PAS - 2 : Assignment - 1.

1. $X \sim Pois\ (\theta)$

$E(x) = \theta$

$v(\theta) = \theta$

$\Rightarrow$ $P[x = 200] = P[199.5 < x < 200.5]$

$= P[-1.96 < x < 1.96]$

$= P[x_1 < \theta < x_2] = 0.95$

$C x : \hat{\theta} = \theta + 1.96 \times \sqrt{\theta}$

$= 200 + 1.96\sqrt{200}$

$= 227.71859$

2. $X_i = 1, 2, \ldots n$

$E(x_i) = \mu$

$V(x_i) = \sigma^2$

i. $\bar{X} = \frac{\Sigma x_i}{n}$

$E(\bar{x}) = \frac{E(\Sigma x_i)}{n}$

$= \frac{\Sigma (x_i)}{n}$

$= \frac{\Sigma \mu}{n}$

$= \frac{n \times \mu}{n}$

$\boxed{\mu = E(x_i)}$

$V(\bar{x}) = \frac{V(\Sigma x_i)}{n}$

$= \frac{\Sigma V(x_i)}{n}$

$= \frac{\Sigma \sigma^2}{n}$

$= \frac{n\sigma^2}{n}$

$\boxed{V(x_i) = \frac{\sigma^2}{n}}$

ii. $E(S^2) = E\left[\frac{1}{(n-1)} \Sigma (x_i - \bar{x})^2\right]^2$

$= E\left[\frac{1}{(n-1)} \cdot \Sigma \left\{(x_i)^2 + (\bar{x})^2 - 2 x_i \times \bar{x}\right\}\right]$

$= \left(\frac{1}{n-1}\right) \Sigma \left[E(x_i^2) + E(\bar{x})^2 - 2(\bar{x})\right]$

$V(x_i) = E(x_i^2) - E(x_i)^2$

$E(x_i^2) = V[x_i] + E[x_i]^2$

$V[x_i] = \sigma^2 \quad E[x_i]^2 - \mu^2$

$E(S^2) = \frac{1}{n-1}\left[\Sigma(\sigma^2 + \mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)\right]$

$= \frac{1}{n-1}\left(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2\right)$

$= \frac{1}{(\cancel{n-1})} \times \sigma^2 (\cancel{n-1})$

$\therefore \boxed{E(S^2) = \sigma^2.}$ //

iii. x_i's are from a normal population

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

$V\left[\frac{(n-1) \cdot S^2}{\sigma^2}\right] = 2(n-1)$

$\therefore \frac{(n-1)^2}{\sigma^4} V(S^2) = 2(n-1)$

$$\therefore V(s^2) = \frac{2\cancel{(n-1)} \cdot \sigma^4}{(n-\cancel{1})^2}$$

$$= \frac{2\sigma^4}{n-1}$$

$$\boxed{V(s^2) = \frac{2\sigma^4}{n-1}}$$ //

iv. $n = 10$
$\sigma^2 = 100$

$E(s^2) = \sigma^2 = 100$ ...... known.

$$\therefore P(50 < s^2 < 100) = P\left(\frac{9 \times 50}{100} < \frac{(n-1)s^2}{\sigma^2} < \frac{9 \times 100}{100}\right)$$

$$= P(4.5 < \chi^2_9 < 9) \quad \because \left(\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)}\right)$$

$$= P(\chi^2_9 < 9) - P(\chi^2_9 < 4.5)$$

$$= 0.5627 - 0.1245$$

$$= 0.4382$$ //

3. $X \sim$ No. of claims
$X \sim \text{Bin.}(4, P)$
A random sample of 180 policies was observed

No. of claims	0	1	2	3	4
No. of policies	86	75	16	2	1

i. MLE
Max. Likelihood Estimate for p.

$$L(P) = \prod f_x(x)$$
$$= \prod P(X=x)$$
$$= P(X=0) \cdot P(X=1) \cdot P(X=2) \cdot P(X=3) \cdot P(X=4)$$
$$= [{}^4C_0 \cdot p^0 (1-p)^4]^{86} \times [{}^4C_1 \cdot p^1 \cdot (1-p)^3]^{75} \times [{}^4C_2 \cdot p^2 \cdot (1-p)^2]^{16} \times$$
$$[{}^4C_3\, p^3 (1-p)^1]^2 \times [{}^4C_4 \cdot p^4 (1-p)^0]$$

Let the combination values be constants.

$$\therefore L(P) = [(1-p)^4]^{86} \cdot [p(1-p)^3]^{75} \cdot [p^2 (1-p)^2]^{16} \cdot [p^3 (1-p)]^2$$
$$= (1-p)^{344} [p^{75} (1-p)^{125}] \cdot [p^{32} (1-p)^{32}] [p^6 (1-p)^2] \cdot p^4$$

$$L(P) = (1-p)^{603} \cdot p^{117} \cdot \text{constant}$$

Taking log on both sides.

$$\ln L(P) = 603 \ln(1-p) + 117 \cdot \ln p + \text{constant}$$

Diff. ~~b/s~~ on both sides w.r.t. p.

$$\frac{d \ln L(P)}{dp} = \frac{603}{(1-p)} \times -1 + \frac{117}{p}$$

$$\frac{d \ln L(P)}{dp} = \frac{-603}{(1-p)} + \frac{117}{p}$$

Eq.

$$\frac{-603}{(1-p)} + \frac{117}{p} = 0$$

$$\frac{117}{p} = \frac{603}{(1-p)}$$

$$117(1-p) = 603p$$
$$117 - 117p = 603p$$
$$117 = 603p + 117p$$
$$720p = 117.$$
$$\boxed{\hat{p} = 0.1625}$$

4. $f(x) = \frac{C\beta^3}{(x+\beta)^4}$ $x>0, \beta>0$

i. $E(x) = \int x \cdot f(x)\, dx.$

$$= \int_0^\infty x \cdot \frac{C\beta^3}{(x+\beta)^4}\, dx.$$

$$= F(x) = \frac{C\beta^3}{(x+\beta)^4} \cdot \frac{\alpha\lambda^\alpha}{(\lambda+\alpha)^{\alpha+1}}$$

$\therefore \lambda = \beta,\ \alpha = 3 = C.$

$\alpha = 3,\ \beta = \lambda,\ E(x) = \frac{\lambda}{\alpha - 1}$

$$\boxed{\frac{\beta}{2} = E(x)}$$

ii. MOM $\rightarrow E(X) = \bar{x}$
Random Sample $\rightarrow X_1, X_2, X_3 \ldots\ldots X_N$

$$\bar{x} = \frac{\sum X_i}{n} = \cancel{\#}\ \bar{x}_n$$

$$E(x) = \frac{\beta}{2}$$

$\therefore \beta = 2\bar{x}_n = 3.$
$a = 3 - {}^{2}/\sqrt{3} \cdot \sqrt{3}.$
$= 5.$

iii. $E(\beta)$

5. i. $E(x) \sim \mu$ if (a, b) $- E(x) = \bar{x}$

$$\frac{b+a}{2} = \bar{x} \qquad \therefore a = 2\bar{x} - b.$$

$$\frac{(b-a)^{12}}{12} = s^2 = (2\bar{x} - 2a)^2 = 12s^2.$$
$2\bar{x} - 2a = 2\sqrt{3}\, s$
$\bar{x} - a = \sqrt{3}\, s$
$\boxed{\hat{a} = \bar{x} - \sqrt{3}\, s}$ //

ii. $\dfrac{(b - 2\bar{x} + b)^2}{12} = s^2. \quad \therefore b = \bar{x} + \sqrt{3}s$

$[2(b - \bar{x})]^2 = 12s^2.$
$2(b - \bar{x}) = \sqrt{3}\, s.$

iii. Sample: 1, 2, 3, 4, 5. $U(1,5)$

$$\bar{x} = \frac{a+b}{2} = \frac{5+1}{2} = 3.$$

$$s^2 = \frac{(b-a)^s}{12} = \frac{16}{12} = \frac{4}{3}$$

$$a = \bar{x} - \sqrt{3}s = 3 - \sqrt{3} \times \frac{2}{\sqrt{3}} = 1$$

$$b = \bar{x} + \sqrt{3}s = 3 + \sqrt{3} \times \frac{2}{\sqrt{3}} = 5$$

8. i. Unbaised Estimator:

Difference between expected value of estimator and estimator is 0.

ii. Bias: When θ used to estimate $\hat{\theta}$, dependent given values, same as θ, then is called bias.

iii. MSE = Variance of Estimator with respect to ~~true~~ parameter value.

$$MSE = \text{Variance} + \text{Bias}^2.$$

v. They would be turned consititent when MSE or greduall becomes 0.

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vi. MOM of Method of Likelihood Estimator.

11]. $n = 12$

$\Sigma x = 213200$

$\Sigma x^2 = 4919860000$

Sample Mean = 17, 767

Sample SD = 10, 144

i. Sample mean = $\frac{\Sigma x}{n}$

ii. Sample SD = $\frac{\Sigma x^2 - n.(\bar{x})^2}{n-1}$

SampleMean = ~~17~~ 17767 = $\frac{\Sigma x_5}{12}$

$\boxed{\therefore \Sigma x_5 = 213204}$