Assignment - 1

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Subject : Probability & Statistics
Section : FY - Bsc - B

Solution 1

$X \sim Pois(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$$

$$P[X_1 < \theta < X_2] = 0.95$$

$X \sim Poi(200)$

$$95\%\ CI = \mu \pm 1.96\sqrt{6}$$
$$= 200 \pm 1.96\sqrt{200}$$

Hence, the interval is $(172.28, 227.7186)$

Solution 2

$$\bar{X} = \sum_{i=1}^{n} xi = \text{sample mean}$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (xi - \bar{x}) = \text{Sample variance}$$

(i) $E(\bar{X}) = E\left(\frac{xi}{n}\right) = \frac{1}{n}(Exi)i = \frac{1}{n}[\sum E(xi)] = \frac{n\mu}{n} =$

$$V(\bar{X}) = V\left[\frac{\sum xi}{n}\right] = \frac{1}{n^2} V[E(\bar{x})] = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

(ii) $E(s^2) = E\left(\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2\right)$

$= \frac{1}{n-1} E\left(\Sigma x_i^2 - \bar{x}^2\right)$

$= \frac{1}{n-1}\left[E\left(\Sigma x_i^2\right) - E\left(n\bar{x}^2\right)\right]$

$= \frac{1}{n-1}\left[E\left(\sigma^2 + \mu^2\right)\right] - n\left(\frac{\sigma^2}{n} + \mu^2\right)\right]$

$= \frac{1}{n-1}\left[n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2\right]$

$= \frac{1}{n-1}\left[(n-1)\sigma^2\right] = \sigma^2$

Now here $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$var\left[\frac{(n-1)s^2}{\sigma^2}\right] = \chi^2_{n-1}$

$\frac{(n-1)^2 \; var[s^2]}{\sigma^4} = 2(n-1)$

$var(s^2) = \frac{2\sigma^4}{n-1}$

(iv) $n = 10$

$\sigma^2 = 100$

$$\frac{(n-1)s^2}{\sigma^2} = 0.5$$

$$\frac{9 \times s^2}{100} = 0.5$$

$$9s^2 = 50$$

$$s^2 = \frac{50}{9}$$

$$s^2 = 5.56$$

Solution (3)

No. of claims	0	1	2	3	4
No. of Policies	86	75	16	2	1

$X \sim Bin(4, p)$

$$L = P(X=0)^{86} \times P(X=1)^{75} \times P(X=2)^{16} \times P(X=3)^{2} \times P(X=4)^{1}$$

$$= \left[{}^4C_0\, p^0 (1-p)^4\right]^{86} \times \left[{}^4C_1\, p^1 (1-p)^3\right]^{75} \times \left[{}^4C_2\, p^2 (1-p)^2\right]^{16}$$

$$\times \left[{}^4C_3\, p^3 (1-p)\right]^{2} \times \left[{}^4C_4\, p^4 (1-p)^0\right]^{1}$$

$$= \left[\text{cons.} \times (1-p)^4\right]^{86} \times \left[\text{cons.} \times p(1-p)^3\right]^{75} \times \left[\text{cons} \times p^2 (1-p)^2\right]^{16}$$

$$\times \left[\text{cons} \times p^3 (1-p)\right]^{2} \times \left[\text{cons} \times p^4\right]$$

$\ln(2) = 344\ln(1-p) + 75\ln(p) + 25\ln(p)$
$+ 32\ln(p) + 321\ln(1-p) + 6\ln p + 2\ln(1-p)$
$+ 4\ln p$

$\ln(2) = 633\ln(1-p) + 117\ln(p)$

$$\frac{d[\ln(2)]}{dp} = \frac{117}{p} - \frac{633}{1-p}$$

let $\frac{d[\ln(2)]}{dp} = 0$

$$\frac{117}{p} = \frac{633}{1-p}$$

$117 - 117p = 633p$

$117 = 750p$

$p = \frac{117}{750}$

$\hat{p} = 0.156$

Solution (4) $\quad f(x) = \dfrac{C\beta^2}{(x+\beta)^4} \quad x>0 \,;\, \beta>0$

(1) $$\int_0^\infty f(x)\,dx = 1$$

$$C\beta^3 = \int_0^\infty \frac{1}{(x+\beta)^4} = 1$$

$$C\beta^3 = \left[\frac{(x+\beta)^{-3}}{-3}\right]_0^\infty = 1$$

$$\frac{C\beta^3}{3\beta^3} = 1$$

$$\frac{C}{3} = 1$$

$$\boxed{C = 3}$$

$$E(X) = \int_0^\infty x\, f(x)\,dx$$

$$= 3\beta^3 \int_0^\infty \frac{x}{(x+\beta)^4}\,dx$$

$$= 3\beta^3 \left[\frac{(x+\beta)^{-2}}{-2} - \frac{\beta(x+\beta)^{-3}}{-3}\right]_0^\infty$$

$$= 3\beta^3 \left[0 - 0 - \left(\frac{\beta^{-2}}{-2} + \frac{\beta^{-2}}{3}\right)\right]$$

$$= 3\beta^3 \left(\frac{1}{2\beta^2} + \frac{1}{3\beta^2}\right) - \frac{\beta}{2}$$

$$E(x^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx$$

$$= 3\beta^3 \int_0^\infty \frac{x^2}{(x+\beta)^4} dx$$

(ii) $\beta = x_1, x_2, x_3 \ldots x_n$

$$E(\bar{x}) = x = \boxed{\hat{\beta} = 2\bar{x}_n}$$

$E(\hat{\beta}) = E(2\bar{x}_n) = 3n \times \frac{\beta}{2} = \beta$ $\therefore$ Unbiased

(iii) MSE $= Var(b\bar{x}_n) + (Bias[b\bar{x}_n])^2$

$$= b^2 x^3/n \times \frac{\beta^2}{n} + (E(b\bar{x}_n) - \beta)^2$$

$$= \beta^2/n \left[\frac{3\beta^2}{n} + n\left(\frac{b}{2} - 1\right)^2\right]$$

(iv) It seem in that $\hat{\beta} = 2\bar{x}_n$ is an unbiased estimator estint $b\sqrt{n}$ in (iii) when $b = \frac{2}{1+3/n}$ is why biased estimation

It is also consistent

Ques. (i)

Ques (i) $X \sim U(a+b)$

$$\bar{X} = \frac{1}{2}(a+b)$$

$$S = \frac{1}{\sqrt{12}}(b-a)$$

$$\sqrt{12} + a - b$$

$$\bar{X} = \frac{1}{2}(a + \sqrt{12}S + a)$$

$$\bar{X} = [a + \sqrt{3}S]$$

$$\hat{a} = \bar{X} - \sqrt{3}S$$

(ii) $$\bar{X} = \frac{1}{2}[b - \sqrt{12}S + b)$$

$$= \frac{1}{2}[2b + 2\sqrt{3}S]$$

$$\hat{b} = \bar{X} + \sqrt{3}S$$

(iii) Sample : 1, 2, 3, 4, 10

$$\bar{X} = 20/5 = 4$$

$$Var = 1/12 \times 81 = 27/4$$

$$\hat{a} = \bar{X} - \sqrt{3} \times 27/4$$

$$= 4 - \sqrt{3} \times 27/4 = -7.6913$$

$\hat{b} = 4 + \sqrt{3} \times (27/4) = 15.69134$, Hence the result is false

(iv) $f(x) = \frac{1}{b-a} = \frac{1}{b}$

$L = -n \log b$

$\frac{dL}{db} = \frac{-n}{b} = 0 \Rightarrow b \to \infty$

$\frac{d^2L}{db^2} = \frac{n}{b^2} > 0$; minimum

$\hat{\theta} = \max x_i$

Ques 8.

(i) $E(\hat{\theta}) = \theta$, it will be unbiased

(ii) $E(\hat{\theta})$ that will be biasness of parameter

(iii) $MSE = var + bias^2$

(iv) $\hat{\theta}$ is more efficient than $\check{\theta}$ since MSE of $\hat{\theta}$ is lower than $\check{\theta}$

(v) when MSE move toward 0, as the n increase, the estimate would be more constitent

(vi) Method of Moments
Maximum likelihood estimate

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Ques 9. $t_K = \dfrac{z}{\sqrt{\dfrac{X_K}{K}}}$

$Z = N(0,1)$

X_K^2 = Chi-square dist. with degree of freedom K.

K = degree of freedom

(ii) $\mu(t_K) = 0$ $\quad$ $var(t_K) = \dfrac{K}{K-2}$

(iii) $n = 10$
$\mu = 50$
$s^2 = 48.667$

$t_0 = 5 \quad 9K = 3.25$

(a) $C.I = 50 \pm 3.25\sqrt{\dfrac{48.667}{10}} = (42.803, 57.1699)$

(b) $C.I = 50 \pm 2.5788\sqrt{\dfrac{48.667}{10}} = (44.3176, 55.6823)$

Question 10 $n = 1000$, $X \sim Pois(\cdot 3)$

$X \sim N(3000, 3000)$

(i) Type 1 error = $P[H_0 \text{ reject} \mid H_0 \text{ is true}]$

$= P[X > 310 \mid X = 3]$

$= P\left[Z > \frac{3100 - 3000}{\sqrt{3000}}\right]$

$= P[Z > 1.8257]$

$= 1 - P[Z < 1.8257]$

$= 3.39\%$

(ii) Type 2 error = $P[\text{do not reject } H_0 \mid H_0 \text{ is false}]$

$= P[X < 3100 \mid \lambda = n\lambda]$

$= P\left[X < \frac{3100 - n\lambda}{\sqrt{n\lambda}}\right]$

(iii) Power = $P[\text{reject } H_0 \mid H_0 \text{ is false}]$

$= P(X > 3100 \mid \lambda = n\lambda^0)$

$= 1 - P\left[\frac{3100 - n\lambda}{\sqrt{n\lambda}}\right]$

(iv) $n = 2900$ $C.I = 3 \pm 2.5758 \sqrt{\frac{3}{2900}}$

$(2.9172, 3.0828)$

Question 11. $X_1, X_2 \dots X_n \sim U(0, \theta)$ $a=0, b=\theta$

(i) $E(x) = \frac{\theta}{2}$ $\quad V(Y) = \frac{\theta^2}{12}$

$$L = \prod_{i=1}^{n} \left(\frac{1}{\theta}\right) = \frac{1}{\theta^n}$$

$$\ln(L) = -\frac{n}{\theta}$$

hence CRLB would not be applicable

(ii) $P[\text{Max } X < y] = \prod_{i=1}^{n} \left[\int_0^y \frac{dx}{\theta}\right] = \left(\frac{y}{\theta}\right)^n$

$$E(Y_k) = \left[y^k \cdot \frac{n}{\theta^n} \cdot y^{n-1}\, dy\right]$$

$$= \frac{n}{n+k} \int \frac{n+k}{\theta^n} \cdot y^{n+k-1} \cdot dy$$

$$= \frac{n\theta^k}{n+k}$$

(ii) $P(X=0) = {}^4C_0\, p^0 (1-p)^4$

$= 91.34$

$P(X=1) = {}^4C_1\, p^1 (1-p)^3$

$= 67.53$

$P(X=2) = {}^4C_2\, p^2 (1-p)^2$
$= 18.72$

$P(X=3) = {}^4C_3\, p^3 (1-p)^3$
$= 2.31$

$P(X=4) = {}^4C_4\, p^4 (1-p)^4$
$= 0.11$

$f(x)$	O_i	E_i	$(O_i - E_i)^2 / E_i$
$P(X=0)$	86	91.34	0.3123
$P(X=1)$	75	67.53	0.3263
$P(X=2)$	19	27.792	0.3982
			39.8563

$\chi^2_{cal} = 39.856$
$\chi^2_{tab} = 3.841$

$\chi_{cal} > \chi_{tab}$

Thus we will not accept H_0 at 5%.