Calculus Assignment 2

Q1)

$$\int_{\frac{1}{50}}^{2} \frac{e^{2/\omega}}{\omega^2} \cdot d\omega$$

$$= \int_{1/50}^{2} e^{2/\omega} \cdot \omega^{-2} \, d\omega$$

$\therefore$ let $x = 1/\omega$ $\qquad$ $x = 1/2, \omega = 2$

$\therefore$ diff. w.r.t. ω $\qquad$ $x = 50, \omega = 1/50$

$$\frac{dx}{d\omega} = -1\omega^{-2}$$

$$\therefore -dx = d\omega \cdot \omega^{-2}$$

$$= -\int_{50}^{1/2} e^{2x} \cdot dx$$

$$= -\left[e^{2x} \cdot \frac{1}{2} \right]_{50}^{1/2}$$

$$= -\left[e^{1} \cdot \frac{1}{2} \right] + \left[e^{100} \cdot \frac{1}{2} \right]$$

$$= 1.344058571 \times 10^{43}$$

Q2) $\int \frac{2t^3+1}{(t^4+2t)^3} \cdot dt$

Let $t^4+2t = x$

diff. w.r.t-

$4t^3+2 = \frac{dx}{dt}$

$\therefore 2(2t^3+1) = \frac{dx}{dt}$

$\therefore (2t^3+1) \cdot dt = \frac{dx}{2}$

$= \int \frac{1}{2x^3} dx$

$= \frac{1}{2}\int x^{-3} \cdot dx$

$= \frac{1}{2}\left(\frac{x^{-2}}{-2}\right) + c$

$= \frac{-1}{4x^2} + c = \frac{-1}{4(t^4+2t)^2} + c$

Q.3) $f'(x) = x^4 + 3x - 9$

$$f(x) = \int f'(x) \cdot dx$$

$$= \int x^4 + 3x - 9 \cdot dx$$

$$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$$

Q4) $f(x) = \begin{cases} 6 & \text{if } x \geq 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$$\int_{-2}^{3} f(x) \cdot dx = \int_{-2}^{1} 3x^2 \cdot dx + \int_{1}^{3} 6 \cdot dx$$

$$= \left[\frac{3 \cdot x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= \left[1 - (-8)\right] + \left[18 - 6\right]$$

$$= [9] + [12]$$

$$= \underline{\underline{21}}$$

Q5) $\int 3(8y-1)\, e^{4y^2-y} \cdot dy$

$\therefore$ Let $4y^2 - y = x$

diff. w.r.t. y

$\therefore 8y - 1 = \frac{dx}{dy}$

$\therefore (8y-1) \cdot dy = dx$

$= \int 3 \cdot e^{x}\, dx$

$= 3 \cdot e^{x} + C$

$= 3 e^{(4y^2 - y)} + C$

Q6) $f(x) = ?$

$f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int f''(x) \cdot dx$

$= \int 15\sqrt{x} + 5x^3 + 6 \; dx$

$\cdot\; f'(x) = 15 \cdot \frac{x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + C_1$

$$f'(x) = 10x^{3/2} + \frac{5x^4}{4} + 6x + C_1$$

$$\therefore f(x) = \int f'(x) \cdot dx$$

$$= \int 10x^{3/2} + \frac{5x^4}{4} + 6x + C_1 \cdot dx$$

$$= \left[\frac{10x^{5/2}}{5/2} + \frac{5x^5}{20} + \frac{6x^2}{2} + C_1 x + C_2 \right]$$

$$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1 x + C_2$$

$$\therefore f(1) = 4(1)^{5/2} + \frac{1^{(5)}}{4} + 3(1)^2 + C_1(1) + C_2$$

$$\therefore \frac{-5}{4} = 4 + \frac{1}{4} + 3 + C_1 + C_2$$

$$\therefore \frac{-5}{4} - \frac{1}{4} - 4 - 3 = C_1 + C_2$$

$$\therefore \frac{-17}{2} = C_1 + C_2 \quad \text{— (2)}$$

$\therefore P(4) = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + 4c_1 + c_2$

$\therefore P(4) = 4(2)^5 + (4)^4 + 3(16) + 4c_1 + c_2$

$\therefore -28 = 4c_1 + c_2 \quad — (1)$

$\therefore (1) - (2)$

$\therefore 4c_1 + c_2 - c_1 - c_2 = -28 + \frac{17}{2}$

$\therefore 3c_1 = \frac{-39}{2}$

$\therefore c_1 = \frac{-13}{2}$

$\therefore c_2 = \frac{-17}{2} - c_1$

$\therefore c_2 = \frac{-17}{2} - \frac{(-13)}{2}$

$\therefore c_2 = \frac{-4}{2} = -2$

Q7) $f(x+iy) + g(x-iy)$

$\therefore f(x) + f(iy) + g(x) - g(iy)$

$\therefore f(x) + i f(y) + g(x) - i g(y)$

Q8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1) = 2$

$\therefore \frac{dr}{r^2} = \frac{d\theta}{\theta}$, $r = (1) = 2$

$\therefore \int \frac{1}{r^2} \cdot dr = \int \frac{d}{\theta} \cdot d\theta$

$\therefore \frac{r^{-1}}{-1} = \log(\theta) + c$

$\therefore \frac{-1}{r} = \log(\theta) + c$

$\therefore \frac{-1}{2} = \log(1) + c$

$\therefore c = -\frac{1}{2}$

$\therefore \frac{-1}{r} = \log(0) - \frac{1}{2}$

Q9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\therefore \frac{dv}{9.8 - 0.196v} = dt$

$\therefore \int \frac{1}{9.8 - 0.196v}\, dv = \int 1 \cdot dt$

$\therefore \log(9.8 - 0.196v)\left(\frac{-1}{0.196}\right) = x + c$

$\therefore \frac{-\log(9.8 - 0.196v)}{0.196} = x + c$

Q10) $ty' + 2y = t^2 - t + 1 \quad \therefore y(1) = 1/2$

$$t \cdot \frac{dy}{dt} + 2y = t^2 - t + 1$$

$$t \cdot \frac{dy}{dt} = t^2 - t + 1 - 2y$$

Q11) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$

$\therefore \dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

$\therefore \dfrac{dy}{y^3} = \dfrac{x}{\sqrt{1+x^2}} \cdot dx$

$\therefore \int \dfrac{1}{y^3} \cdot dy = \int \dfrac{x}{\sqrt{1+x^2}} \cdot dx$

$\therefore$ Let $1+x^2 = t$

d.w.r.t. x

$\therefore 2x = \dfrac{dt}{dx}$

$\therefore x \cdot dx = \dfrac{dt}{2}$

$\int \dfrac{1}{y^3} \cdot dy = \int \dfrac{1}{2\sqrt{t}} \cdot dt$

$\therefore \dfrac{y^{-2}}{-2} = \dfrac{1\,t^{+1/2}}{2(1/2)} + c$

$\therefore \dfrac{-1}{2y^2} = \sqrt{t} + c \Rightarrow \sqrt{1+x^2} + c = \dfrac{-1}{2y^2}$

$\therefore y(0) = -1$

$$\therefore \sqrt{1+0^2} + c = \frac{-1}{2(-1)^2}$$

$$\therefore 1 + c = \frac{-1}{2}$$

$$c = \frac{-3}{2}$$

$$\sqrt{1+x^2} - \frac{3}{2} = \frac{-1}{2y^2}$$

Q1)

$f(x) = x^3 - 10x^2 + 6 \quad ; x = 3$

$f(3) = -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$

$f''(x) = 6x - 20 \qquad f''(3) = -2$

$f'''(x) = 6 \qquad f'''(3) = 6$

$f^{iv}(x) = 0$

$$\therefore f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{iv}(a)}{4!}(x-a)^4$$

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$$\therefore f(x) = -57 + \frac{(-33)(x-3)}{1} + \frac{(-2)(x-3)^2}{2\times 1} + \frac{6(x-3)^3}{3\times 2\times 1} + \frac{0\times(x-4)^4}{4!}$$

$$\therefore f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$$

Q13) $f(x) = (1+x)^4 \qquad \therefore f(0) = 1$

$\therefore f'(x) = 4(1+x)^{4-1} \qquad \therefore f'(1) = 4$

$\therefore f''(x) = 4(4-1)(1+x)^{4-2} \qquad \therefore f''(1) = 4(4-1)$

$\therefore f'''(x) = 4(4-1)(4-2)(1+x)^{4-3} \qquad f'''(3) = 4(4-1)(4-2)$

$\therefore f''''(x) = 4(4-1)(4-2)(4-3)(1+x)^{4-4}$

$f''''(3) = 4(4-1)(4-2)(4-3)$

$$\therefore f(x) = f(0) + \frac{x f'(0)}{1} + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \frac{x^4 f'''(0)}{4!}$$

$$\therefore f(x) = 1 + x(4) + \frac{x^2(4)(4-1)}{2!} + \frac{x^3(4)(4-1)(4-2)}{3!} + \frac{x^4(4)(4-1)(4-2)(4-3)}{4!}$$

Q14) $f(x) = \sqrt{1+x} \quad f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}} \quad f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}} \quad f''(0) = \dfrac{-1}{4}$

$f'''(x) = \dfrac{3}{8(1+x)^{5/2}} \quad f'''(0) = \dfrac{3}{8}$

$f^{iv}(x) = \dfrac{-15}{16(1+x)^{7/2}} = f^{iv}(0) = \dfrac{-15}{16}$

$\therefore f(x) = f(0) + x \cdot f'(0) + \dfrac{x^2 \cdot f''(0)}{2!} + \dfrac{x^3 \cdot f'''(0)}{3!} + \dfrac{x^4 \cdot f^{iv}(0)}{4!}$

$\therefore f(x) = 1 + x \cdot \left(\dfrac{1}{2}\right) + \dfrac{x^2(-1/4)}{2} + \dfrac{x^3(3/8)}{3 \times 2} + \dfrac{x^4(-15/16)}{4 \times 3 \times 2 \times 1}$

$f(x) = 1 + \dfrac{x}{2} - \dfrac{x^2}{8} + \dfrac{x^3}{16} - \dfrac{5x^4}{96}$

Q15)

$f(x) = e^{-6x}$ $\quad f(-4) = e^{24}$

$f'(x) = -6 \cdot e^{-6x}$ $\quad f'(-4) = -6 \cdot e^{24}$

$f''(x) = 36 \cdot e^{-6x}$ $\quad f''(-4) = 36 \cdot e^{24}$

$f'''(x) = -216 \cdot e^{-6x}$ $\quad f'''(-4) = -216 \cdot e^{24}$

$f^{iv}(x) = 1296 \cdot e^{6x}$ $\quad f^{iv}(-4) = 1296 \cdot e^{24}$

$$\therefore f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2 f''(a)}{2!} + \frac{(x-a)^3 f'''(a)}{3!} + \frac{(x-a)^4 f^{iv}(a)}{4!}$$

$$\therefore f(x) = e^{24} + (x+4)e^{24}_{\times -6} + \frac{(x+4)^2 e^{24} \times 36}{2} + \frac{(x+4)^3 e^{24}_{\times -216}}{3\times 2\times 1} + \frac{(x+4)^4 e^{24}_{\times 1296}}{4\times 3\times 2\times 1}$$

$$\cancel{= e^{24}\left[(x+4) + \frac{(x+4)^2}{2} + \frac{(x+4)^3}{3\times 2\times 6} + (x+4)^4\right.}$$

$$\therefore f(x) = e^{24} - 6e^{24}(x+4) + 18 \cdot e^{24}(x+4)^2 - 36 \cdot e^{24}(x+4)^3 + 54e^{24}(x+4)^4$$

$$\therefore f(x) = e^{24}\left(1 - 6(x+4) + 18(x+4)^2 - 36(x+4)^3 + 54(x+4)^4\right)$$

Q16) $f(x) = \ln(3+4x)$, $f(0) = \ln(3)$

$f'(x) = \dfrac{4}{(3+4x)}$, $f'(0) = \dfrac{4}{3}$

$f''(x) = \dfrac{-16}{(3+4x)^2}$, $f''(0) = \dfrac{-16}{9}$

$f'''(x) = \dfrac{128}{(3+4x)^3}$, $f'''(0) = \dfrac{128}{27}$

$f^{iv}(x) = \dfrac{-384\times 4}{(3+4x)^4}$, $f^{iv}(0) = \dfrac{-1536}{81}$

$$\therefore f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \frac{f^{iv}(a)(x-a)^4}{4!}$$

$$\therefore f(x) = \ln(3) + \frac{4}{3}(x) - \frac{16}{9\times 2}(x)^2 + \frac{128}{27\times 3\times 2}(x)^3 - \frac{1536}{4\times 3\times 2\times 1}(x)^4$$

$$\therefore f(x) = \ln(3) + \frac{4x}{3} - \frac{8x^2}{9} + \frac{64x^3}{81} - 64x^4$$