x) = 3x^2 - 6x$

Differentiate w.r.t. x

$$f'(x) = 6x - 6$$

$$\text{put } f'(x) = 0$$

$$6x - 6 = 0$$

$$x - 1 = 0$$

$$x = 1$$

$$f(x) = 3x^2 - 6x$$

$$f(2) = 3(2)^2 - 6(2)$$

$$= 12 - 12 = 0$$

$$f(0) = 3(0)^2 - 6(0) = 0$$

Differentiate w.r.t. x again

$$f''(x) = 6x$$

$$\text{put } f''(1) = 6$$

$\therefore f(x)$ attains minimum value at $x=1$

$$\text{put } f''(0) = 6$$

$f(x)$ attains minimum value at $x=0$

$\therefore$ The function $f(x)$ is minimum

