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# Assignment

1. Development year

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1245 | 999 | 610 | 356 |
| 2002 | 1455 | 1088 | 723 | |
| 2003 | 1567 | 1079 | | |
| 2004 | 1491 | | | |

Inflation Index

| | |
|---|---|
| 2001 | $(1.021)(1.012)(0.992) = 1.02498$ |
| 2002 | $1.012 \times 0.992 = 1.003904$ |
| 2003 | 0.992 |
| 2004 | 1 |

Incremental claim mid 2004 prices

DY

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| | $1245 \times 1.02498$ | | | |
| 2001 | 1276.11 | 1002.9 | 605.415 | 356 |
| 2002 | 1460.68 | 1079.30 | 723 | |
| 2003 | 1554.46 | 1079 | | |
| 2004 | 1491 | | | |

Cummulative claim 2004 prices

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1276.11 | 2279.01 | 2884.13 | 3240.13 |
| 2002 | 1460.68 | 2539.98 | 3262.98 | |
| 2003 | 1554.46 | 2633.46 | | |
| 2004 | 1491 | | | |
| Total claim | 5782.25 | 7452.45 | 6147.11 | 3240.13 |
| −(last claim) | (1491) | (2638.40) | (3262.98) | (0) |

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Assignment

1. Pdevopment year

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1245 | 999 | 610 | 356 |
| 2002 | 1455 | 1088 | 723 | |
| 2003 | 1567 | 1074 | | |
| 2004 | 1491 | | | |

Inflation Index

| | |
|---|---|
| 2001 | $(1.021)(1.012)(0.992) = 1.02493$ |
| 2002 | $1.012 \times 0.992 = 1.003904$ |
| 2003 | 0.992 |
| 2004 | 1 |

Incremental claim mid 2004 prices

PY

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | $1245 \times 1.02493$<br>1276.11 | 1002.9 | 605.415 | 356 |
| 2002 | 1460.68 | 1079.30 | 723 | |
| 2003 | 1554.46 | 1079 | | |
| 2004 | 1491 | | | |

Cummulative claim 2004 prices

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1276.11 | 2279.01 | 2884.13 | 3240.13 |
| 2002 | 1460.68 | 2539.98 | 3262.98 | |
| 2003 | 1554.46 | 2633.46 | | |
| 2004 | 1491 | | | |
| Total claim | 5782.25 | 7452.45 | 6147.11 | 3240.13 |
| −(last claim) | (1491) | (2638.40) | (3262.98) | (0) |
| | 4291.25 | 4818.94 | 2884.13 | 3240.13 |

| | | | | |
|---|---|---|---|---|
| Df | – | 1.73666 | 1.2756 | 1.12343 |

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | 1276.11 | 2279.01 | 2884.13 | 3240.13 |
| 2002 | 1460.68 | 2539.98 | 3262.98 | 3665.74 |
| 2003 | 1554.46 | 2633.46 | 3359.245 | 3723.89 |
| 2004 | 1491 | 2589.36 ~~2589.36~~ | 3302.99 | 3710.69 |

Incremental claim mid 2004 prices

| AY | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2001 | | | | |
| 2002 | | | | |
| 2003 | | | | |
| 2004 | | 1098.36 | 712.64 | 407.7 |
| Inflation | | 1.025 | 1.050625 | 1.076891 |
| Inflation adjusted | | 1125.819 | 748.72 | 439.05 |

Total claim = 2313.5875 (in thousands)

(ii) Ultimate ~~clm~~ amount = 525000 × 0.75

= 393,750

| | | | | |
|---|---|---|---|---|
| Cumulative DF | – | 2.48841 | 1.43364 | 1.12343 |
| Reverse | 1 | 1.12343 | 1.43364 | 2.48841 |
| 1/f | 1 | 0.89017 | 0.6978 | 0.4018 |
| 1 − 1/f | 0 | 0.1098 | 0.3022 | 0.5982 |

| DY | Emerging liability |
|---|---|
| 1 | $393.75 \times \left(\frac{1}{1.43364} - \frac{1}{2.48841}\right) = 116.51 \times 1.02 = 119$ |
| 2 | $393.75 \times \left(\frac{1}{1.12343} - \frac{1}{1.433}\right) = 757.24 \times 1.0504 = 7$ [illegible] |
| 3 | $393.75 \times \left(1 - \frac{1}{1.12343}\right) = 432.61 \times 1.07(?) = 465$ |

Total emerging liability = 2456.09 (in arrears)

(ii) Ultimate ~~clt~~ amount = 525000 × 0.75

= 3929.500

| Cumulative | | | | |
|---|---|---|---|---|
| DF | - | 2.48891 | 1.43304 | 1.12343 |
| Reverse | 1 | 1.2343 | 1.43304 | 2.48891 |
| 1/f | 1 | 0.81017 | 0.6978 | 0.4018 |
| 1 - 1/f | 0 | 0.18983 | 0.3022 | 0.5982 |

| DY | Emerging liability |
|---|---|
| 1 | $393.75 \times \left(\frac{1}{1.43304} - \frac{1}{2.48897}\right) = 1165.51 \times 1.025 = 1194.6$ |
| 2 | $393.75 \times \left(\frac{1}{1.12343} - \frac{1}{1.433}\right) = 757.24 \times 1.050625 = 795.5$ |
| 3 | $393.75 \times \left(1 - \frac{1}{1.2343}\right) = 432.61 \times 1.076891 = 465.8$ |

Total emerging liability = 2456.09 (in thousand)

Development year

| | 1 | 2 | 3 |
|---|---|---|---|
| 2005 | 3512 | 7210 | 9563 |
| 2006 | 2889 | 6723 | |
| 2007 | 3876 | | |

loss ratio = 87%

| Premium | Income |
|---|---|
| 2005 | 11041 |
| 2006 | 11314 |
| 2007 | 12549 |

Total claim paid = 20103000

| | 1 | 2 | 3 | ultimate |
|---|---|---|---|---|
| 2005 | 3512 | 7210 | 9563 | 9563 |
| 2006 | 2889 | 6723 | | |
| 2007 | 3876 | | | |
| r | 2.1769 | 1.3264 | 1 | |
| f | 2.8872 | 1.3264 | 1 | |
| $1-\frac{1}{f}$ | 0.6536 | 0.24608 | 0 | |

| AY | 2007 | 2006 | 2005 |
|---|---|---|---|
| EP | 12549 | 11314 | 11041 |
| IUL | 10917.63 | 9843.18 | 9605.67 |
| EL | 7135.76 | 2422.21 | 0 |
| RL | 3870 | 6723 | 9563 |
| UL | 110576 | 9145.21 | 9563 |

Total UL = 29713.97 × 1000

Reserves = 29713970 − 20103000

= 9610970

PY

| AY | 0 | 1 | 2 |
|---|---|---|---|
| 2005 | 82 | 121 | 156 |
| 2006 | 128 | 163 | |
| 2007 | 98 | | |

Total amount claim

| | 0 | 1 | 2 |
|---|---|---|---|
| 2005 | 43512 | 87740 | 129563 |
| 2006 | 62889 | 116723 | |
| 2007 | 73876 | | |

ACPC method

| AY | 0 | 1 | 2 | ultimate |
|---|---|---|---|---|
| 2005 | 530.634 (63.29%) | 720.744 (86.78%) | 836.532 (100%) | 836.532 |
| 2006 | 491.320 (59.543%) | 716.092 (86.781%) | | 825.1314 |
| 2007 | 753.836 (61.216) | | | 1221.4595 |

Ultimate claim no.

| AY | 0 | 1 | 2 | ultimate |
|---|---|---|---|---|
| 2005 | 82 (52.56%) | 121 (77.564%) | 156 (100%) | 156 |
| 2006 | 128 (60.964%) | 163 (77.564%) | | 210.109 |
| 2007 | 98 (56.738%) | | | 172.723 |

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Projected ult. loss

| A/Y | ACPC × no of claim | | |
| --- | --- | --- | --- |
| 2006 | 825.1714 × 210.945 | = | 173,908.9 |
| 2007 | 1221.4595 × 172.7239 | = | 210975.00 |
| 2008 | 830.532 × 156 | = | 129562.99 |

Total = 513946.9407

Assuming given data is incurred

outstanding claim = 513946.9407 − (73876 + 116723 + 129563)

= 193784.93

5.(i) $X \sim \exp(\lambda)$

$Y = kX \sim$ ~~$\lambda$~~ $\exp$

$P(Y < y) = P(kX < y)$

$= P(X < y/k)$

$= \int_0^{y/k} \lambda e^{-\lambda s}\, ds$

~~put $x = ks$~~

put $x = k$

$= \cancel{\int^{y/k}} = \int_0^{y} \lambda e^{-\frac{\lambda}{k}x} \frac{dx}{k}$

$= \int_0^{y} \lambda e^{-\lambda/k\, x} \frac{dx}{k}$

$= \int_0^{y} \frac{\lambda}{k} e^{+\lambda/k\, x}\, dx$

$Y \sim \exp(\lambda/k)$

(ii) $P(\text{loss in } 2006 > m) = e^{-\lambda m}$

$P(\text{loss in } 2007 > m) = e^{-\lambda m/k}$

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$$L = c \times e^{-4\lambda M} \times \prod_i (\lambda e^{-\lambda x_i}) \times e^{-6\lambda M/k} \times \prod \left(\frac{\lambda}{k} e^{-\lambda y_i/k}\right)$$

$$\cancel{L = C \times e^{-4\lambda M} \times \prod (\lambda e^{-\lambda x_i}) e^{-6\lambda M/k} \times}$$

$$l = \log L = -4\lambda M + 10\log\lambda - \lambda\Sigma x_i - 6\lambda M/k + 12\log\lambda - \lambda/k\,\Sigma y_i$$

$$= c' - 4\lambda M + 22\log\lambda - 13500\lambda - 6\lambda M/k - 17000\lambda/k$$

$$\frac{\partial l}{d\lambda} = -4M + \frac{22}{\lambda} - 13500 - \frac{6M}{k} - 17000/k$$

$$-4M + \frac{22}{\lambda} - 13500 - 6M/k - \frac{17000}{k} = 0$$

$$\lambda = \frac{22}{13500 + 17000/k + 4M + 6M/k}$$

$$\frac{\partial^2 l}{d\lambda} = \frac{-22}{\lambda^2} < 0$$

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(iii) a) $\hat{\lambda} = \dfrac{22}{13500 + 17000/1.1 + 4 \times 1600 + \frac{6 \times 1600}{1.1}}$

$= 0.000499$

(b) Expected payment

$$\int_0^m x \lambda e^{-\lambda x} dx + m \int_m^\infty \lambda e^{-\lambda x} dx$$

$$\left[-xe^{-\lambda x}\right]_0^m + \int_0^m e^{-\lambda x} dx + m\left[-e^{-\lambda x}\right]_m^\infty$$

$$= -me^{-\lambda m} + \left[-\frac{1}{\lambda} e^{-\lambda x}\right]_0^m + me^{-\lambda m}$$

$$= \frac{1}{\lambda}\left(1 - e^{-\lambda m}\right)$$

$$= \frac{1}{0.000499}\left(1 - e^{-0.000499 \times 1600}\right)$$

$= 1102.107$

Claim for 2008 ~ $\exp\left(\lambda / 1.05 \times 1.1\right)$

$$11.02.107 = \int_0^R x\mu e^{-\mu x}\,dx + R\int_R^{\infty} \mu e^{-\mu x}\,dx$$

$$= \frac{1}{\mu}\left(1 - e^{-\mu R}\right)$$

$$= \frac{1.05 \times 1.1}{0.0004999}$$

$$0.47614892 = 1 - e^{-0.00043223463R}$$

$$R = -\frac{1}{0.00043203462} \log\left(1 - 0.47014892\right)$$

$$= 1496.52$$

6. (i) Heterogeneous group as the parameters vary by patient. not at the overall level

(ii) (a) $E(\lambda) = 0.5 \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{6} \times 0.9$

$= 0.3$

$E(\lambda_i^2) = 0.5 \times 0.1^2 + \frac{1}{3} \times 0.3^2 + \frac{1}{6} \times 0.9^2$

$= 0.17$

$Var(\lambda_i) = 0.17 - 0.3^2 = 0.08$

(b) $E(S_i | \lambda_i) = (\lambda_i m_i) \cdot (Var(S_i | \lambda_i)) = \lambda_i m_2$

(c) $E(S_i) = E[E(S_i / \lambda_i)] = E(\lambda_i m_1)$

$= 0.3 m_1$

$= 0.3 \times 250 = 75$

(d) Mean $= 100 \times 75 = 7500$

Var $= 100 \times 23810$

$= 2381000$

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(iii) $var(\lambda_1) = 0.5 \times (0.2^2 + 0.4^2) - 0.3^2$

$= 0.01$

$E\left[\sum_{i=1}^{n} S_i\right] = nE(S_1) = 0.3 \times 100 \times 250 = 7500$

$Var\left(\sum_{i}^{n} S_i\right) = E\left[var\left(\sum_{i=1}^{n} S_i / \lambda\right)\right] +$

$Var\left[E\left[\sum^{n} S_i / \lambda\right]\right]$

$= E(n\lambda m_2) + var(n\lambda m_1)$

$= 0.3 \times 100 \times 200 + 0.01 \times 100^2 \times 250^2$

$= 6256000$

(iv) Mean is same but variance is much higher.

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7. Claim $\sim$ Poi (50)

Claim amount $\sim 100X$

$$f(x) = \begin{cases} \frac{3}{32}(6x - x^2 - 5) & 1 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

(i) $E(100X) = 100\,E(X)$

$$= 100 \times \frac{3}{32} \int_1^5 x \cdot (6x - x^2 - 5)\, dx$$

$$100 \times \frac{3}{32} \times \int_1^5 6x^2 - x^3 - 5x \, dx$$

$$\frac{300}{32} \times \left[ 2x^3 - \frac{x^4}{4} - \frac{5}{2}x^2 \right]_1^5$$

$$\frac{300}{32} \times [31.25 + 0.75]$$

$$= 300$$

$$Var(100X) = E(100X)^2 - (E(100X))^2$$

$$E((100X)^2) = 100^2 \; E(X^2)$$

$$100^2 \times \frac{3}{32} \cdot \int_1^5 x^2 \cdot (6x - x^2 - 5)\, dx$$

$$\frac{30000}{32} \left[ \frac{6x^4}{4} - \frac{x^5}{5} - \frac{5x^3}{3} \right]_1^5$$

$$98000$$

$$Var(100X) = 98000 - (300)^2$$

$$= 8000$$

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$Y$ = ~~Amount~~ ~~of~~ repair

$E(Y) = 0.3 \times 200 = 60$

$E(Y^2) - (E(Y))^2$

$Var(Y) = 0.3 \times 200^2 - (60)^2$

$= 8400$

$Z$ = Amount + repair

$E(Z) = E(X+Y)$

$= E(X) + E(Y)$

$= 300 + 60$

$= 360$

$Var(Z) = Var(X+Y)$

$= Var(X) + Var(Y)$

$= 8000 + 8400$

$= 16400$

$E(S) = E(N).E(Z)$

$Var(S) = E(N).Var(Z) + Var(N).E(Z)^2$

$E(S) = 50 \times 360$

$= 18000$

$Var(S) = 50 \times 16400 + 50 \times 360^2$

$= 7300000$