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Numerical methods & Algebra
~~Assing~~ Assignment 1.

1) To find : $\sqrt{7+\sqrt{7+\sqrt{7}}} \dots$

Let $x = \sqrt{7+\sqrt{7+\sqrt{7}}} \dots$ — (1)
Squaring both sides.
$x^2 = 7+\sqrt{7+\sqrt{7}} \dots$ — (2)
Substituting (1) in (2)
$x^2 = 7+x$
$x^2 - x - 7 = 0$
$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = 3.1926, -2.1926$
$\therefore x = 3.1926$
$\therefore \sqrt{7+\sqrt{7+\sqrt{7}}} = 3.1926$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$
$7x^2 + 11x = -4y^2 + 6y^2$
$18x^2 = 2y^2$
$9x^2 = y^2$
$\dfrac{x^2}{y^2} = \dfrac{1}{9}$
$\dfrac{x}{y} = \dfrac{1}{3}$

3) $x + 3y = 4, \quad x^2 + y^2 - 4y = 12$

$x + 3y = 4$
$x = 4 - 3y$ — (1)

Subsituting ① in the equation:

$x^2 + y^2 - 4y = 12$

$(4-3y)^2 + y^2 - 4y = 12$

$16 - 24y + 9y^2 + y^2 - 4y = 12$

$10y^2 - 28y + 4 = 0$

$5y^2 - 14y + 2 = 0$

$$y = \frac{14 \pm \sqrt{(-14)^2 - 4(5)(2)}}{10}$$

$= 2.648996, \; 0.1510004.$

when $y = 2.648996$

$x = -3.946998$

when $y = 0.1510004$

$x = 3.546998.$

∴ The ordered pairs that satisfy the intger ar $(-3.94070, 2.~~46~~6490)$ & $(~~0.151~~\; 3.547, 0.151)$

4) $d = 2x^2 - 16x + 34$

d = density of smoke

x = speed of the engine.

a) $x = 3.5$:

$d = 2(3.5)^2 - 16(3.5) + 34$

$= 2.5$ million particles per cubic foot.

b) for minimum smoke $\frac{d}{d(x)} > 0$

$\frac{d}{dx} = 4x - 16 > 0 \rightarrow 4x > 16$

$\therefore x \geq 4.$

no. of revelovtions for min smoke = 4/min.

min out put = 2

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c) $d = 100$

$100 = 2x^2 - 16x + 34$

$2x^2 - 16x - 66 = 0$

$x^2 - 8x - 33 = 0$

$x^2 - 11x + 3x - 33 = 0$

$(x-11)(x+3) = 0$

$x = 11$ or $x = -3$ (Invalid)

Speed of engine is 1100 revolutions/min.

5) Company 1 : 1500/day + 25/km

Company 2 : 2100/day + 22/km

n = total no. of km driver in a day.

a) Total cost of a car from company 1 = $1500 + 25n$

b) " " " " 2 = $2100 + 22n$

c) $1500 + 25n > 2100 + 22n$

d) $1500 + 25n > 2100 + 22n$

$25n - 22n > 2100 - 1500$

$3n > 600$

$n > 200$ //

It is less expensive to rent from Company # 2 when the no. of km driver in a day is greater than 200.

6) $f(x) = x^3 - 7x^2 + 14x - 6 = 0$

a) considering $a = 0$, $b = 1$

$f(a) = f(0) = -6$

$f(b) = f(1) = 2$

Since $f(a)$ & $f(b)$ have opposite ~~sign~~ signs the ~~inter~~ interval is appropriate.

$x = 0.58$ //

b) Considering $a=1$ $b=3.2$

$f(a) = f(1) = 2$

$f(b) = f(3.2) = -0.112$

The interval is appropriate as $f(a)$ & $f(b)$ have opposite signs.

~~x = 0~~ $x = 3.0011$

c) Considering $a = 3.2$, $b = 4$

$f(a) = f(3.2) = -0.112$

$f(b) = f(4) = 2$

$\therefore x = 3.42$

7) $f(x) = 230x^2 + 18x^3 + 9x^2 - 221x - 9$

$\therefore f'(x) = 920x^2 + 54x^2 + 18x - 227$

Since $f(x)$ has real zero in $(-1, 0)$ we consider the midpoint ~~as~~ -0.5 as x_0

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x)} = -0.5 - \frac{f(0.5)}{f'(0.5)} = \cancel{-0.5}\ -0.15045$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.0418168$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.040659343 5$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = -0.040659 28832.$$

Since $f(x)$ has a real zero in $(0, 1)$ we consider the midpoint 0.5 as x_0

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{f(0.5)}{f'(0.5)} = -0.705089200$

$\therefore x_2 = -0.323791142$

$\therefore x_3 = -0.06460313103$

$\therefore x_4 = -0.04068615115$

$\therefore x_5 = -0.04065928835.$

8) 70, 86, 81, 83

Grade on final exam: x

a) Converse average $= \frac{70+86+81+83+x}{5} = \frac{320+x}{5}$

b) $80 \leq \frac{320+x}{5} < 90$

c) $80 \leq \frac{320+x}{5} < 90$

$400 \leq 320 + x < 450$

$80 \leq x \leq 130$

If we assume that all marks are out of 100:-

$80 \leq x \leq 100$ //

9) $x_1 = 8.54$

$x_2 = -11.81$

~~y_1~~ $x = 0$

$y_1 = 4\% = 0.04$

$y_2 = 20\% = 0.20$

Using linear interpolation.

$y - 0.04 = \frac{0.20 - 0.04}{-11.81 - 8.54}(0 - 8.54)$

$y = 10.7144963\%$

$IRR = 10.7145\%$

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10. Revenue = $R(x) = 32x - 0.21x^2$
Cost = $C(x) = 195 + 12x$

To stay ~~to~~ profitable $R(x) > C(x)$
$32x - 0.21x^2 > 195 + 12x$
$+0.21x^2 - \cancel{32x}\ 20x + 195 < 0$

Equating the inequality to 0

$0.21x^2 - 20x + 195 = 0$
$x = 84.2114275\,3,\ 11.0266771.$

Since $x < 11.02666771$ & $x > 84.211428$ do not satisfy the original inequality:
$11.0266771 < x < 84.2114275$ → Satisfy Original inequa
$12 < x < 84$

No. of orders to remain profitable should be greater than 12 & less than 84, with minimum orders being 12.