Q1) The nominal rate of discount per annum convertible quaterly is 8%.

i) Calculate the equivalent force of interest

ii) Calculate the equivalent effective rate of interest per annum.

iii) Calculate the equivalent nominal rate of discount per annum convertible monthly.

→

~~$(i)^{(4)} = 8\%$ p.a.~~

~~$i = 2\%$ per quarter.~~

$d^4 = 0.08$

$d = \frac{d^4}{4} = \frac{0.08}{4} = 0.02$ q.e.dr

1) equivalent force of interest = δ

$\therefore \delta = \ln(1+i) \rightarrow (1)$

$\because d = \frac{1}{1+i}$

$\therefore (1+i) = \frac{1}{d}$

$\therefore i = \frac{1}{d} - 1$

$i = 49$

$\therefore$ Sub $i = 49$ in (1)

$\therefore \delta = \ln(1+49)$

$= 3.91202$

2) Equivalent effective rate of interest per annum

$i_e = \left(1 + \frac{i}{m}\right)^m - 1 \rightarrow$ here m = period

→ ∴ annuity

here $m=1$

$\therefore i_e = (1+0.02)-1$

$i_e = 0.02$

∴ Equivalent effective rate of interest

$= 0.02 = 2\%$

3) $d^{(12)} = 8\%$

$d = \frac{d^{12}}{12}$ per month

$= \frac{0.08}{12}$

$= 6.667$

Q2) The force of interest $\delta(t) = 0.04(t) + 0.0002t^2$ all t.

(i) Find PV of sum of rupees 1000 payable end of 10 years from now

ii) Find constant annual effective rate of interest over a 10 year period.

→ i) $(A.V) = 1000$

$t = 10$ year

$(A.V) = P A(0, 10)$

$1000 = P.V\, e^{\int_0^{10} 0.004t + 0.0002t^2\, dt}$

$P.V\, e^{\left[0.004t^2 + \frac{0.0002t^3}{3}\right]_0^{10} dt}$

$1000 = P.V \times 1.5946697$

P.V =

iii) eff Rate of Interest.

$$AV = (1+i)^t$$

$$1000 = (1+i)^{10}$$

$$i = 0.995\%$$

4) (i) Explain the relationship $d = iv$ by general reasoning where d is the effective annual rate of discount and i is the effective annual rate of ~~discount~~ interest.

ii) Given $i = 0.08$ calculate

a) $d^{(12)}$

b) $i^{(365)}$

c) δ

d) $i^{(0.5)}$

→ i) Let i be effective annual rate of ~~discount~~ interest

d be effective annual rate of discount.

v be discounting factor.

$\because$ Rate $= \dfrac{\text{Accumulated}}{\text{Principal}}$

$\therefore i = \dfrac{d}{1-d}$ → ~~(i is)~~ when loan is

$\therefore d = \dfrac{i}{1+i}$ → ① when loan is

$\because V = \dfrac{1}{1+i}$ → ②

$\therefore$ Sub ② in ①

$\therefore d = i \cdot V$

ii) Given $i = 0.08$

a) $\cancel{d^{(12)} = \dfrac{i}{1+i}}$ $\qquad \because d = \dfrac{d^{12}}{12}$

$\therefore \dfrac{i}{1+i} = \dfrac{d^{(12)}}{12}$

$\Rightarrow \dfrac{0.08}{1.08} = \dfrac{d^{12}}{12}$

$d^{(12)} = 0.8889$

b) $\because \quad i = \frac{i^{365}}{365}$

$\therefore \quad 0.08 \times 365 = i^{365}$

$\therefore \quad i^{(365)} = 29.2$

c) $\delta = \ln(1+i)$

$\delta = 0.07696$

d) $i^{(0.5)}$

$\because \quad i = \frac{i^{(0.5)}}{0.5}$

$\therefore \quad i^{(0.5)} = 0.08 \times 0.5$

$\boxed{i^{(0.5)} = 0.04}$

Q5 i) The rate of discount per annum convertible quarterly is 6%. Calculate :-

a) The equivalent rate of interest per annum convertible half yearly.

b) The equivalent rate of discount per annum convertible monthly.

i) On 15th April 2005 Amit borrowed Rs 2,00,000 to be repaid one year later by single payment of Rs 2,20,000. Amit repaid the loan early on 17th July 2005.

a) Find sum paid by Amit to terminate the contract assuming that the interest is reduced proportionately for early settlement.

$\Rightarrow$ $d^{(4)} = 6\%$

$\therefore d' = \frac{d^4}{4} = \frac{0.06}{4}\% = \frac{3}{2} = 1.5\%$ p.

a) $i = \frac{d}{1-d}$

$= \frac{1.5}{1-4.5}$

$= \frac{1.5}{0.5}$

$i = 3$ p.q

b) $d^{(12)} = 6\%$

$d = \frac{d^{(12)}}{12}$ p.monthly

$= \frac{6}{12} = 0.5\%$

ii) $C = PV =$ Rs, 200,000 on 15th April 2005

$AV = 2,20,000$ on 17th July 2005

$t = 3$ months

$AV = C \cdot (1 - i)^{t}$

$2,20,000 = 200,000 (1 - i)^{3}$

$1.1 = (1 - i)^{3}$

$1.03228 = 1 - i$

$i = 1 - 1.03228$

$\boxed{i = 0.03228}$

Q6 i) The manufacturer of a certain toy sells to retailers on either of the following terms
a) cash payment :- 30% below the recommended retail price.
b) Six month credit : 25% below the recommended retail
Find the effective annual rate of discount offered by the manufacturer to the retailers who pay cash as opposed to those retailers who accept the credit terms.
ii) Find $d^{(12)}$, $i^{(2)}$ and δ

→ Assume cash payment = 100

At time $t = 0$

$$\text{Discounted value} = 100 \times \frac{70}{100}$$

$$= 70$$

At six month $= 100 \times 25\%$

$= 75$

$$PV = AV(1-d)^n$$

$$70 = 75(1-d)^{1/2}$$

$$\sqrt{\frac{70}{75}} = 1-d$$

$$d = 0.12889$$

$$d = 12.889\%$$

$$d^{(12)} = 12\left(1-(1-0.12889)^{1/12}\right)$$

$$= 13.7183\%$$

$$i = \frac{0.12889}{1 - 0.12889}$$

$$= 0.1479596$$

$$\delta = -\ln(1-d)$$

$$\delta = 13.7987\%$$

Q7 If an investment fund offers to increase INR 20,000 to INR 25,000 in 17 month.

i) The nominal rate of interest per annum convertible quaterly.

ii) The nominal rate of discount per annum convertible half yearly.

iii) Simple rate of interest per annum

iv) Comment on the results.

→ (i) $PV_0 = 20000$; $AV_{17} = 25000$

$$20000 = 25000 \left(1 + \frac{i^{(4)}}{4}\right)^{4 \times \frac{17}{12}}$$

$$\sqrt[17/3]{\frac{20000}{25}} = 1 + \frac{i^{(4)}}{4}$$

$$i^{(4)} = 18.96\ \%$$

ii) $d^{(2)} = ?$

$$i = \left(\cancel{1+}\ \frac{0.1896}{4}\right)^4 - 1$$

$$= (4.1896)^4 - 1$$

$$= 20.3454$$

$$d = \frac{0.20345}{1 + 0.203454}$$

$$= 16.90559$$

iii) $2\cancel{5}\cancel{0}\cancel{0}\cancel{0} = 20\cancel{0}\cancel{0}\cancel{0}(1+i)^{17/12}$

$$\left(\frac{25}{20}\right)^{12/17} = 1+i$$

$$i = \left(\frac{25}{20}\right)^{12/17} - 1$$

$$i = 21.8\%$$

9) i) Define force of interest.

→ The measure of interest at individual moment of time which measures the intensity with which interest is operating over a infinitesimally small interval of time. It is called force of interest.

Given.

$i^{(2)} = 0.0775$

$i = \frac{i^{(2)}}{2}$

$i = 0.03375$

$\delta = \cancel{1+i}\ \ln(1+i)$

$= 0.03319298$

$d(1/2)$

$d = \frac{d^{(12)}}{12}$

$\frac{i}{1+i} = \frac{d^{(12)}}{12}$

$\frac{0.03375}{1.03375} \times 12 = d^{(12)}$

$d^{(12)} = 0.391777$

$i^{(1/2)}$

$\therefore \quad i = \frac{i^{(1/2)}}{\cdot} \times 2$

$\therefore \quad 0.03375 \times 2 = i^{(1/2)}$

$\therefore \quad i^{(1/2)} = 0.0675$