Financial Mathematics Assignment
Unit 1 & 2 (Ch-1, 2, 3, 4)

Ques 1 $d^{(4)} = 8\%$

$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$

$= 1 - (1 - 2\%)^4$

$d = 7.763184\%$

(i) $\delta = -\ln(1-d)$

$= 8.081083\%$

(ii) $i = \frac{d}{1-d}$

$= 8.416578\%$

(iii) $d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$= 8.053934\%$

Ques 2 $\delta(t) = 0.004t + 0.0002t^2$

(i) Amt. due = $C = £1000$

$PV_0 = C \times v(t)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2}$

$= 1000 \times e^{-\left[0.002t^2 + 0.0002\frac{t^3}{3}\right]_0^{10}}$

$= 1000 \times e^{-0.266666667}$

$= 765.93$

(ii) $(1+i)^{10} = e^{0.266666667}$

$i = 2.7025403\%$

Ques 3(i) The relationship $d = iv$, has an interesting verbal intepretation. Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

(ii)(a) $\frac{d^{(4)}}{4} = 2.25\%$

$d = 1 - (1 - 2.25\%)^4$

$d = 8.7007806\%$

$d^{(2)} = 2\left[1 - (1-d)^{\frac{1}{2}}\right]$

$d^{(2)} = 8.89875\%$

(b) $d^{(1/2)} = 5\%$

$d = 1 - \left(1 - \frac{d^{(1/2)}}{(1/2)}\right)^{1/2}$

$d = 1 - (1 - 10\%)^{1/2}$

$d = 5.1316702\%$

$d^{(2)} = 2\left[1 - (1-d)^{1/2}\right]$

$d^{(2)} = 5.1992507\%$

(iii) $v\left(0, \frac{91}{365}\right) = \frac{1}{(1+i)^{91/365}} = \left(1 - \frac{91}{365}d\right)$

$$\frac{1}{(1.05)^{91/365}} = \left(1 - \frac{91}{365}d\right)$$

$$d = \frac{365}{91}\left(1 - \frac{1}{(1.05)^{91/365}}\right)$$

$$d = 4.849462\%$$

Ques4(i) The relationship $d = iv$, has an interesting verbal interpretation. Interest earned on an investment, paid at the ~~beginin~~ beginning of the period is d. Interest earned on an investment, paid at the end of a period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

(ii) $i = 0.08$

(a) $d = \frac{i}{1+i} = 7.407407407\%$

$$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$

$$d^{(12)} = 7.671478\%.$$

(b) $i^{(365)} = 365\left((1+i)^{1/365} - 1\right)$

$$= 7.696915\%.$$

(c) $\delta = \ln(1+i)$

$= 7.696104\ \%$

(d) $$i^{(0.5)} = \frac{1}{2}\left[(1+i)^{\frac{1}{1/2}} - 1\right]$$

$$= \frac{1}{2}\left[(1+i)^{2} - 1\right]$$

$$= 8.32\ \%$$

Ques 5(i) $d^{(4)} = 6\ \%$

$d = 1 - (1 - 1.5\%)^4$

$d = 5.866344937\ \%$

(a) $i = \dfrac{d}{1-d} = 6.231931538\ \%$

$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$

$i^{(2)} = 6.137752\ \%$

(b) $d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$d^{(12)} = 6.030253\ \%$

(ii) $i = \dfrac{\cancel{20,000}}{\cancel{2,00,000}} \times 10\cancel{0} = 10\ \%$

No. of days = 93 (excluding end date)

$AV_{93\text{ days}} = 2,00,000 \times (1.1)^{93/365}$

$= 2,04,916.3564$

Ques6 Let the retail price be x

If retailer pays cash $= 70\% (x)$

then he has to pay

$$PV = 0.7x$$

If retailer uses the six month credit

$$= 75\% (x)$$

Mode, that he has to pay

$AV = 0.75x$ in 6 months time

We need to find the discounting done on 6 month credit mode for the retailers who pay in cash {- ie cash is preferred rather than credit mode}

Time 6 months $= \frac{1}{2}$ years

$$PV_0 = AV(1-d)^n$$

$$0.7x = 0.75x(1-d)^{1/2}$$

$$d = 1 - \left(\frac{0.7}{0.75}\right)^2$$

$$d = 12.888889\%$$

(ii) (a) $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$

$d^{(12)} = 13.719544\%$

(b) $i = \frac{d}{1-d} = 14.79591837\%$

$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right]$

$i^{(2)} = 14.285714\%$

(c) $\delta = -\ln(1-d)$
$\delta = 13.798574\ \%$

Ques 7 Amt inv $= C = 20000$
$AV_{17\text{ months}} = 26000$
$17 \text{ months} = 1 + \frac{5}{12}$ yrs

$= 1.416666667$ yrs

$AV = C(1+i)^n$
$26000 = 20000(1+i)^{1.416666667}$
$i = 20.34570672\ \%$

(i) $i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$
$i^{(4)} = 18.955255\ \%$

(ii) $d = \frac{i}{1+i} = 16.9060514\ \%$
$d^{(2)} = 2\left[1-(1-d)^{1/2}\right]$
$d^{(2)} = 17.688235\ \%$

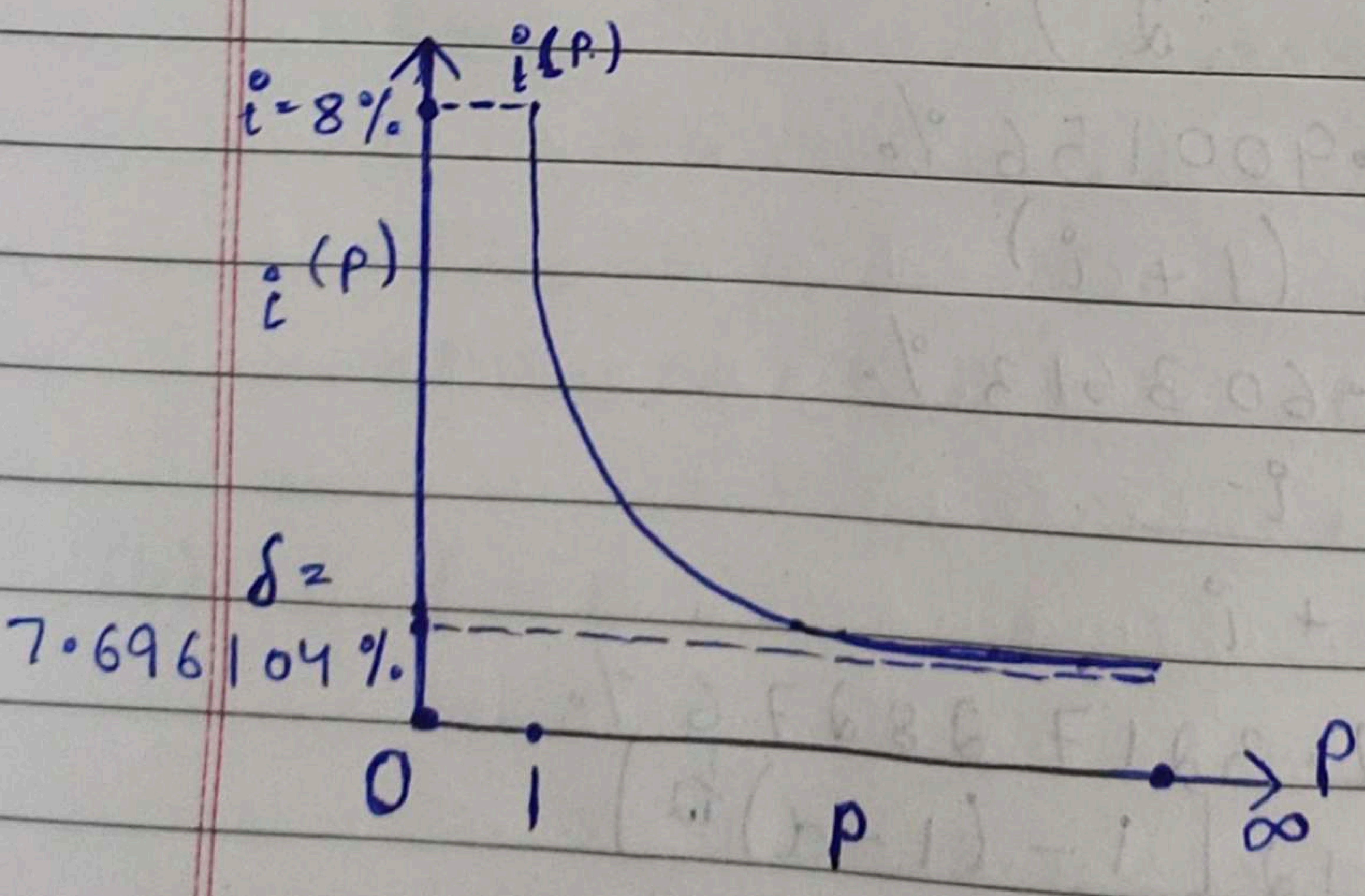
(iii) $26000 = 20000(1 + 1.416666667\, i_s)$
$i_s = 21.1764706\ \%$

(iv) Numerically, in terms of compounding, i is the highest numerical value and d is the lowest numerical value. This is just numerically, as both will yield the same accumulated and present value. The simple interest, is greater than i because, since there is no interest on interest accumulation is simple interest it has to compensate for the compounding effect and hence it has to be greater than i.

Ques 8 $i = 8\%$

$i^{(p)} = p\left[(1.08)^{1/p} - 1\right]$

p	$i^{(p)}$
1	8%
2	7.84609%
3	7.79567%
4	7.77062%
12	7.72084%
100	7.69907%
365	7.69691%
∞	7.696104%



→ More frequently the compounding takes place (i.e. as p increases), the smaller is the equivalent nominal annual rate

$i^{(p)} \propto \frac{1}{p}$

→ When continuous compounding takes place i.e. when p approaches infinity then $i^{(p)}$ has a limiting value known as the force of interest or δ.

Ques 9(i) Force of interest: The measure of interest at individual moments of time or at any instant of time is known as force of interest.

(ii) $i^{(2)} = 0.0775$

(a) $i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$

$i = 7.900156\,\%$

(b) $\delta = \ln(1+i)$

$= 7.603613\,\%$

(c) $d = \frac{i}{1+i}$

$= 7.321728276\,\%$

$d^{(12)} = 12\left[1-(1-d)^{\frac{1}{12}}\right]$

$d^{(12)} = 7.579575\,\%$

(d) $i^{(1/2)} = \frac{1}{2}[(1+i)^2 - 1]$

$= 8.212219\%$

Ques 10 $\delta(t) = \begin{cases} 0.08 & 0 \le t \le 5 \\ 0.06 & 5 \le t \le 10 \\ 0.04 & t \ge 10 \end{cases}$

When, $0 \le t \le 5$

$v(t) = e^{-\int_0^t 0.08}$

$= e^{-0.08t}$

When $t \ge 10$

$v(t) = e^{-\int_0^5 0.08 - \int_5^{10} 0.06 - \int_{10}^t 0.04}$

$= e^{{}_0^5[0.08t] + {}_{10}^5[0.06t] + {}_t^{10}[0.04t]}$

$= e^{-\cancel{0.4} + 0.3 - 0.6 + \cancel{0.4} - 0.04t}$

$= e^{-0.04t - 0.3}$

Ques 11 (a) $t = \frac{\cancel{9}\ 3}{\cancel{12}\ 4} = \frac{3}{4}$ years

Amt. Due $= C = 50{,}000$

$d = 18\%$

$PV_0 = C(1-d)^t$

$PV_0 = 43{,}085.42623$

$= 43{,}085.43$

(b) (i) $\delta = 6\%$

$d = 1 - e^{-\delta}$

$= 5.823546642\%$

$d^{(12)} = 12[1-(1-d)^{1/12}]$

$d^{(12)} = 5.985025\%$

(ii) $d^{(4)} = 6\%$

$$d = 1 - \left[1 - \frac{d^{(4)}}{4}\right]^4$$

$d = 5.866344937\%$

$$i = \frac{d}{1-d}$$

$i = 6.231931537\%$

$\rightarrow$ $i^{(4)} = 4\left[(1+i)^{\frac{1}{4}} - 1\right]$

$i^{(4)} = 6.0913705\%$

$\rightarrow$ $i^{(12)} = 12\left[(1+i)^{1/12} - 1\right]$

$i^{(12)} = 6.060709\%$

(c) Ascending order: $d < \delta < i^{(4)}$

d will always be the smallest numerical value as it is charged in the beginning of the term. So due to time value of money $d < \delta$ and $d < i^{(4)}$. δ refers to continuous compounding i.e more frequency than quarterly compounding. Due to more compounding than $i^{(4)}$ if $\delta = i^{(4)}$. Thus, δ has to be smaller than $i^{(4)}$ to yield the same accumulated value. Thus, $\delta < i^{(4)}$ and hence $d < \delta < i^{(4)}$.

(d) The relationship $d = iv$, has an interesting verbal interpretation. Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

Ques 12 Amt invested $= C = 7049$

$$AV_6 = C\, A(0,6)$$

$$= 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$$

→ For $A(0,2)$

$$i^{(12)} = 6\%$$

$$\frac{i^{(12)}}{12} = 0.5\%$$

$$A(0,2) = (1.005)^{24}$$

→ For $A(2,4.5)$

is per quarter $= 1.5\%$

$$A(2,4.5) = [1 + 10(0.015)] = 1.15$$

→ For $A(4.5,6)$

$$d^{(12)} = 6\%$$

$$\frac{d^{(12)}}{12} = 0.5\%$$

$$A(4.5,6) = \frac{1}{(1-0.005)^{18}}$$

$$AV_6 = 7049 \times A(0,2) \times A(2,4.5) \times A(4.5,6)$$
$$= 7049 \times (1.005)^{24} \times (1.15) \times \frac{1}{(1-0.005)^{18}}$$

$$AV_6 = 9999.894$$

Ques 13 Let amt. invested = $C = x$

$$AV_n = 2x$$
Time = n years

(i) $$AV_n = C(1+i)^n$$
$$i = 10\%$$
$$2\cancel{x} = \cancel{x}(1.1)^n$$
$$\ln 2 = n \ln(1.1)$$
$$n = \frac{\ln(2)}{\ln(1.1)}$$
$$n = 7.27 \text{ years}$$

(ii) $$AV_n = \frac{C}{(1-d)^n}$$
$$d^{(12)} = 10\%$$
$$\frac{d^{(12)}}{12} = 0.833333333\%$$
$$AV_n = \frac{C}{\left(1-\frac{d^{(12)}}{12}\right)^{12n}}$$

$$2x = \frac{x}{\left(1-\frac{d^{(12)}}{12}\right)^{12n}}$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12n} = \frac{1}{2}$$

$$12n \ln\left(1 - \frac{d^{(12)}}{12}\right) = \ln\left(\frac{1}{2}\right)$$

$$n = 6.902 \text{ years}$$