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(28/4/22)

Probability & Statistics - 2

Assignment - 1

Q1) $X \sim Poi(\theta)$ $\qquad \frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$ $\qquad X \sim 200$

$\therefore \theta = X \pm 1.96 \left(\sqrt{\theta / n}\right)$

Q2) $X_i \sim N(\mu, \sigma^2)$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

(i) $Var(\bar{x}) = \frac{Var(\Sigma X_i)}{n} = \frac{1}{n^2}(n\sigma^2) \cdot \left(\frac{\sigma^2}{n}\right)$

$E(\bar{x}) = \frac{E(\Sigma X_i)}{n} - \frac{n\mu}{n} = \mu$

~~(ii)~~ $\cancel{\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}}$ $\quad \cancel{E(S^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2}$

$\cancel{Var(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{n-1}}$

$\cancel{(80 < S^2 < 150)}\left(\cancel{\frac{9 \times 50}{100}}\right. \chi^2 \left.\cancel{4\left(\frac{9 \times 150}{100}\right)}\right)$
$\quad\quad 2 \quad\quad 3 \quad 2$

$= \cancel{4.8, 13.5}$

(ii) $E(S^2) = \sigma^2$,

$E(s^2) = E\left[\frac{1}{n-1}\left(\Sigma x_i^2 - n\bar{X}^2\right)\right]$

$= \frac{1}{n-1}\left[E\left(\Sigma x_i^2 - n\bar{x}^2\right)\right]$

$$= \frac{1}{n-1}\left[\sum E(x_i)^2 - n E(\bar{x}^2)\right]$$

$$= \frac{1}{n-1}\left[\sum(\mu^2+\sigma^2) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right]$$

$V(X) = E(x_i^2) - E(x)^2$
$E(x_i)^2 = \mu^2 + \sigma^2$
$V(\bar{x}) = E(\bar{x})^2 - E(\bar{x})^2$
$E(\bar{x})^2 = \mu^2$

$$= \frac{1}{n-1}\left[n\mu^2 + n\sigma^2 - n\mu^2 - \sigma^2\right]$$

$$= \frac{1}{n-1}[\sigma^2](n-1)$$

$$= \sigma^2$$

iii) $X \sim N(\mu, \sigma^2)$

$$\frac{(n-1)s^2}{\sigma} \sim \chi^2_{n-1}$$

$$V\left(\frac{(n-1)s^2}{\sigma^2}\right) = \frac{(n-1)^2}{\sigma^4} V(s^2)$$

$$= V(\chi^2_{n-1}) = \frac{(n-1)^2}{\sigma^4} V(s^2)$$

$$2(n-1) = \frac{(n-1)^2}{\sigma^4} V(s^2)$$

$$V(s^2) = \frac{2\sigma^4}{n-1}$$

iv) $P(0.5\sigma^2 < s^2 < 1.5\sigma^2)$

$$= P(s^2 < 1.5\sigma^2) - P(s^2 < 0.5\sigma^2)$$

$n = 10$

$\sigma^2 = 100$

i) $P(S^2 < 1.5\sigma^2) = P(S^2 < 150)$

$= P\left(\frac{9S^2}{\sigma^2} < \frac{150 \times 9}{100}\right)$

$= P(\chi^2_9 < 13.5)$

$= 0.8587$

i) $P(S^2 < 0.5\sigma^2) = P(S^2 < 0.5\sigma^2)$

$= P\left(\frac{9S^2}{\sigma^2} < \frac{50 \times 9}{100}\right)$

$= P(\chi^2_4 < 4.5)$

$= 0.1245$

$P(50 < S^2 < 150) = 0.8587 - 0.1245$
$= 0.7342$

Q3) i) $n = 4$ | $Bin(4, P)$ 180 policies

No. of claim	0	1	2	3	4
No. of policies	86	75	16	2	1

$L(P) = ((1-P)^4)^{86} (4P(1-P)^3)^{75} (6P^2(1-P)^2)^{16} (4P^3(1-P))^2 (P^5)$

$= (1-P)^{344} P^{75} (1-P)^{225} P^{32} (1-P)^{32} P^6 (1-P)^2 P^5$

$= (1-P)^{603} P^{117}$ Constant

$\ln L(P) = 603 \ln(1-P) + 117 \ln P$

$\frac{d \ln L(P)}{dP} = \frac{-603}{1-P} + \frac{117}{P} = 0$

$117 - 117P - 603P$

$q = 0.8375$ $P = 0.1625$

ii) $P(X=0) = (1-p)^4 = 0.492$ $\quad P(X=2) = 6p^2(1-p)^2 = 0.111$

$P(X=1) = 4p(1-p)^3 = 0.3818$ $\quad P(X=3) = 4p^3(1-p) = 0.0144$

$p(X=4) = 0.0007$

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.59	0.126

	0	1	2
O_i	86	75	19
E_i	88.56	68.72	22.71

$\sum C_i = 1.2557 \sim \chi^2_1$

$C_i = \sum \frac{(O_i - E_i)^2}{E_i}$	0.0757	0.5739	0.6061

$\therefore 1.2557 < 3.841$

$\therefore$ dont reject H.

Q4) $\times$ Doubt

Q5) $U(a,b)$ $\quad E(X) = \frac{a+b}{2}$ $\quad Var(X) = \frac{(b-a)^2}{12}$

$a = \bar{x} - \sqrt{3}S$ $\quad a = 2\bar{x} - b$ $\quad a = b - 2\sqrt{3}S$

$2\bar{x} - b = b - 2\sqrt{3}S$

$[b = \bar{x} + \sqrt{3}S]$ $\quad [a = \bar{x} - \sqrt{3}S]$

Sample $= 2, 4, 6, 8, 10$ $\quad \bar{x} = 6$ $\quad S = 3.16$

$a = 6 - \sqrt{3}(3.16)$
$= 0.527$

$b = 6 + \sqrt{3}(3.16) = 11.47$

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$U(0,b)$ $f(x) = \frac{1}{b}$ $L = \frac{1}{b^n}$ $\log L = -n \log b$

$\frac{d \log L}{dL} = \frac{-n}{b}$ $\therefore b > \alpha$

(Q6) H_0 $P = 0.1$ H_1 $P > 0.1$

12 missiles fired H_0 3 or more hits observed otherwise,

$\frac{1}{4} + \frac{1}{4}$

24 missiles fired detail H_0 : 5 or more hits observed

Q7) $\bar{X}_{pub} = 30$ $X_{poi} = 35.056$

i) $S^2_{pub} = 64.3125$ $S^2_{poi} = 67.403$ $S_p^2 = 58.123$

$$\frac{(\bar{X}_{pub} - \bar{X}_{poi}) - (\mu_{pub} - \mu_{poi})}{S_p / \left(\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)} \sim t_{n_1+n_2-2}$$

ii) $H_0 : \mu_{pub} = \mu_{poi}$ $H_1 : \mu_{pub} \neq \mu_{poi}$

$$\frac{(30 - 35.056)}{\sqrt{58.123}\sqrt{\frac{2}{9}}} = -1.407$$

$\therefore$ accept H_0

iii) Private Hospital does not result in higher claim.

Q8) i) unbias when $[E(\hat{\theta}) = \theta)]$
ii) bias is when $[E(\hat{\theta}) - \theta \neq 0]$
iii) MSE is when $\theta = Var + Bias^2$

Q9) i) $t_k = \frac{N(0,1)}{\sqrt{\frac{t_k}{k}}}$ $\qquad \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$

ii) $\mu(t_k) = 0$ $\qquad Var(t_k) = \frac{k}{k-2}$

iii) $n = 10$ $\quad \bar{x} = 50$ $\quad S^2 = 48.667$ $\quad CI = 99\%$

a) $t_{0.005, 9} = 3.25$

$\therefore \mu = \bar{x} \pm t_{05\%, 9} \frac{S}{\sqrt{n}} = 50 \pm 3.25 \sqrt{\frac{48.667}{10}}$

b) $t_{0.005, 9} = 2.5750$ $\quad \mu = 50 \pm 2.5758 \left(\sqrt{\frac{48.667}{10}}\right)$

$= 55.68, 44.32$

Q10) $n = 1000$ $\quad X \sim Poi(3)$ $\qquad X \sim N(3000, 3000)$

if $P(X < 3100)$ then $X \sim Poi(3)$

else X does not follow $Poi(3)$.

(i) Type error = $P(H_0$ is reject / H_0 is true$)$

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$= P(X > 3100, \lambda = 3) \quad \therefore X \sim N(3000, 3000)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right) = P(Z > 1.826) = 3.39\%$

ii) Typ 2 error $\Rightarrow$ P(H₀ accept / H₀ is false)

iii) Power = P(reject H₀/H₀ false) $= 1 - \left(\frac{Z(3100-\mu)}{\sqrt{n\mu}}\right)$

Q11) $n = 12 \quad \Sigma x = 213200 \quad \Sigma x^2 = 4919860000$

$\bar{X} = 17767 \quad S = 10144$

$X_1 = 11000 \quad X_2 = 48000 \quad X_1' = 27500 \quad X_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 48000$ A (sum of 10 terms)

$A = 154200$

$\Sigma X' = A + x_1' + x_2' = 213200 \quad \boxed{\bar{X}' = 17767}$

$\Sigma x^2 = 4919860000 = B + 11000^2 + 48000^2$

$B = 2494860000$

$\Sigma x'^2 = B + x_1'^2 + x_2'^2 = 4243360000$

$S = \frac{1}{\sqrt{11}}\left(\sqrt{4243360000 - 12(17767)^2}\right)$

$= 6434.031$