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Calculus Assignment.

A 1. $\lim_{h \to 0} \frac{2(-3+h)^2 - 18}{h}$

$= \lim_{h \to 0} \frac{2(9-6h+h^2) - 18}{h}$

$= \lim_{h \to 0} \frac{\cancel{18} - 12h + 2h^2 - \cancel{18}}{h}$

$= \lim_{h \to 0} \frac{-12h + 2h^2}{h}$

$= \lim_{h \to 0} \frac{2\cancel{h}(-6+h)}{\cancel{h}}$

$= \lim_{h \to 0} 2(-6+h)$

$= 2(-6+0)$

$= -12.$

a.) $g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$

a) $x = 4$,

b.) $x = 6$.

a.) $g(4) = 2(4)$
$= 8.$ $\therefore$ function is continuous at $x = 4$

b.) $g(6) = 6-1$
$= 5$ $\therefore$ function is continuous at $x = 6$

3.) $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}} = \lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}} \times \dfrac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x \to 9} \dfrac{(x-9)}{(3-\sqrt{x})}$

$= \lim_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim_{x \to 9} \dfrac{(x-9)(3+\sqrt{x})}{9-x}$

$= \lim_{x \to 9} \dfrac{-1(9-x)(3+\sqrt{x})}{9-x}$

$= \lim_{x \to 9} -1(3+\sqrt{x})$

$= -1(3+\sqrt{9})$

$= -(3+3)$

$= -6$

4.) $f(x) = \dfrac{4x+5}{9-3x}$

a.) $f(-1) = \dfrac{4(-1)+5}{9-3(-1)}$

$= \dfrac{-4+5}{9+3}$

$= \dfrac{1}{12}$ $\therefore$ function is continous at $x=-1$

a.) $f(0) = \dfrac{4(0)+5}{9-3(0)}$

$= \dfrac{5}{9}$ $\therefore$ function is continuous at $x=0$

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$(3+\sqrt{x})$
$(3+\sqrt{x})$

c.) $f(3) = \frac{4(3)+5}{9-3(3)}$

$= \frac{17}{0}$ $\therefore$ function is discontinuous at $x=3$.

5.) $\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$= \lim_{x \to \infty} \frac{e^{4x}(6 - e^{-6x})}{e^{4x}(8 - e^{-2x} + 3e^{-6x})}$

$= \frac{6 - \frac{1}{e} 6(\infty)}{8 - \frac{1}{2} e(\infty) + 3\left(\frac{1}{e5\infty}\right)}$

$= \frac{6 - 1/\infty}{8 - 1/\infty + 3/\infty}$

$= \frac{3}{4}$

6.) $g(y) = \begin{cases} y^2+5 & y < -2 \\ 1-3y & y \geq -2 \end{cases}$

a.) $\lim_{y \to 6} g(y) = 1-3(6)$
$= 1-18$
$= -17$

b.) $\lim_{y \to -2} g(y) = 1-3(-2)$
$= 1+6$
$= 7$

7.) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

$f'(t) = (4t^2 - t) \cdot \frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12) \cdot \frac{d}{dt}(4t^2 - t)$

$= (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$

$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$

$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$

$= 4(5t^4 - 33t^3 + 6t^2 + 24t - 3)$

8.) $R(\omega) = \dfrac{3\omega + \omega^4}{2\omega^2 + 1}$

$R'(\omega) = \dfrac{2\omega^2 + 1 \frac{d}{d\omega}(3\omega + \omega^4) - (3\omega + \omega^4) \cdot \frac{d}{d\omega}(2\omega^2 + 1)}{(2\omega^2 + 1)^2}$

$= \dfrac{2\omega^2 + 1 (3 + 4\omega^3) - (3\omega + \omega^4)(4\omega)}{(2\omega^2 + 1)^2}$

$= \dfrac{6\omega^2 + 8\omega^5 + 3 + 4\omega^3 - 12\omega^2 - 4\omega^5}{(2\omega^2 + 1)^2}$

$= \dfrac{4\omega^5 + 4\omega^3 - 6\omega^2 + 3}{(2\omega^2 + 1)^2}$

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9.) $f(x) = 2e^{x} - 8^{x}$

$f'(x) = 2e^{x} - 8^{x}\log 8.$

10.) $y = x^{5} - e^{x}\ln(x)$

$$\frac{dy}{dx} = 5x^{4} - \left[e^{x}\cdot\frac{d}{dx}(\ln x) + \ln x\cdot\frac{d}{dx}e^{x}\right]$$

$$= 5x^{4} - \left[e^{x}\left(\frac{1}{x}\right) + \ln x\,(e^{x})\right]$$

$$= 5x^{4} - \frac{e^{x}}{x} - \ln x(e^{x})$$

11.) a.) $f(x) = (6x^{2} + 7x)^{4}$

$$f'(x) = 4(6x^{2}+7x)^{3}\cdot\frac{d}{dx}(6x^{2}+7x)$$

$$= 4(6x^{2}+7x)^{3}\,(16x+7)$$

b.) $f(t) = 5 + e^{4t + t^{7}}$

$$f'(t) = e^{4t+t^{7}}\cdot\frac{d}{dt}(4t + t^{7})$$

$$= e^{4t+t^{7}}\cdot(4 + 7t^{6})$$

12.) a.) $z = \ln(7 - x^{3})$

$$\frac{dz}{dx} = \frac{1}{7-x^{3}}\cdot\frac{d}{dx}(7-x^{3})$$

$$= \frac{1}{7-x^{3}}(-3x^{2})$$

$$= \frac{-3x^{2}}{7-x^{3}}$$

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$$\frac{d^2 x}{dx^2} = \frac{(7-x^3)\cdot\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

b.) $g(v) = \frac{2}{(6+2v-v^2)^{-4}}$

$\cancel{g'(v)} = 2(6+2v-v^2)^{-4}$

$g'(v) = 2(-4)(6+2v-v^2)^{-5} \frac{d}{dv}(6+2v-v^2)$

$= -8(6+2v-v^2)^{-5}(2-2v)$

$g''(v) = -8\left[(16+2v-v^2)^{-5} \frac{d}{dv}(2-2v) + (2-2v)\frac{d}{dv}(6+2v+v^2\right.$

$= -8\left[(6+2v-v^2)^{-5}(-2) + (2-2v)(2-2v)\right]$

$= 16\left[(6+2v-v^2)^{-5} + 40(6+2v-v^2)^{-6}(2-2v)\right.$

c.) $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{(1+t^2)} \cdot \frac{d}{dt}(1+t^2)$

$= \frac{2t}{1+t^2}$

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$$f''(t) = \frac{(1+t^2)(2)-(2t)(2t)}{(1+t^2)^2}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

$$= \frac{2-2t^2}{(1+t^2)^2}$$

$$= \frac{2(1-t^2)}{(1-t^2)^2}$$

13.) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9.$

Put $f'(x) = 0$.

$3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0.$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0$ or $x + 1 = 0.$

$x = 3$ $\quad$ $x = -1$

$f''(x) = 6x - 6$

Put $x = 3$ in $f''(x)$

$f''(3) = 6(3) - 6.$

$= 18 - 6$

$= 12$

$\therefore f(x)$ is minimum at $x = 3.$

Put $x = -1$ in $f''(x)$

$f''(x-1) = 6(-1) - 6$

$= -6 - 6$

$= -12$

$\therefore f(x) =$ is maximum at $x = -1$

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Put $x = 3$ in $f(x)$.

$\therefore$ Minimum value $= (3)^3 - 3(3)^2 - 9(3) + 12$.

$= 27 - 3(9) - 27 + 12$

$= 27 - 27 - 27 + 2$

$= -15$

Put $x = 1$ in $f(x)$

$\therefore$ Maximum value $= (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3(1) + 9 + 12$.

$= 17$.

14.) $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

Put $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$12(x^2 - 3x + 2) = 0$.

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-2)(x-1) = 0$

$x - 2 = 0$ or $x - 1 = 0$

$x = 2$ $\quad$ $x = 1$

$f''(x) = 24x - 36$.

Put $x = 2$ in $f''(x)$ $\quad$ Put $x = 1$ in $f''(x)$

$f''(2) = 24(2) - 36$ $\quad$ $f''(1) = 24(1) - 36$

$= 48 - 36$ $\quad$ $= 24 - 36$

$= 12$. $\quad$ $= -12$.

$\therefore$ $f(x)$ is maximum at $x = 2$ $\quad$ $\therefore$ $f(x)$ is maximum at $x = 1$

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Put $x = 2$ in $f(x)$

$\therefore$ Maximum value $= 4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 4(8) - 18(4) + 42 - 7$

$= 32 - 72 + 48 - 7$

$= 1$

Put $x = 1$ in $f(x)$

$\therefore$ Maximum value $= 4(1)^3 - 18(1)^2 + 24(1) - 7$

$= 4 - 18 + 24 - 7$

$= 3$

15) $f(x) = x^2(x+3)$

$f'(x) = 2x(x+3) + (1)x^2$

$= 2x^2 + 6x + x^2$

$= 3x^2 + 6x$

Put $f'(x) = 0$

$3x^2 + 6x = 0$

$3(x^2 + 2x) = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0.$ or $x = -2.$

$f''(x) = 6x^2 + 6,$

Put $x = 0$ in $f''(x)$ $\therefore f''(0) = 6(0) + 6 = 6$

Put $x = -2$ in $f''(x)$ $\therefore f''(-2) = 6(-2) + 6 = -6$

$\therefore$ $f(x)$ is maximum at $x = 4$ & minimum at $x = 6$

Put $x = -1$ in $f'(x)$

$\therefore$ $f'(-7) = 3(-1)^2 + 6(-1)$

$= 3 - 6$

$= -3.$

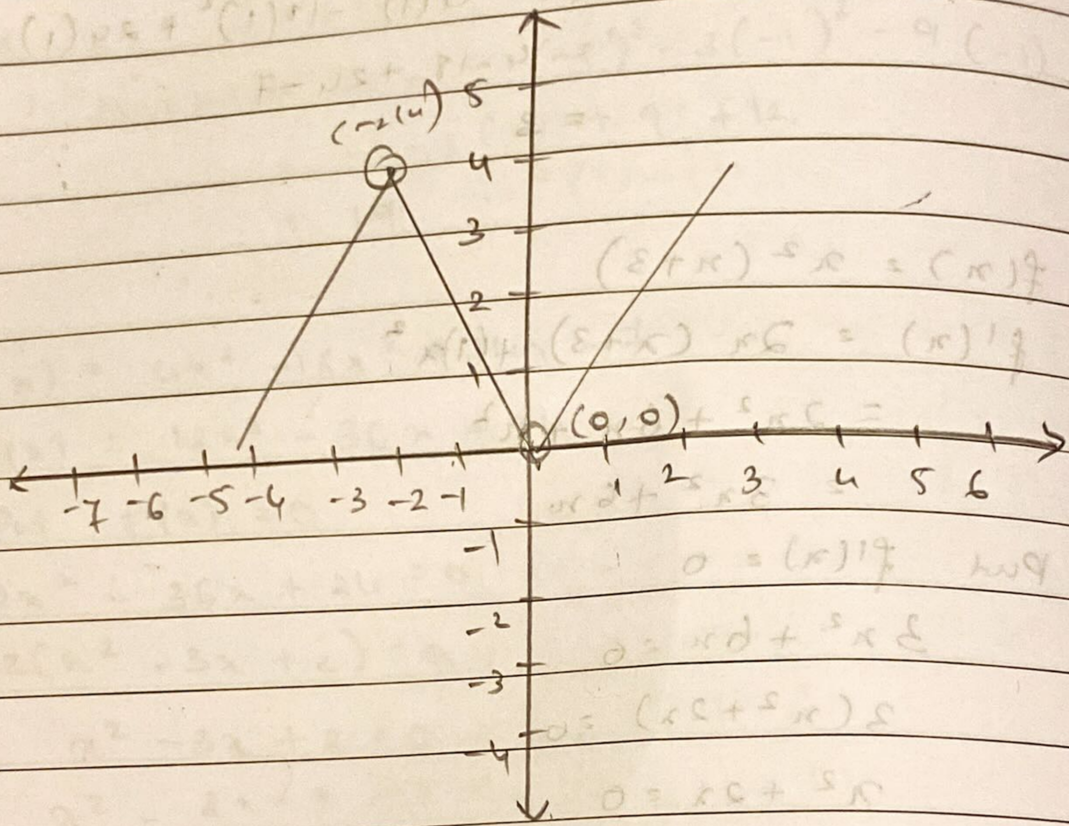
the given function is decreasing between -2 & 0

put $x=0$ in $f(x)$

Minimum value $= f(0) = 0(0+3) = 0 \therefore x = 0, y = 0.$

Put $x=-2$ in $f(x)$

Maximum value $= f(-2) = (-2)^2(-2+3) = 4(1) = 4 \therefore x = -2$



6.) $f(x) = 3x^2 - 6x$

i.e $y = 3x^2 - 6x$

Put $x=0$ in $f(x)$

$y = 3(0)^2 - 6(0) = 0$

Put $x=-1$ in $f(x)$

$y = 3(-1)^2 - 6(-1)$

$= 3+6$

$= 9$

Put $x=1$ in $f(x)$

$\therefore y = 3(1)^2 - 6(-1) = 3 - 6 = -$

Put $x=2$ in $f(x)$

$\therefore y = 3(2)^2 - 6(2)$

$= 12 - 12$

$= 0$

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Put $x = -2$ in $f(x)$

$\therefore y = 3(2)^2 - 6(2)$

$= 12 - 12$

$= 0$

Put $x = 3$ in $f(x)$

$y = 3(3)^2 - 6(3)$

$= 3(9) - 18.$

$= 9$

Put $x = -3$ in $f(x)$

$y = 3(-3^2 - 6(-3)$

$= 3(9) + 18$

$= 45$