DATE / /

Calculus HW

1] $$\lim_{h\to 0} \frac{2(-3+h)^2-18}{h}$$

$$= \lim_{h\to 0} \frac{2(9+h^2-6h)-18}{h}$$

$$= \lim_{h\to 0} \frac{18+2h^2-12h-18}{h} \Rightarrow \lim_{h\to 0} \frac{\cancel{2(h^2-12)}\,2h^2-12h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2h-12)}{\cancel{h}} \Rightarrow \lim_{h\to 0} 2h-12$$

substituting value of h -

$= 2(0)-12$

$= -12$

2] $g(x) = \begin{cases} 2x & , x<6 \\ x-1 & , x\geq 6 \end{cases}$

(a) at $x = 4$,

$g(x) = 2x$ (since $x<6$)

$\therefore g(x) = 8.$

(b) at $x = 6$

For function to be continuous at x, its, left hand limit = right hand limit

$\therefore \lim_{x\to 6^+} x-1 = \lim_{x\to 6^-} 2x$

LHS

$= 6-1$

$= 5$

RHS

$= 2(6)$

$= 12$

$\because$ LHL $\neq$ RHL

Function is discontinuous.

3] $\lim_{x \to 9} \dfrac{x-9}{3-\sqrt{x}}$

$= \lim_{x \to 9} \dfrac{(\sqrt{x}+3)(\sqrt{x}-3)}{3-\sqrt{x}}$

$= \lim_{x \to 9} \dfrac{-(\sqrt{x}+3)\cancel{(3-\sqrt{x})}}{\cancel{(3-\sqrt{x})}} \Rightarrow \lim_{x \to 9} -(\sqrt{x}+3)$

= ~~to~~ applying limits -

$= -(\sqrt{9}+3) \Rightarrow -(3+3)$

$= -6$

6.] $g(y) = \begin{cases} y^2+5 & , y < -2 \\ 1-3y & , y \geqslant -2 \end{cases}$

(a) $\lim_{y \to 6} g(y)$

$\because y > -2,$

$\lim_{y \to 6} 1-3y \Rightarrow 1-3(6)$

$= -17$

(b) $\lim_{y \to -2} g(y)$

(1) $\lim_{y \to -2^+} 1-3y \Rightarrow 1-3(-2)$

$= 7$

(11) $\lim_{y \to -2^-} y^2+5 \Rightarrow (-2)^2+5$

$\Rightarrow 9$

7.] $f(t) = (4t^2-t)(t^3-8t^2+12)$

differentiating w.r.t t -

$f'(t) = (4t^2-t)\left[\frac{d}{dt}(t^3-8t^2+12)\right] + (t^3-8t^2+12)\left[\frac{d}{dt}(4t^2-t)\right]$

$f'(t) = [(4t^2-t)(3t^2-16t)] + [(t^3-8t^2+12)(8t-1)]$

$f'(t) = (12t^4-64t^3-3t^3+16t^2)+(8t^4-64t^3+96t-t^3+8t^2-12)$

$f'(t) = (12t^4-67t^3+16t^2)+(8t^4-65t^3+8t^2+96t-12)$

Ans — $f'(t) = 20t^4-132t^3+24t^2+96t-12$

PAGE No.
DATE / /

8.] $R(w) = \dfrac{3w + w^4}{2w^2+1}$

differentiating w.r.t w -

$$R'(w) = \frac{(2w^2+1)\left[\frac{d}{dw}(3w+w^4)\right] - (3w+w^4)\left[\frac{d}{dw}(2w^2+1)\right]}{(2w^2+1)^2}$$

$$R'(w) = \frac{\left[(2w^2+1)(3w^3+3)\right] - \left[(3w+w^4)(4w)\right]}{(2w^2+1)^2}$$

$$R'(w) = \frac{(6w^5+6w^2+3w^3+3) - (12w+4w^5)}{(2w^2+1)^2}$$

$$R'(w) = \frac{6w^5+3w^3+6w^2+3-4w^5-12w}{(2w^2+1)^2}$$

Ans - $R'(w) = \dfrac{2w^5+3w^3+6w^2+3}{(2w^2+1)^2}$

9.] $f(x) = 2e^x - 8^x$

diff. wrt x -

Ans→ $f'(x) = 2e^x - 8^x \log 8$

10.] $y = z^5 - e^z \log z$

diff. wrt z -

$$\frac{dy}{dz} = 5z^4 - \left(e^z\left(\frac{1}{z}\right) + e^z \log z\right)$$

Ans - $\dfrac{dy}{dz} = 5z^4 - \dfrac{e^z}{z} + e^z \log z$

DATE / /

11.]

(a) $f(x) = (6x^2 + 7x)^4$

diff. wrt x -

Ans- $f'(x) = 4(6x^2 + 7x)^3 \cdot (12x + 7)$

(b) $f(t) = 5 + e^{4t + t^7}$

diff. wrt t -

Ans- $f'(t) = e^{4t + t^7} \cdot (4 + 7t^6)$

12.]

(a) $z = \log(7 - x^3)$

diff. wrt x -

$$\frac{dz}{dx} = \frac{1 \cdot -3x^2}{7 - x^3} \Rightarrow \frac{-3x^2}{7 - x^3}$$

diff. wrt x -

$$\frac{d^2z}{dx^2} = \frac{(7 - x^3)(-6x) - (-3x^2)(-3x^2)}{(7 - x^3)^2}$$

$$\frac{d^2z}{dx^2} = \frac{-42x + 6x^4 - 6x^4}{(7 - x^3)^2}$$

Ans- $$\frac{d^2z}{dx^2} = \frac{-42x}{(7 - x^3)^2}$$

b.) $Q(v) = \dfrac{2}{(6+2v-v^2)^4} \Rightarrow 2(6+2v-v^2)^{-4}$

diff. w.r.t. v -

$Q'(v) = 2\left[-4(6+2v-v^2)^{-5}\right].(2-2v)$

$= -16(6+2v-v^2)^{-5}(1-2v)$

diff. w.r.t. v -

$$Q''(v) = -16\left[(1-2v)\{-5(6+2v-v^2)^{-6}.(2-2v)\}\right.$$
$$\left.+ (6+2v-v^2)^{-5}(-2)\right]$$

$$Q''(v) = -16\left[(1-2v)^2\{-5(6+2v-v^2)^{-6}.2\}\right.$$
$$\left. + (-2)(6+2v-v^2)^{-5}\right]$$

Ans- $Q''(v) = -32\left[-5(1-2v)^2(6+2v-v^2)^{-6} - (6+2v-v^2)^{-5}\right]$

c) $f(t) = \log(1+t^2)$

diff. wrt t -

$f'(t) = \dfrac{2t}{1+t^2}$

diff. wrt t -

$f''(t) = \dfrac{(1+t^2).(2) - (2t)(2t)}{(1+t^2)^2}$

$f''(t) = \dfrac{2+2t^2-4t^2}{(1+t^2)^2} \Rightarrow \dfrac{2(1-t^2)}{(1+t^2)}$ (Ans)

13) $f(x) = x^3 - 3x^2 - 9x + 12$

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

when $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$3x^2 + 3x - \cancel{8}9x - 9 = 0$

$3x(x+1) - 9(x+1) = 0$

$(3x - 9)(x+1) = 0$

$\therefore \quad x = -1, \cancel{or} \quad x = 3$

$f''(-1) = 6(-1) - 6$
$= -12$ (-ve)

$f''(3) = 6(3) - 6$
$= 12$ (+ve)

$\therefore$ maximum value is at -1,
min. value at 3.

$\therefore f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 12$
$= -1 - 3 + 9 + 12$
$= 17$ [max. value]

$f(3) = (3)^3 - 3(3)^2 - 9(3) + 12$
$= 27 - 27 - 27 + 12$
$= -15$ [min. value]

14. $f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

when $f'(x) = 0$

$$12x^2 - 36x + 24 = 0$$
$$\therefore x^2 - 3x + 2 = 0$$
$$x^2 - x - 2x + 2 = 0$$
$$x(x-1) - 2(x-1) = 0$$
$$(x-2)(x-1) = 0$$
$$\therefore x = 2, \quad x = 1$$

$f''(x) = 24x - 36$

for $x = 2$	for $x = 1$
$f''(2) = 24(2) - 36$	$f''(1) = 24 - 36$
$= 12$ (+ve)	$= -12$

$\therefore$ max at 1,
min at 2

$\therefore$ max value $\Rightarrow f(1) = 4 - 18 + 24 - 7$
$= 3$

min value $\Rightarrow f(2) = 32 - 72 + 48 - 7$
$= 1$