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Subject : Probability & Statistics 2
Category : Assignment 1

Q1. $X \sim Poisson(\theta)$

$$\frac{X-\theta}{\sqrt{\theta}} \sim \text{approx. } N(0,1)$$

$x_1 = 200$
Use continuity correction
95% CI :

$\cancel{P(X \leq 200) = \alpha/2}$
$\cancel{P(X \leq 200.5) = \alpha/2}$

$$\hat{\lambda} \pm Z_{\alpha/2} \sqrt{\frac{\hat{\lambda}}{n}}$$

$$C.I \in \left(\frac{200}{1} \pm 1.96 \sqrt{\frac{200}{1}}\right)$$

$$C.I \in (200 \pm 27.719)$$

$$C.I \in (172.28, 227.719)$$

[as we have been given single value & have to find approximate C.I so n = 1 in the approximate C.I expression]

Q2 Sample mean be $\overline{x}$

(i) $\overline{x} = \frac{\Sigma x_i}{n}$

$E[\Sigma x_i] = \Sigma E[x_i] = \Sigma \mu = n\mu$

$Var[\Sigma x_i] = \Sigma Var[x_i] = n\sigma^2$; since they are identically distributed

$E[\overline{x}] = \mu$

$Var[\overline{x}] = \frac{1}{n^2} \times n\sigma^2 = \frac{\sigma^2}{n}$.

(ii) $s^2 = \frac{1}{(n-1)}\left[\Sigma x_i^2 - n\overline{x}^2\right]$

$$E[s^2] = \frac{1}{(n-1)}\left[\Sigma E(x_i^2) - n E[\overline{x}^2]\right]$$
$$= \frac{1}{(n-1)}\left[\Sigma(\sigma^2+\mu^2) - n\left(\frac{\sigma^2}{n}+\mu^2\right)\right]$$
$$= \frac{1}{(n-1)}\left[n(\sigma^2+\mu^2) - \sigma^2 - n\mu^2\right]$$
$$= \frac{1}{(n-1)}\left[(n-1)\sigma^2\right]$$
$$= \sigma^2$$

(iii) $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

mean = μ , variance = σ^2 of Normal dist.

$$Var\left[\frac{(n-1)s^2}{\sigma^2}\right] = 2(n-1)$$

$$\frac{(n-1)^2}{\sigma^4} Var[s^2] = 2(n-1)$$

$$Var[s^2] = \frac{\sigma^4}{(n-1)^2} \times 2(n-1)$$

$$= \frac{2\sigma^4}{(n-1)}$$

(iv) $\sigma^2 = 100$

$n = 10$

$$P(50 < S^2 < 150) = P\left(\frac{9 \times 50}{100} < \frac{9S^2}{100} < \frac{150 \times 9}{100}\right)$$

$$= P(4.5 < Y < 13.5)$$

$$= P(Y < 13.5) - P(Y < 4.5)$$

$$= 0.8587 - 0.1245$$

$$= 0.7342$$

Q3

i) $L(p) = [P(x=0)]^{86} \times [P(X=1)]^{75} \times [P(X=2)]^{16} \times [P(X=3)]^2 \times [P(x=4)]^1$

$P(X=0) = 4C_0\, p^0 (1-p)^4$

$P(X=1) = 4C_1\, p^1 (1-p)^3$

$P(X=2) = 4C_2\, p^2 (1-p)^2$

$P(X=3) = 4C_3\, p^3 (1-p)^1$

$P(X=4) = 4C_4\, p^4 (1-p)^0$

$L(p) = \text{const} \times [p^0 (1-p)^4]^{86} \times [p^1 (1-p)^3]^{75} \times [p^2 (1-p)^2]^{16} \times [p^3 (1-p)^1]^2 \times [p^4 (1-p)^0]^1$

$= \text{const} \times p^{75+32+6+4} (1-p)^{344+225+32+2}$

$= \text{const} \times p^{117} \times (1-p)^{603}$

$\ln L(p) = \ln c + 117 \ln p + 603 \ln(1-p)$

$\frac{d}{dp} \ln L(p) = 0 + \frac{117}{p} + \frac{603}{(1-p)}(-1)$

Equating to 0

$\frac{117}{p} = \frac{603}{(1-p)}$

$117 - 117p = 603p$

$\hat{p} = 0.1625$

FOR EDUCATIONAL USE

(ii) Goodness of fit test : $\hat{p} = 0.1625$

formula : $\sum \frac{(O_i - E_i)^2}{E_i}$

$C \cdot V = \chi^2_{n-1}$

$P(X=0) = {}^4C_0\, p^0 (1-p)^4 = 0.49197$

$P(X=1) = {}^4C_1\, p^1 (1-p)^3 = 0.38182$

$P(X=2) = {}^4C_2\, p^2 (1-p)^2 = \cancel{0.22226}$ 0.11113

$P(X=3) = {}^4C_3\, p^3 (1-p)^1 = 0.014375$

$P(X=4) = {}^4C_4\, p^4 (1-p)^0 = 0.000697$

No. of claims	O_i	E_i	~~$(O_i-E_i)^2$~~
0	86	$0.49197 \times 180 = 88.5546$	
1	75	$0.38182 \times 180 = 68.7276$	
2	16	~~0.22226~~ 0.11113 $\times 180 = 20.0034$	
3	2	$0.014375 \times 180 = 2.5875$	
4	1	$0.000697 \times 180 = 0.12546$	
		180	

~~Since~~ ~~E_i~~

No. of claims	E_i	$(O_i - E_i)^2/E_i$
0	88.5546	0.07369
1	68.7276	0.572448
2+	22.7164	0.60800
	180	1.254138

$\chi^2_{cal} = 1.254138$ $\quad \chi^2_{tab} = \chi^2_{2,0.05} = 5.991$

Conclusion : $\chi^2_{cal} < \chi^2_{tab}$ $\therefore$ Ho is not rejected.

At LOS 5%, we donot reject Ho.

At LOS 10%, we still do not reject Ho.

Q4. $f(x) = \dfrac{c\beta^3}{(x+\beta)^4}$

$X \sim$ Pareto $(3, \beta)$

i) On comparing with Pareto Distribution $f(x) = \dfrac{\alpha\lambda^\alpha}{(\lambda+x)^{\alpha+1}}$,

$\beta = \lambda$

& $\alpha = 3$

$c = \alpha = 3$

$\therefore \; c = 3$

Mean $= \dfrac{\lambda}{\alpha-1} = \dfrac{\beta}{3-1} = \dfrac{\beta}{2}$

Variance $= \dfrac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} = \dfrac{3\beta^2}{(\cancel{\beta}3-1)^2(3-2)}$

$= \dfrac{3\beta^2}{4}$

ii) <u>Using Methods of Moments</u>

$E(X) = \overline{X}_n = \dfrac{\beta}{2}$

$\hat{\beta} = 2\overline{X}_n$

Biasedness $= E(\hat{\theta}) - \theta$

$= E(\hat{\beta}) - \hat{\beta} = 2\overline{X}_n - 2\overline{X}_n = 0$

$\therefore$ It is unbiased~~ness~~

Consistency

$$MSE = Var(\hat{\beta}) + bias^2$$
$$= Var(\hat{\beta})$$
$$= Var(2\overline{X}_n)$$
$$= 4\overline{X}_n^2$$
$$= 4\left(\frac{\Sigma X}{n}\right)^2$$

if $MSE \to 0$ then $n \to \infty$

$\therefore$ It is consistent

iii) set of estimators $= b\overline{x}_n$

$$b = \frac{2}{(1+3/n)} \quad \text{for min MSE.}$$

$$MSE = Var(\hat{\beta}) + (Bias[b\overline{x}_n])^2$$
$$MSE = b^2 \cancel{\frac{\Sigma X}{n}} \times \frac{3}{4}\frac{\beta^2}{n} + (E[b\overline{X}_n] - \beta)^2$$
$$= b^2 \times \frac{3}{4}\frac{\beta^2}{n} + \beta^2\left(\frac{b}{2} - 1\right)^2$$
$$= \frac{\beta^2}{n}\left(\frac{3}{4}b^2 + n\left(\frac{b}{2} - 1\right)^2\right)$$
$$\frac{d\,MSE}{db} = \frac{\beta^2}{n}\left(\frac{3}{2}b + n\left(\frac{b}{2} - 1\right)\right)$$
$$\frac{\beta^2}{n}\left(\frac{3}{2}b + n\left(\frac{b}{2} - 1\right)\right) = 0$$
$$b = \frac{2n}{n+3} = \frac{2}{(1+3/n)}$$

Differentiating the MSE a second time,

$$\frac{d^2}{db^2}(MSE) = \frac{\beta^2}{n}\left(\frac{3}{2}+\frac{n}{2}\right) > 0$$

$$\therefore \; b = \frac{2}{(1+3/n)} \text{ is minimum.}$$

(iv) Comment :

$b\bar{X}_n$ in (iii) when $b = \frac{2}{(1+3/n)}$ is negatively biased because $b<2$ when $n \geq 1$

Also as $n \to \infty$, $b \to 2$
so estimator in (iii) is also consistent.

Q5.

i) $E[X] = \frac{a+b}{2}$

$b = 2E[X] - a$
$= 2\bar{x} - a$

$Var(X) = \frac{(b-a)^2}{12}$

$s^2 = \frac{(2\bar{x}-a-a)^2}{12} = \frac{(\bar{x}-a)^2}{3}$

$(\bar{x}-a)^2 = 3s^2$

$\hat{a} = \bar{x} - \sqrt{3}s$

ii) $\hat{b} = 2\bar{x} - (\bar{x} - \sqrt{3}s)$

$\hat{b} = \bar{x} + \sqrt{3}s$

iii) Sample: 2, 4, 6, 8, 10

$\bar{x} = \frac{30}{5} = 6$

$s^2 = \frac{1}{4}(220 - 5\times 6)$

$= \frac{190}{4} = 47.5$

$s = 6.892$

$\hat{a} = 6 - \sqrt{3}\times 6.892$
$= -5.937$

$\hat{b} = 6 + \sqrt{3}\times 6.892$
$= 17.937$

The probability of sample point being less than 'a' or greater than 'b' is zero & we have a sample value 10 greater than 'b'. This shows MOM is not efficient here.

(iv) $L(b) = \frac{1}{b^n}$ if $b \geq \max(x_i)$ or $L(b) = 0$

We cannot use method of differentiation w.r.t 0 & equate it with 0 as range depends on b.

We must find maximum L(b).

We would want to be as small as possible subject to constraint as $L(b) = \frac{1}{b^n}$

$$\therefore \hat{b} = \max(x_i)$$

Q6

i) Type I error is when H_0 is rejected when it is actually true.

$P(H_0 \text{ is true}) = 0.1$

Let 1st step be X_1 & 2nd step be X_2

$P(\text{Type I error}) = P(X_1 \geq 3 \mid P = 0.1) +$

$P(X_1 = 0 \mid P = 0.1) \times P(X_2 \geq 5 \mid P = 0.1)$

$+ P(X_1 = 1 \mid P = 0.1) \times P(X_2 \geq 4 \mid P = 0.1) +$

$P(X_2 = 2 \mid P = 0.1) \times P(X_2 \geq 3 \mid p = 0.1)$

$= (1 - 0.8891) + (0.2824)(1 - 0.9957) +$

$(0.6590 - 0.2824)(1 - 0.9744) +$

$(0.8891 - 0.6590)(1 - 0.8891)$

[From Binomial tables, Pg 188]

$= 0.14728$

$\approx 15\%$ or ~~0.0~~ 0.15

ii) $P(\text{Rejecting } H_0 / p=0.3)$

$$= P(X_1 \geq 3 / p=0.3) + P(X_1=0 / p=0.3) \times P(X_2 \geq 5 / p=0.3) + P(X_1=1 / p=0.3) \times P(X_2=4 / p=0.3) + P(X_1=2 / p=0.3) \times P(X_2 \geq 3 / p=0.3)$$

$$= (1-0.2528) + (0.0138)(1-0.7237) + (0.085-0.0138)(1-0.4925) + (0.2528-0.085)(1-0.2528)$$

$$= 0.91253$$

iii) $P(\text{Type II error}) = P(\text{Accepting } H_0 / p=0.3)$

$$= 1 - P(\text{Rejecting } H_0 / p=0.3)$$

$$= 1 - 0.91253$$

$$= 0.08747$$

iv) $n = 120$ missles

$X = \text{hits} = 40$

$$\hat{p} = \frac{40}{120} = \frac{1}{3} = 0.333$$

$$Z_{0.1} = 1.2816$$

Right tailed lower bound C.I $\in \hat{p} - 1.2816\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$$\in \left(0.333 - 1.2816\sqrt{\frac{0.333(1-0.333)}{120}}\right) = 0.2782$$

(v) $H_0 : p = 0.278$
$H_1 : p > 0.278$

$$T.S.V = \frac{(X \pm 0.5) - np}{\sqrt{npq}}$$ By continuity correction

$$\therefore TSV = \frac{39.5 - 120(0.278)}{\sqrt{120 \times 0.278 \times 0.722}} = 1.2511$$

We are carrying out a one-sided test.
$C.V = 1.2816$

$\therefore T.S.V < C.V$
Hence we cannot reject H_0
$\therefore p = 0.278$

(vi) The C.I. upperlimit in part (iv) is 0.2782 and contains $p = 0.2780$ we ~~f~~ proved in part (v). There is minor change of value is due to binomial ~~co~~ distribution (a discrete distribution) is ~~connect~~ converted by continuity distribution ~~No~~ to Normal distribution (continuous distribution).

(vii) $H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)} = \frac{11(2916) + 14(4900)}{25}$$
$$= 4027.04$$

$$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$\bar{X}_1 = 65$ $\quad \bar{X}_2 = 70$

$s_1 = 54$ $\quad s_2 = 70$

$n_1 = 12$ $\quad n_2 = 15$

$$\therefore \frac{(65-70) - 0}{63.459 \times \sqrt{\frac{1}{12} + \frac{1}{15}}} = -0.2034$$

and t_{25} = ~~0.2~~ 2.06 at ~~2~~ 95% LOS

Therefore Ho is not rejected.

Therefore it is reasonable to conclude that ~~there is~~ the means of the two populations are equal.

Q7. Public

i) $\Sigma X_{Public} = 270$

$\overline{X}_{Public} = \frac{270}{9} = 30.$

Private

$\Sigma X_{Private} = 315.5$

$\overline{X}_{Private} = 35.06$

$\therefore$ sample mean of public $= 30$

sample mean of private $= 35.06$

$$s^2_{public} = \frac{1}{n-1}\left(\Sigma X^2_{Public} - n \times \overline{x}_{public}\right)$$

$$= \frac{1}{8}\left(8614.5 - 9 \times 30^2\right)$$

$$= 64.31$$

$$s^2_{private} = \frac{1}{n-1}\left(\Sigma X^2_{private} - n\,\overline{x}_{private}\right)$$

$$= \frac{1}{8}\left(11599.25 - 9 \times 35.06^2\right)$$

$$= 67.42$$

ii) Primary condition for testing equal mean is sample taken from populations with equal variances.

$H_0 : \sigma_1^2 = \sigma_2^2$ $\qquad$ $H_1 : \sigma_1^2 \neq \sigma_2^2.$

Test Statistic is s_p^2/σ.

$$\frac{s^2_{public} / \sigma^2_{public}}{s^2_{private} / \sigma^2_{private}} \sim F_{n_1-1, n_2-1}$$

$$\text{Test statistics} = \frac{s^2_{public}}{s^2_{private}}$$

$$= \frac{64.31}{67.42}$$

$$= 0.9542$$

$$F_{8,8,0.05} = 0.2256 \ \& \ 4.433$$

Test statistics lies in this interval
Therefore we have insufficient evidence
to reject the hypothesis
$\therefore \sigma^2_{public} = \sigma^2_{private}$.

iii) $H_0 : \mu_{public} = \mu_{private}$
$H_1 : \mu_{public} < \mu_{public}$

$$\text{Test statistic} = \frac{(X_{private} - X_{public}) - (\mu_{private} - \cancel{\mu_{public}} - \mu_{public})}{\sqrt{Sp^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$Sp^2 = \frac{(n_1-1)s^2_{public} + (n_2-1)s^2_{private}}{(n_1+n_2-2)}$$

$$= (8 \times 64.31 + 8 \times 67.42)\ 1/16 = 65.86$$

Test statistic : $\frac{35.06 - 30}{\sqrt{65.86\left(\frac{1}{9}+\frac{1}{9}\right)}} = 1.32$

$t_{n_1+n_2-2, 0.05} = t_{16, 0.05} = 1.746$

Test statistic is lower than t value
∴ This gives insufficient evidence to reject H_0.
∴ $\mu_{public} = \mu_{private}$

Q8.
(i) $\hat{\theta}$ is an estimator of parameter θ

Unbiased estimator is when :

$$\text{bias} = E(\hat{\theta}) - \theta$$
$$0 = E(\hat{\theta}) - \theta$$
$$E(\hat{\theta}) = \theta$$

(ii) ~~Mean~~ Bias is mathematically defined as:

$$\text{bias} = E(\hat{\theta}) - \theta$$

(iii) Mean square error is mathematically defined as :

$$MSE(\hat{\theta}) = [E(\hat{\theta}) - \theta]^2$$
$$= Var(\hat{\theta}) + \text{bias}^2$$

(iv) $\hat{\theta} \rightarrow$ no bias
high MSE

$\tilde{\theta} \rightarrow$ positive bias
low MSE

Rather than biasness, MSE is a better ~~parame~~ factor to check efficiency of p estimators.
Lower MSE, higher efficiency of the estimator.
$\therefore \tilde{\theta}$ is better estimator (efficient)

(v) Either of the two estimators would be termed ~~efficient if~~ consistent if $MSE \rightarrow 0$, $n \rightarrow \infty$.

(vi) Estimating θ:
a. Method of Moments: The sample and population mean are equated to estimate θ.
b. Maximium likelihood method: We find ~~maximum~~ likelihood function: $L(\lambda)$, differentiate it to get an expression, equate it to 0 and estimate θ.

Q9

i) The variable t_K used in the t-test for sampling distribution is:

$$t_K = \frac{N(0,1)}{\sqrt{\frac{\chi^2_K}{K}}}$$ k denotes degree of freedom

N(0,1) denotes standard normal distribution

χ^2_K denotes chi-square distribution with k df.

Both N(0,1) & χ^2_K are independent.

ii) Mean of $t_K = 0$ for $k > 2$

Variance of $t_K = \frac{K}{K-2}$ for $K > 2$

iii) $n = 10$

$\bar{x} = 50$ $s^2 = 48.667$

(a) 99% CI $\Rightarrow \left(\bar{x} \pm t_{n-1}\sqrt{\frac{s^2}{n}}\right)$

99% CI $\in \left(50 \pm 3.25\sqrt{\frac{48.667}{10}}\right)$

99% CI $\in (42.83, 57.17)$

(b) $t_{n-1} \sim N\left(0, \sqrt{\frac{n-1}{n-3}} = \sqrt{\frac{9}{7}}\right)$ $Z = 2.58$

99% CI $\in \left(50 \pm 2.58\sqrt{\frac{48.667}{10} \times \frac{9}{7}}\right) = (43.54, 56.45)$

Q10

i) Type I error is when Ho is rejected when it is actually true.

Let N be no. of claims in a year.

$N \sim \text{Poisson}(1000 \times 3 = 3000)$

$H_0: \quad N \sim \text{Poisson}(3000)$

Using Normal Approximation,

$X \sim N(3000, 3000)$

$P(H_0 \text{ rejected} \mid H_0 \text{ is true}) = \text{Type I error}$

$= P(N > 3100) = P\left(Z > \frac{3100-300}{\sqrt{3000}}\right) = 1 - P(Z < 1.825)$

$= 0.0339$

ii) Type II error is when Ho is accepted when it is actually false.

iii) Power of test is the probability that the hypothesis is rejected when the hypothesis is actually false.

$P(H_0 \text{ is rejected} \mid H_0 \text{ is false}) = P(N > 3100)$

$= 1 - P\left(Z < \frac{3100 - n\mu}{\sqrt{n\mu}}\right)$

The value of power of Test will depend on value of parameter under alternative hypothesis

iv) $\hat{\mu} \Rightarrow$ estimator of μ

C.I for poisson parameter :

$$\hat{x} \pm Z_{\alpha/2} \sqrt{\frac{\hat{\mu}}{n}}$$

$$\hat{\mu} = \frac{2900}{1000} = 2.9$$

$$\therefore 99\% \text{ CI} : \left(2.9 \pm 2.5758 \sqrt{\frac{2.9}{1000}}\right)$$

$$= (2.7613, 3.0387$$

Q11 $\Sigma x = 213200$

$\Sigma x^2 = 4919860000$

Sample Mean = 17767

Sample Standard Deveiance = 10144

Salary of 2 tempory employees:
11,000 and 48,000

Salary of 2 randomly selected permanent:
27500 and 31,500

$$\text{Revised Sample Mean} = \frac{(213200 - 11{,}000 - 48{,}000 + 27{,}500 + 31{,}500)}{12}$$

$$= 17767$$

$$\text{Revised } \Sigma x^2 = 4919860000 - 121000000 - 2304000000 + 756250000 + 992250000$$

$$= 4243360000$$

$$\text{Revised sample } s^2 = \sqrt{\frac{1}{11}\left(\text{Revised } \Sigma x^2 - \frac{(\text{Revised } \bar{x})^2}{n}\right)}$$

$$= \sqrt{\frac{1}{11} \times \left(4243360000 - \frac{213200^2}{12}\right)}$$

$$\doteq \sqrt{\frac{1}{11} \times 455506666.7}$$

$$= (41409696.97)^{1/2}$$

$$= 6435$$

Comment :

Sample mean remains same.
This is because revised Σx is same as initial Σx
The sample standard deviation is reduced because the revised observation is near sample mean

Q12.

(i) The Cramer - Rao Lower Bound result for variance of unbiased estimators ~~is~~ of θ does not apply in this case because the interval of Uniform Distribution contains parameters.

(ii) $Y = \max X$
For $0 < y < \theta$
$P[Y < y] = P[\max X < y]$
If we assume X_i values are independent.

$$= \prod P[X_i < y]$$

$$= \frac{y}{\theta} \times \frac{y}{\theta} \ldots$$

$$= \left(\frac{y}{\theta}\right)^n$$

$$\text{p.d.f} = \frac{d}{dy}\left(\frac{y}{\theta}\right)^n = \frac{n}{\theta^n} y^{n-1}$$

$$E[Y^k] = \int_0^\theta y^k \frac{n}{\theta^n} y^{n-1} dy$$

$$= \frac{n}{n+k} \int_0^\theta \frac{n+k}{\theta^n} y^{n+k-1} dy$$

$$= \frac{n}{n+k} \frac{\theta^{n+k} - 0}{\theta^n}$$

$$= \frac{n\theta^k}{n+k}$$

(iii) $\text{Bias}[\hat{\theta}] = E[\hat{\theta} - \theta]$

$$= E[cY - \theta]$$

$$= c \cdot E(Y) - \theta$$

$$= c \frac{n\theta}{n+1} - \theta$$

$$= \left(\frac{cn}{n+1} - 1\right) \theta$$

$$MSE = Var(\hat{\theta}) + bias[\hat{\theta}]$$

$$= E\left[(\hat{\theta} - \theta)^2\right]$$

$$= E[(cY - \theta)^2]$$

$$= E[c^2Y^2 - 2c\theta Y + \theta^2]$$

$$= c^2 E(Y^2) - 2c\theta E(Y) + \theta^2$$

$$= c^2 \frac{n\theta^2}{n+2} - c \frac{2n\theta^2}{n+1} + \theta^2$$

(iv) Bias $[\hat{\theta}(c_u)] = 0$

$$\left(\frac{c_u n}{n+1} - 1\right) \cdot \theta = 0$$

(v) Minimize : MSE $[\hat{\theta}(c)]$

$$\frac{d}{dc} \text{MSE} [\hat{\theta}(c)] = 0$$

$$\frac{d}{dc}\left[c^2 \frac{n(\theta)^2}{n+2} - c\,\frac{2n\theta^2}{n+1} + \theta^2\right] = 0$$

$$2c \cdot \frac{n\theta^2}{n+2} - \frac{2n\theta^2}{n+1} = 0$$

$$c = \frac{2n\theta^2}{n+1} \times \frac{(n+2)}{n\theta^2}$$

$$= \frac{n+2}{n+1}.$$

$$\frac{d^2}{dc^2} \text{MSE}[\hat{\theta}(c)] = \frac{d}{dc}\left[2c \cdot \frac{n\theta^2}{n+2} - \frac{2n\theta^2}{n+1}\right]$$

$$= \frac{2n\theta^2}{n+2} > 0$$

$\therefore$ It is minimum

$$c = \frac{n+2}{n+1}$$

(vi) lower MSE means better estimator.
Therefore $\hat{\theta}(Cm)$ is better than $\hat{\theta}(Cu)$

$$\hat{\theta}(cu) = \left(\frac{n+1}{n}\right)Y$$

$$\hat{\theta}(cm) = \left(\frac{n+2}{n+1}\right)Y$$

if $n \to \infty$ (n is large) then $\hat{\theta}(cu)$ &
$\hat{\theta}(cm)$ ~~is~~ tends to Y