Assignment

1) X = claim Size of home insurence.
$X \sim$ Normal $(800, 100^2)$
Y = Claim Size on Car insurence.
$Y \sim$ Normal $(1200, 300^2)$

$= P(Y_1 + Y_2 + Y_3) > X_1 + X_2 + X_3 + X_4 + 800)$
$= P((Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3(1200) - 4(800), (3 \times 300^2 + 4 \times 100^2) \sim N(400, 310000)$

$P((Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$
$= P(Z > 800 - 400) = P(Z > 0.718)$

$= 1 - P(Z < 0.718)$
$\cong 1 - 0.76424$
~ 0.23576

2) N = Negetive claim
$N \sim$ Negetive binomial $(3, 0.9)$

X = Claim amount.
$X \sim$ gamma $(6, 2)$

Y = total claim amount.

$$M_Y(t) = M_N(\log M_X(t))$$

$$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

$$M_Y(t) = M_N\left(\log\left(1 - \frac{t}{2}\right)^{-6}\right)$$

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$$M_Y(t) = \left(\frac{p e^{\log(1-\frac{t}{2})^{-6}}}{1-qe^{\log(1-\frac{t}{2})^{-6}}} \right)^3$$

$$M_Y(t) = \left(\frac{p(1-\frac{t}{2})^{-6}}{1-q(1\cancel{z}-t/2)^{-6}} \right)^3$$

$$M_Y(t) = \left(\frac{0.9\,(1-t/2)^{-6}}{1-(0.1)(1-t/2)^{-6}} \right)^3$$

$$E(N) = \frac{3}{0.9} = \frac{10}{3}$$

$$V(N) = \frac{K(0.1)}{(0.9)^2} = \frac{3}{0.9}\left(\frac{1}{9}\right) = \frac{10}{27}$$

$$E(X) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3$$

$$V(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = \frac{3}{2}$$

$$V(Y) = E(N)\,V(X) + E(X)^2\,V(N)$$

$$= 5\,\frac{\cancel{10}}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{2}}\right) + \cancel{9}\left(\frac{10}{\cancel{27}_{3}}\right)$$

$$= 5 + \frac{10}{3} = \frac{25}{3}$$

$$S.D(Y) = \sqrt{\frac{25}{3}}$$

$$= 2.88675$$

③ $f_X(n) = \lambda e^{-\lambda(x-\alpha)}$

$M_X(t) = E(e^{tX})$

$= \int_{\alpha}^{\infty} e^{tn} \cdot \lambda \cdot e^{-\lambda(x-\alpha)} \cdot dx$

$= \lambda \int_{\alpha}^{\infty} e^{tx} \cdot e^{-\lambda x} \cdot e^{\lambda \alpha} dx$

$= \lambda e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{x(t-\lambda)} dx$

$= \frac{\lambda e^{\lambda\alpha}}{t-\cancel{\alpha}\lambda} \left[e^{(t-\lambda)n} \right]_0^{\infty}$

$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{-(\lambda-t)n} \right]_0^{\infty}$

$M_X(t) = \frac{\lambda \cancel{e^{\lambda\alpha}}}{t-\lambda} \left[0 \cdot e^{\cancel{-\lambda\alpha}} \quad e^{t\alpha} \right]$

$M_X(t) = \frac{\lambda e^{t\alpha}}{\lambda - t}$

ii) $M_X'(t) = \lambda(e^{t\alpha})(\lambda-t)^{-2}(-1)(-1) +$
$\lambda(\lambda-t)(e^{t\alpha})(\alpha)$

$= \frac{\lambda e^{t\alpha}}{(\lambda-t)^{t2}} + \frac{\lambda e^{t\alpha}\alpha}{\lambda-t}$

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$$E(X) = M'_X(0) = \frac{\lambda(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda}$$

$$= \frac{1+\lambda\alpha}{\lambda} = \frac{1+\lambda\alpha}{\lambda}$$

$$M''_\lambda(t) = \lambda e^{t\alpha}(-2)(\lambda-t)^{-3}(-1) + \lambda(\lambda-t)^{-2}(e^{t\alpha})(\alpha) + \lambda\alpha e^{t\alpha}(\lambda-t)^{-2}(-1)(-1) + \lambda\alpha(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\alpha^2\lambda e^{t\alpha}}{\lambda-t}$$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{2\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda\alpha^2 e^{t\alpha}}{\lambda-t}$$

$$E(X^2) = M''_X(0) = \frac{2\cancel{\lambda}(1)}{\lambda^{\cancel{3}}} + \frac{2\cancel{\lambda}\alpha}{\lambda^{\cancel{2}}} + \frac{\cancel{\lambda}\alpha^2}{\cancel{\lambda}}$$

$$= \frac{2}{\lambda^2} + \frac{2\alpha}{\lambda} + \alpha^2$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1+\lambda^2\alpha^2+2\alpha\lambda}{\lambda^2}\right)$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha^2 - \frac{2\alpha}{\lambda}$$

$$V(X) = \frac{1}{\lambda^2}$$

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④ $G_v(1) = \left(\frac{p}{1-qt}\right)^k$, $p+q=1$

$\therefore$ Given distribution is negetive binomial type 2.

$$p(v=4) = \frac{\sqrt{k+4}}{\sqrt{5}\,\sqrt{k}} p^4 q^4$$

⑤ $f(x,y) = ax^2 + axy$ $\quad 0<x<5$

$10<y<20$

$$\Rightarrow \quad 1 = \int_{10}^{20}\int_0^5 ax^2 + axy \, dx\, dy$$

$$1 = \int_{10}^{20} \left[\frac{ax^3}{3}\right]_0^5 + \left[ay\frac{x^2}{2}\right]_0^5 dy$$

$$1 = \int_{10}^{20} \frac{125a}{3} + \frac{25}{2} ay \, dy$$

$$1 = \frac{125a}{3}\left[y\right]_{10}^{20} + \frac{25a}{2}\left[y\right]_{10}^{20}$$

$$1 = \frac{1250a}{3} + 1875a$$

$$a = \frac{3}{6875}$$

$$f_X(x) = \int_{10}^{20} (x^2 + xy)\, a \cdot dy$$

$$= \frac{3}{6875} \int_{10}^{20} (x^2 + xy) \cdot dy$$

$$= \frac{3}{6875} \left[x^2 \left[y \right]_{10}^{20} + x \left[\frac{y^2}{2} \right]_{10}^{20} \right]$$

$$= \frac{3}{6875} (10x^2 + 150x)$$

$$E(Y/x) = y \int (Y/x) = y \int_{10}^{20} \frac{f(x,y)}{f_X(x)}$$

$$= \int_{10}^{20} \frac{\frac{y(\cancel{3})}{\cancel{6875}} (x^2 + xy)}{\frac{\cancel{3}}{\cancel{6875}} (10x^2 + 150x)} dy$$

$$= \frac{1}{10} \int_{10}^{20} \frac{y(x+y)}{(x+15)} dy$$

$$= \frac{1}{10(x+15)} \left[\left[x\frac{y^2}{2} + \frac{y^3}{3} \right]_{10}^{20} \right]$$

$$= \frac{1}{10(x+15)} \left(150x + \frac{7000}{3} \right)$$

$$E(Y/x) = \frac{45x + 700}{3(x+15)}$$

$$P\left(Y > \frac{1}{3}X + 10\right) \Rightarrow$$

$$f_Y(y) = \int_0^5 \frac{3}{6875}(x^2 + xy)\,dx$$

$$= \frac{3}{6875}\left[\frac{x^3}{3} + \frac{x^2}{2}y\right]_0^5$$

$$= \frac{3}{6875}\left[\frac{125}{3} + \frac{25Y}{2}\right]$$

$$= \frac{1}{6875}\left[\frac{250 + 75Y}{2}\right]$$

$$P\left(Y > \frac{1}{3}X + 10\right) = \int_{10}^{20} \frac{1}{6875}\frac{(250 + 75Y)}{2}\,dy$$

$$= \frac{1}{6875(2)}\left[(250)\left(20 - \frac{X}{3} - 10\right) + 75\left(750 - \frac{X^2}{9} - 100 - \frac{20X}{3}\right)\right]$$

$$= \frac{1}{6875(2)}\left[5000 - \frac{250X}{3} - 2500 + 15000 - \frac{75X^2}{3} - 7500 - \frac{1500X}{3}\right]$$

$$= \frac{1}{6875(2)}\left[10000 - \frac{1750X}{3} - \frac{25X^2}{3}\right]$$

(6) $X \sim Poi(\lambda)$ $\qquad Y \sim poi(\mu)$

$M_x(t) = e^{\lambda(e^t-1)} \qquad M_y(t) = e^{\mu(e^t-1)}$

$M_{x-y}(t) = E(e^{t(x-y)})$
$= E(e^{xt-yt})$

$= \frac{E(e^{xt})}{E(e^{yt})} = \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}} = e^{(\lambda-\mu)(e^t-1)}$

$\therefore \quad X-Y \sim poi(\lambda-\mu)$ .... Hence proved.

$M_{x+y}(t) = E(e^{t(x+y)}) = E(e^{tx+yt})$
$= E(e^{tx}) \cdot E(e^{ty})$
$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$
$= e^{(\lambda+\mu)(e^t-1)}$

$\therefore \quad X+Y \sim poi(\lambda+\mu)$ .... hence proved

(7) $var(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$

$= \sum_x x^2 \frac{P(X=x, Y=2)}{P(Y=2)} -$

$\left(\sum_x x \frac{P(X=x, Y=2)}{P(Y=2)}\right)^2$

$= \frac{1^2(0)+2^2(0.3)+3^2(0.1)+4^2(0.2)}{0.6} -$

$\left[\frac{1(0)+2(0.3)+3(0.1)+4(0.2)}{0.6}\right]^2$

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$$= \frac{53}{6} - \frac{289}{36} = \frac{29}{36}$$

ii) $f(u, v) = \frac{48}{67}(2uv - u^2)$, $0 < u < 1$, $\frac{u}{2} < v < 2$

$$E(U/V=v) = \int_0^1 u\, f(u/v=v)\, du$$

$$= \int_0^1 \frac{u f(u, V=v)}{f(V=v)}\, du \quad — ①$$

$$\rightarrow f_v(V=v) = \int_0^1 \frac{48}{67}(2uv - u^2)\, du$$

$$= \frac{48}{67}\left[\frac{\cancel{2}u v}{\cancel{2}} - \frac{u^3}{3}\right]_0^1$$

$$= \frac{48}{67}\left[v - \frac{1}{3}\right]$$

$$= \frac{48[3v-1]}{67(3)} = \frac{16(3v-1)}{67}$$

$$E(U/V=v) = \int_0^1 \frac{u\left(\frac{48}{67}\right)(2uv - u^2)}{\frac{16}{67}(3v-1)}\, du$$

$$= \frac{3}{(3v-1)} \int_0^1 (2u^2 v - u^3)\, du$$

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$$= \frac{3}{(3v-1)} \left[\frac{2u^3 v}{3} - \frac{u^4}{4} \right]_0^1$$

$$= \frac{3}{(3v-1)} \left[\frac{2y}{3} - \frac{1}{4} \right]$$

$$E(U / V = v) = \frac{(8v - 3)}{4(3v-1)}$$

8) $X \sim \text{gamma} \ (3, 2)$

$E(Y / X = x) = 3x + 1 \qquad Var(Y / X = x) = 2x^2+5$

$E(E(Y / X = x)) = E(Y) \qquad var(var(Y / X = x)) = V(Y$

$E(Y) = E(3x + 1) \qquad V(Y) = E(var(Y / X = x)) + V(E(Y / X = x))$

$= 3E(X) + 1 \qquad = E(2x^2 + 5) + V(3x+1)$

$= 3\left(\frac{3}{2}\right) + 1 \qquad = 2E(x^2) + 5 + 9V(X)$

$= \frac{11}{2} \qquad = 2(3) + 5 + 9\left(\frac{3}{4}\right)$

$= 6 + 5 + 6.75$

$= 17.75$

9) $Z = X + Y$

$E(X) = 3 \qquad E(Y) = 4$

$\sqrt{V(X)} = 2 \qquad \sqrt{V(Y)} = 1$

i) $cov(X, Z) = cov(X, X+Y) \qquad corr(X, Y) = -0.3$

$= cov(X, X) + cov(X, Y)$

$= V(X) + (-0.6)$

$= 4 - 0.6 = 3.4$

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ii) $$\begin{aligned} Var(Z) &= var(X+Y) \\ &= V(X) + V(X) + 2\,Cov(X,Y) \\ &= 4 + 1 + 2\,(-0.6) \\ &= 5 \div 1.2 \\ &= 3.8 \end{aligned}$$

10) $$f(x,y) = k x^{-\alpha} e^{-y/13}, \quad 1 < x < \infty$$
$$1 < y < \infty$$
$$\alpha > 1, \quad \beta > 0$$

$$1 = k \int_1^{\infty} x^{-\alpha}\, dx \int_1^{\infty} e^{-y/\beta}\, dy$$

$$1 = k \left[\frac{x^{-\alpha+1}}{-\alpha+1} \right]_1^{\infty} \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^{\infty}$$

$$1 = k \left[0 - \frac{1}{-\alpha+1} \right] \left[0 \cancel{\beta} - \beta \left(e^{-1/\beta} \right) \right]$$

$$1 = k \left[\frac{1}{-\alpha+1} \right] \left[\left(-\beta e^{-1/\beta} \right) \right]$$

$$k = \frac{-1+\alpha}{\beta e^{-1/\beta}}$$

$$k = \frac{\alpha - 1}{\beta e^{-1/\beta}}$$