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Assignment - 1

1) ~~Radius~~ Radius $= 12.65$

~~axis~~ Actual Radius $= 12.5$

i) Absolute Error $= 12.65 - 12.5$

$= 0.15$

ii) Relative Error $= \dfrac{(12.65 - 12.5)}{12.5}$

$= 0.012$

iii) Percentage Error $= \dfrac{(12.65 - 12.5)}{(12.5)} \times 100$

$= 0.012 = 1.2\,\%$

2)

x	$f(x) = \log x$
4	0.60206
4.5	0.6532
5.5	0.7403
6.0	0.7782

i) $\dfrac{x - x_1}{x_1 - x_2} = \dfrac{y - y_1}{y_1 - y_2}$

$\dfrac{5-4}{4-6} = \dfrac{y - 0.60206}{0.60206 - 0.7782}$

$y = 0.69011$

2) ii) $\dfrac{x-x_1}{x_1-x_2} = \dfrac{y-y_1}{y_1-y_2}$

$\dfrac{5-4.5}{4.5-5.5} = \dfrac{y-0.65321}{0.65321-0.74036}$

$y = 0.69679$

3) $x^4 - x - 10 = 0$

$a\ \cancel{x} = 1.5, \overset{b}{\cancel{y}} = 2$

$\cancel{a}\ x$	$\cancel{b}\ y$
1.5	-6.4375
2	4

$x_1 = \dfrac{1.5+2}{2} = 1.75$

$\therefore y = -2.37109$

x	y
1.75	-2.371
2	4

$x_2 = \dfrac{1.75+2}{2} = 1.875$

$\therefore y = 0.48462$

x	y
1.75	-2.371
1.875	0.4862

$x_3 = \dfrac{1.75+1.875}{2} = 1.8125$ | $\therefore y = -1.0202$

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3) ~~x_4~~

x	y
1.8125	-1.0202
1.875	0.48462

$$x_4 = \frac{1.8125 + 1.875}{2} = 1.84375$$

$y = -0.287734$

x	y
1.84375	-0.287734
1.875	0.48462

$$x_5 = \frac{1.84375 + 1.875}{2} = 1.859375$$

$\therefore y = 0.093378$

4) $f(x) = x^3 - x - 1$; $f'(x) = 3x^2 - 1$

x	y
1	-1
2	5

$$x_1 = 1 - \frac{(-1)}{2}$$

$x_1 = 1.5$

$f(x_1) = 0.875$

$f'(x_1) = 5.75$

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$$x_2 = 1.5 - \frac{0.875}{5.75}$$

$x_2 = 1.347826$ 6)

$f(x_2) = 0.100682$

$f'(x_2) = 4.44991$

$$x_3 = 1.3478 - \frac{0.100682}{4.44991}$$

$= 1.3252$

$f(x_3) = 0.002058$

$f'(x_3) = 4.26847$

$$x_4 = 1.3252 - \frac{0.002058}{4.26847}$$

$= 1.3247182$

$f(x_4) = 0.000000924$

5) $A^2 = A$

$7A - (I + A)^3 = 7A - (I^3 + 3I^2 A + 3IA^2 + A^3)$

$= 7A - I^3 - \cancel{3I^2} - 3AI^2 - 3IA^2 - A^2.A$

$= 7A - I^3 - 3A - 3A - A.A$

$= 7A - I - 6A - A^2$

$= 7A - I - 7A$

$= -I$

6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$= 1(-6) + 1(-4-0) + 2(4) = -2$

$A_{minor} = \begin{bmatrix} -6 & -4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix}$

&

$A_{cofactor} = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$

$\Rightarrow$

$\text{Adjacent } A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$\therefore A^{-1} = -\frac{1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -0.5 & 3 \\ -2 & 0.5 & -1 \\ -2 & 0.5 & -2 \end{bmatrix}$

Subject :

7) $A = 8\hat{i} - 9\hat{j}$

$B = 7\hat{i} - 9\hat{j}$

$|A| = \sqrt{8^2 + 9^2}$
$= \sqrt{145}$
$= 12.04159$

$|B| = \sqrt{7^2 + 9^2}$
$= \sqrt{130}$
$= 11.40175$

$A \cdot B = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$
$= (8 \times 7) + (-9 \times -9)$
$= 56 + 81$
$= 137$

$A \cdot B = |A||B| \cdot \cos\theta$
$137 = |12.041| \cdot |11.40175| \cdot \cos\theta$
$137 = 5\sqrt{754} \times \cos\theta$
$\cos\theta = \frac{137}{5\sqrt{754}}$
$\cos\theta = 0.997849$
$\theta = \cos^{-1}(0.997849)$
$\theta = 3.6243$

8) $\text{magnitude} = \sqrt{(75)^2 + (25)^2}$
$= \sqrt{5625 + 625}$
$= \sqrt{6250}$
$= 79.057$

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9) $a = 101$; $a_n = 98$; $d = 3$

$\therefore a_n = a + (n-1)d$

$98 = 101 + (n-1)3$

$897 = (n-1)3$

$n-1 = 299$

$n = 300$

$S_{300} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{300}{2}(2(101) + \cancel{(n-1)}(299)(3))$

$= 164850$

10) $a_6 = ar^5 = 32$

$a_8 = a.r^7 = 128$

$\frac{a_8}{a_6} = \frac{a.r^7}{a.r^5}$

$\frac{128}{32} = \frac{a.r^7}{a.r^5}$

$4 = r^2$

$r = 2$

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11) $(4+3x)^5$

$= {}^5C_0 . 4^5 + {}^5C_1 .(4)^4 .(3x) + {}^5C_2 .(4)^3 .(3x)^2 + {}^5C_3 .(4)^2 (3x)^3$

$+ {}^5C_4 . 4 . (3x)^4 + {}^5C_5 (3x)^5$

$= 1024 + 1280(3x) + 10(64)(9x^2) + 10(16)(27x^3)$

$+ 5(4)(81x^4) + \cancel{243(} \; 243x^5$

$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

12) $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} = A$

$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = 0$

$(2-\lambda)\left[(-5-\lambda)(3-\lambda)\right] + 3\left[2(3-\lambda)\right] = 0$

$(2-\lambda)(\overset{-5-\lambda}{\cancel{-5+\lambda}})(3-\lambda) + 6(3-\lambda) = 0$

$(3-\lambda)\left[(2-\lambda)(-5-\lambda) + 6\right] = 0$

$(3-\lambda)\left[-10-2\lambda+5\lambda+\lambda^2+6\right] = 0$

$(3-\lambda)\left[\lambda^2+3\lambda-4\right] = 0$

$(3-\lambda)\left[\lambda^2+4\lambda-\lambda-4\right] = 0$

$(3-\lambda)\left[(\lambda+4)(\lambda-1)\right] = 0$

$\therefore (3-\lambda)(\lambda+4)(\lambda-1) = 0$

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12) $3-\lambda=0$, $\lambda=3$ | $\lambda+4=0$, $\lambda=-4$ | $\lambda-1=0$, $\lambda=1$

Eigen values of A are $3, -4, 1$

Eigen vectors of A ~~when~~ are :

$(A-\lambda I)x=0$

$$\left(\begin{bmatrix}2 & -3 & 0\\ 2 & -5 & 0\\ 0 & 0 & 3\end{bmatrix} - \begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix}\right)\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

As we know, $\lambda=-4$

$$\begin{bmatrix}2+4 & -3 & 0\\ 2 & -5+4 & 0\\ 0 & 0 & 3+4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

$$\begin{bmatrix}6 & -3 & 0\\ 2 & -1 & 0\\ 0 & 0 & 7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

$R_1 = R_1 - 3R_2$

$$\begin{bmatrix}0 & 0 & 0\\ 2 & -1 & 0\\ 0 & 0 & 7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = 0$$

$$\begin{bmatrix}0\\ 2x_1-x_2\\ 7x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$7x_3=0$

$\therefore x_3 = 0 \quad -(1)$

~~Substitute $x_3=0$~~

$2x_1 - x_2 = 0$

$2x_1 = x_2 \quad -(2)$

$\therefore$ Eigen vectors for $\lambda=-4$ are $\begin{bmatrix}x_1\\ 2x_1\\ 0\end{bmatrix}$

12) ~~Assume~~ $\lambda = 1$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$R_2 = R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} x_1 - 3x_2 \\ 0 \\ 2x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 3x_2 = 0$

$x_1 = 3x_2 \quad -①$

$2x_3 = 0$

$x_3 = 0 \quad -②$

$\therefore$ Eigen vectors when $\lambda = 1$ are $\begin{bmatrix} 3x_2 \\ x_2 \\ 0 \end{bmatrix}$

12) $\lambda = 3$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 - 3x_2 = 0$

$x_1 = -3x_2 \quad - ①$

$2x_1 - 8x_2 = 0$

Substitute $x_1 = -3x_2$ in ~~equation ①~~ this

$2(-3x_2) - 8x_2 = 0$

$-6x_2 - 8x_2 = 0$

$-14x_2 = 0$

$x_2 = 0 \quad - ②$

Substitute $x_2 = 0$ in equation ①

$x_1 = -3x_2$

$x_1 = -3(0)$

$x_1 = 0 \quad - ③$

$\therefore$ Eigen vectors when $\lambda = 3$ are $\begin{bmatrix} 0 \\ 0 \\ \text{undefined} \end{bmatrix}$

Subject :

13) $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(x_0) = (5)^3 - 7(5)^2 + 8(5) - 3 = -13$

$f'(x_0) = 3(5)^2 - 14(5) + 8 = 13$

$$x_{Formula} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 5 - \frac{(-13)}{13} = 6$$

$f(x_1) = (6)^3 - 7(6)^2 + 8(6) - 3 = 9$

$f'(x_1) = 3(6)^2 - 14(6) + 8 = 32$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - \frac{9}{32} = 5.71875$$

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14) $(1-x+x^2)^4$

Use Binomial Therom

$= [(x^2-x)+1]^4$

$[(x^2-x)+1]^4 = {}^4c_0(x^2-x)^4(1)^0 + {}^4c_1(x^2-x)^3(1)^1 + {}^4c_2(x^2-x)^2(1)^2$
$+ {}^4c_3(x^2-x)^1(1)^3 + {}^4c_4(x^2-x)^0(1)^4$

$= (x^2-x)^4 + 4(x^2-x)^3 + 6(x^2-x)^2 + 4(x^2-x) + 1$

$= [{}^4c_0(x^2)^4(-x)^0 + {}^4c_1(x^2)^3(-x)^1 + {}^4c_2(x^2)^2(-x)^2 + {}^4c_3(x^2)^1(-x)^3$
$+ {}^4c_4(x^2)^0(-x)^4] + 4[{}^3c_0(x^2)^3(-x)^0 + {}^3c_1(x^2)^2(-x)^1 + {}^3c_2(x^2)^1(-x)^2 \cancel{{}^3c_3(x^2)^0}$
$+ {}^3c_3(x^2)^0(-x)^3] + 6[x^4 - 2x^3 + x^2] + 4x^2 - 4$

$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4(x^6 - 3x^5 + 3x^4 - x^3) + 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4$

$= x^8 - 4x^7 + 6x^6 - 4x^5 + x^4 + 4x^6 - 12x^5 + 12x^4 - 4x^3 + 6x^4 - 12x^3 + 6x^2 + 4x^2 - 4$

$= x^8 - 4x^7 + \overset{10}{\cancel{6}}x^6 - \overset{16}{\cancel{4}}x^5 + \cancel{x}\,19x^4 - 16x^3 + 10x^2 - 4$

$\therefore (1-x+x^2)^4 = x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4$

15) $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

x	y
1	0.368
2	-0.768

$x_1 = \frac{x_1 + x_2}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$y_1 = e^{-1.5} - 3\log(1.5) = -0.30514$

x	y
1	0.368
1.5	-0.305

Subject:

15) $x_2 = \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$

$y_2 = e^{-1.25} - 3\log(1.25) = -0.00423$

$x_3 = \frac{1+1.25}{2} = 1.125$

$y_3 = e^{-1.125} - 3\log(1.125) = 0.1712$

$x_4 = \frac{1.125+1.25}{2} = 1.1875$

$y_4 = e^{-1.1875} - 3\log(1.1875) = 0.08108$

$x_5 = \frac{1.1875+1.25}{2} = 1.21875$

$y_5 = e^{-1.21875} - 3\log(1.21875) = 0.037856$