Assignment II Calculus

1) $$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

Sol^n: let $e^{2/w} = t$

$w = 1/50 \Rightarrow t=$
$w = 2 \Rightarrow t=$

$$-2/w^2 = dt/dw$$

$$\frac{dw}{w} = -\frac{dt}{2}$$

$$\int_{100}^{1} -e^t \frac{dt}{2}$$

$$= \frac{-1}{2}\left[e^1 - e^{100}\right]$$

$$= \frac{1}{2}\left[e^{100} - e^1\right]$$

2) $$\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$$

$t^4 + 2t = x$

$4t^3 + 2 = dx/dt$

$(2t^3+1)\,dt = \frac{dx}{2}$

$$= \int \frac{dx}{2x^3} = \frac{1}{2}\left[\frac{x^{-2}}{-2}\right]$$

$$= \frac{-1}{4(t^4+2t)^2}$$

3) $f(x) = \int f''(x)\,dx$

Sol^n $f(x) = \int (x^4 + 3x - 9)\,dx$

$f(x) = \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + C$

4) $f(x) = \begin{cases} 6, & \text{if } x > 1 \\ 3x^2, & \text{if } x \leq 1 \end{cases}$

Sol^n $\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

$= [x^3]_{-2}^{1} + [6x]_{1}^{3}$

$= 9 + 12$

$= 21$

5) $\int 3(8y-1)\, e^{4y^2-y}\, dy$

Solⁿ: let $4y^2 - y = t$

$(8y-1)\,dy = dt$

$= 3\int e^t\, dt$

$= 3e^{4y-y^2} + C$

6) $f''(x) = 15\sqrt{x} + 5x3 + 6$

Solⁿ: $f(x) = \int\int (15\sqrt{x} + 5x^3 + 6)\, dx\, dx$

$= \int\left(\frac{15x^{3/2}}{3/2} + \frac{5}{4}x^4 + 6x\right) dx$

$= \int\left(10x^{3/2} + \frac{5}{4}x^4 + 6x\right) dx$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C$

classmate

Date

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8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r = 1$

$$\int \frac{dr}{x^2} = \int \frac{d\theta}{\theta}$$

$$\frac{-1}{r} = \log \theta + C$$

$r = 1$, $\theta = 2$

$$\frac{-1}{(1)} = \log(2) + C$$

$$C = -3.321928$$

$$\frac{-1}{r} = \log \theta - 3.321928$$

9) ~~Find~~

Sol^n $\frac{dv}{dt} = 9.8 - 0.196v$

$$\int \frac{dv}{9.8 - 0.196v} = \int dt$$

$$\int \frac{1}{(9\sqrt{5}/1)^2 - (4/10\sqrt{10})^2} dv = t + C$$

$$\frac{1}{2 + \sqrt{5}/\sqrt{5}} \ln\left[\frac{7\sqrt{5}/5 + 7\sqrt{104}/50}{7\sqrt{5}/5 - 7\sqrt{104}/50}\right] = t + C$$

$$\log\left[\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}}\right] = \frac{14}{\sqrt{5}} t + C$$

10) $ty' + 2y = t^2 - t + 1$ $\quad y(1) = 1/2$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$$

$$\frac{dy}{dt} = t - 1 + \frac{1}{t} - \frac{2t}{t}$$

11) $\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\cancel{\int \frac{dy}{dx}}$ $\int \frac{dy}{y^3} = \int \frac{x\,dx}{\sqrt{1+x^2}}$, let $1+x^2 = t$

$x\,dx = dt$

$\frac{-1}{2y^2} = \frac{1}{2}\int \frac{t}{t^{1/2}}$

$\frac{-1}{2y^2} = \sqrt{1+x^2} + C$

$\frac{-1}{2(-1)^2} = 1 + C$ $y(0) = -1$

$C = -3/2$ $\therefore \frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$

12) $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

Sol^n

$f(x) = x^3 - 10x^2 + 6$ $f(3) = 27 - 90 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $f'(3) = 3(3)^2 - 20(3) = -33$

$f''(x) = 6x - 20$ $f''(3) = 18 - 20 = -2$

$f'''(x) = 6$ $f'''(3) = 6$

$x^3 - 10x^2 + 6 = (-57) + (x-3)(-33) + (x-3)^2(2) + (x-3)^3$

13) Maclaurin series for $(1+x)^n$

Sol^n

$f(x) = (1+x)^n$	$f(0) = 1$
$f'(x) = n(1+x)^{n-1}$	$f'(0) = n$
$f''(x) = n(n-1)(1+x)^{n-2}$	$f''(0) = n(n+1)$
$f'''(x) = n(n-1)(n-2)(1+x)^{n-3}$	$f'''(0) = n(n+1)(n-2)$

$$f(x) = 1 + \sum_{1=1}^{n} \frac{n\,x^n(n-n)}{n!}$$

14) $f(x) = \sqrt{1+x}$ .. Maclaurian series

Sol^n

$f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}}$ $\qquad f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}}$ $\qquad f''(0) = \dfrac{-1}{4}$

$f'''(x) = \dfrac{3}{8(1+x)^{5/2}}$ $\qquad f'''(0) = \dfrac{3}{8}$

$$f(x) = 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{16} + \dots$$

15) Taylor Series

$f(x) = e^{-6x}$

$f'(x) = (-6)e^{-6x}$

$f''(x) = 36e^{-6x}$

$f(-4) = e^{24}$

$f'(x) = -6e^{24}$

$f''(x) = 36e^{24}$

$$f(x) = e^{24} - (x+4)6e^{24} + (x+4)^2 \times 18e^{24} + \ldots$$

16) Taylor Series

Solⁿ

$f(x) = \ln(3+4x)$

$f(0) = \ln(3)$

$f'(x) = \dfrac{1}{3+4x}$

$f'(x) = \dfrac{1}{3}$

$f''(x) = \dfrac{-4}{(3+4x)^2}$

$f''(0) = \dfrac{-4}{9}$

$f'''(x) = \dfrac{8}{(3+4x)^3}$

$f'''(0) = \dfrac{8}{27}$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{\cancel{4}^{2}x^2}{\cancel{2}\times 9} + \frac{\cancel{8}^{4}x^3}{27\times\cancel{6}3} + \ldots$$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{2x^2}{9} + \frac{4x^3}{81} + \ldots$$