

Home work 1

Lagrange's Interpolation

1.	x	7	8	9	10
	f(x)	3	1	1	9

$$f(9.5) = ?$$

~~$$f(9.5) = \frac{(9.5-1)(9.5-4)(9.5-9)}{(3-1)(3-1)(3-9)} \times 3$$

$$+ \frac{(9.5-3)(9.5-8)(9.5-10)}{(1-3)(1-8)(1-10)} \times 1$$~~

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \times 3$$

$$+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \times 1$$

$$+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \times 9$$

$$= \frac{3}{16} - \frac{5}{16} + \frac{5}{16} + \frac{45}{16}$$

$$= \underline{\underline{3.625}}$$

2.	x	3	7	9	10	$f(x)$
	$f(x)$	108	120	72	63	

$$\begin{aligned}
 f(6) &= \frac{(6-7)(6-9)(6-10)(108)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)} \\
 &+ \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)} \\
 &= 12 + 180 + 27 - 72 \\
 &= 147
 \end{aligned}$$

3.	x	1	2	7	8	$f(x)$
	$f(x)$	4	5	5	4	

$$\begin{aligned}
 f(6) &= f(1) \frac{(6-2)(6-7)(6-8)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)} \\
 &+ \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)} \\
 &= \frac{-16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{40}{21} \\
 &= \underline{\underline{5.6667}}
 \end{aligned}$$

4.	x	1	2	3	4	7	$f(x)$
	$f(x)$	2	4	8	16	128	

$$\begin{aligned}
 f(5) &= \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} \\
 &+ \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} \\
 &+ \frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)} \\
 &+ \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)} \\
 &= \underline{\underline{32.9334}}
 \end{aligned}$$

5.	x	10	20	14	16	$f(x)$
	$f(x)$	0.25	0.2	0.15	0.10	

$$\begin{aligned}
 f(15) &= \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)(0.2)}{(20-10)(20-14)(20-16)} \\
 &+ \frac{(15-10)(15-20)(15-16)(0.15)}{(14-10)(14-20)(14-16)} + \frac{(15-10)(15-20)(15-14)(0.10)}{(16-10)(16-20)(16-14)} \\
 &= \underline{\underline{0.1208}}
 \end{aligned}$$

Home work - 2

Regula Falsi Method

1. $x^3 + x - 1 = 0$ 0.5 and 1

x	0.5	0.6	0.7
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y	-0.375	-0.184	0.043
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Using ~~$x_1 y_2 - x_2 y_1$~~

$$\frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

Taking $x_1 = 0.6$ $x_2 = 0.7$

$y_1 = -0.184$ $y_2 = 0.043$

$$x_n = \frac{(0.6)(0.043) - (0.7)(-0.184)}{(0.043) - (-0.184)}$$

$$x_n = 0.681057269$$

$$y_n = (0.681057269)^3 + (0.681057269) - 1$$

$$= -0.003041807$$

Taking $x_1 = 0.681057269$ $x_2 = 0.7$

$y_1 = -0.0030401807$ $y_2 = 0.043$

$$x_n = \frac{(0.681057269)(0.043) - (0.7)(-0.0030401807)}{0.043 + 0.0030401807}$$

$$= \underline{\underline{0.682308118}}$$

$$x_n = 0.682308118$$

$$y_n = -0.00042180$$

$\therefore$ Root of $x^3 + x - 1 = 0$ is 0.682308118

$$2. \quad x^3 - 2x + 0.5 = 0$$

0 and 1

x	0	0.1	0.2	0.3
y	0.5	0.301	0.108	-0.073

$$x_1 = 0.3$$

$$x_2 = 0.2$$

$$y_1 = -0.073$$

$$y_2 = 0.108$$

$$x_n = \frac{(0.3)(0.108) - (0.2)(-0.073)}{(0.108) - (-0.073)}$$

$$x_n = 0.259668508$$

$$y_n = \cancel{1.0368458} - 0.001828157$$

$$x_1 = 0.2597$$

$$x_2 = 0.2$$

$$y_1 = -0.00183$$

$$y_2 = 0.108$$

$$x_n = \frac{(0.2597)(0.108) - (0.2)(-0.00183)}{0.108 + 0.00183}$$

$$x_n = 0.258705272$$

$$y_n = -0.00095809$$

$\therefore$ The root of $x^3 - 2x + 0.5 = 0$ is 0.2587

$$3. \quad x^3 + 2x - 20 = 0$$

2 and 2.5

x	2	2.1	2.2	2.3	2.4	2.5
y	-8	-6.539	-4.952	-3.233	-1.376	0.625

$$x_1 = 2.4$$

$$x_2 = 2.5$$

$$y_1 = -1.376$$

$$y_2 = 0.625$$

$$x_n = \frac{(2.4)(0.625) - (2.5)(-1.376)}{0.625 + 1.375}$$

$$= 2.47$$

$$y_n = 0.0092$$

$$x_1 = 2.47$$

$$x_2 = 2.5$$

$$y_1 = 0.0092$$

$$y_2 = 0.625$$

$$x_n = \frac{(2.47)(0.625) - (2.5)(0.0092)}{0.625 - 0.0092}$$

$$= 2.469$$

$$y_n = 0.00124$$

The root of $x^3 + 2x - 20$ is 0.00124

4. $3x - \cos x - 1 = 0$

0 and 1

x 0 ~~0.01~~ 1 ~~0.2~~

y -2 1.001523

$x_1 = 0$

$x_2 = 1$

$y_1 = -2$

$y_2 = 1.001523$

$x_n = 0.66663$

$y_n = -0.0004231$

Root is 0.66663

5. $x^3 - 3x - 5 = 0$ 2 and 2.5

x	2	2.1	2.2	2.3
y	-3	-2.039	-0.952	0.267

$x_1 = 2.2$ $x_2 = 2.3$

$y_1 = -0.952$ $y_2 = 0.267$

$$x_n = \frac{(2.2)(0.267) - (2.3)(-0.952)}{0.267 + 0.952}$$

$= 2.279$

$y_n = -0.011$

$x_1 = 2.279$ $x_2 = 2.3$

$y_1 = -0.011$ $y_2 = 0.267$

$$x_n = \frac{(2.279)(0.267) - (2.3)(-0.011)}{0.267 + 0.011}$$

$= 2.271$

$y_n = -0.098$

$x_1 = 2.271$ $x_2 = 2.3$

$y_1 = -0.098$ $y_2 = 0.267$

$x_n = 2.278$

$y_n = -0.0029$

$x_1 = 2.278$ $x_2 = 2.3$

$y_1 = -0.0029$ $y_2 = 0.267$

$x_n = 2.278$ $y_n = -0.0098$

$x_1 = 2.278$ $x_2 = 2.3$

$y_1 = -0.0098$ $y_2 = 0.267$

$x_n = 2.278$ $y_n = -0.00301$

$x_1 = 2.278$ $x_2 = 2.3$

$y_1 = -0.003$ $y_2 = 0.267$

$x_n = 2.278$

$x_1 = 2.278$

$y_1 = -0.009$

$x_n = 2.277$

$x_2 = 2.3$

$y_2 = 0.267$

$y_n = -0.00017$

Root = 2.279

6. $x \log_{10} x - 1.2 = 0$

2 and 3

$x \quad 2 \quad 3$

$y \quad -0.59744 \quad 0.23136$

$x_1 = 2$

$x_2 = 3$

$y_1 = -0.59744$

$y_2 = 0.23136$

$x_n = \frac{2(0.23136) - (3)(-0.59744)}{0.23136 + 0.59744}$

$= \frac{2.23136 + 1.79232}{0.8288}$

$= 2.721018$

$y_n = -0.0170871$

$x_1 = 2.721018$

$x_2 = 3$

$y_1 = -0.017$

$y_2 = 0.231$

$x_n = 2.740206$

$y_n = -0.00038381$

$\therefore$ Root of $x \log_{10} x - 1.2 = 0$ is 2.740206

-10
 5
 $(-17)(1) -$

7. $x^3 + 4x + 1 = 0$

~~| | | | |
|-----|---|----|----|
| x | 0 | 1 | 2 |
| y | 1 | 6 | 17 |
| x | 0 | -1 | |
| y | 1 | -4 | |~~

~~$x_1 = 0 - 1$ $x_2 = 0$
 $y_1 = -4$ $y_2 = 1$
 $x_n = -0.2$ $y_n = 0.192$
 $x_1 = -0.2$ $x_2 = 0$
 $y_1 = 0.192$ $y_2 = 1$
 $x_n = -0.247$ $y_n = 9.192$
 $x_1 = -0.247$ $x_2 = 0$
 $y_1 = 9.192$ $y_2 = 1$
 $x_n = 0.03$ $y_n = 1.12$~~

7. $x^3 - 4x + 1 = 0$

x	0	0.1	0.2	0.3
y	1	0.6	0.208	-0.173

$x_1 = 0.2$ $x_2 = 0.2$
 $y_1 = -0.173$ $y_2 = 0.208$
 $x_n = 0.2546$ $y_n = -0.0018$
 $x_1 = 0.2546$ $x_2 = 0.2$
 $y_1 = -0.0018$ $y_2 = 0.208$
 $x_n = 0.2541$ $y_n = -0.0002$

∴ Root of $x^3 - 4x + 1 = 0$ is 0.2541

$$8. \quad x^3 + 2x^2 - 8 = 0$$

x	0	1	2
y	-8	-5	8

$$x_1 = 1 \quad x_2 = 2$$

$$y_1 = -5 \quad y_2 = 8$$

$$x_n = \frac{(1)(8) - (2)(-5)}{8 + 5}$$

$$= \frac{18}{13} = 1.384$$

$$y_n = -1.5115$$

$$x_1 = 1.384 \quad x_2 = 2$$

$$y_1 = -1.5115 \quad y_2 = 8$$

$$x_n = 1.481$$

$$y_n = -1.5115157$$

$$x_1 = 1.481 \quad x_2 = 2$$

$$y_1 = -1.5115157 \quad y_2 = 8$$

$$x_n = 1.563$$

$$y_n = 0.710$$

$$x_1 = 1.563 \quad x_2 = 2$$

$$y_1 = 0.71 \quad y_2 = 8$$

$$x_n = 1.599 \quad y_n =$$

q. $x^3 - 9x + 1 = 0$

$$\begin{array}{ccc} x & 0 & 1 \\ y & 1 & -7 \end{array}$$

$$x_1 = 1 \quad x_2 = 0$$

$$y_1 = -7 \quad y_2 = 1$$

$$x_n = \frac{(1)(1) - (0)(-7)}{1+7}$$

$$= 0.125$$

$$y_n = -0.123$$

$$x_1 = 0.125 \quad x_2 = 0$$

$$y_1 = -0.123 \quad y_2 = 1$$

$$x_n = 0.1113$$

$$y_n = -0.000401$$

$\therefore$ Root of $x^3 - 9x + 1 = 0$ is 0.1113

$$(1)^3 - (1) - 4$$

10. $x^3 - x - 4 = 0$

$$x \quad 0 \quad 1 \quad 2$$

$$y \quad -4 \quad -4 \quad 2$$

$$x_1 = 1$$

$$x_2 = 2$$

$$y_1 = -4$$

$$y_2 = 2$$

$$x_n = \frac{(1)(2) - (2)(-4)}{2 + 4}$$

$$= \frac{2 + 8}{6}$$

$$= 1.6667$$

$$y_n = -1.037$$

$$x_1 = 1.667$$

$$x_2 = 2$$

$$y_1 = -1.037$$

$$y_2 = 2$$

$$x_n = \frac{(1.667)(2) - (2)(-1.037)}{2 + 1.037}$$

$$= 1.780704643$$

$$y_n = -0.1343$$

$$x_1 = 1.78$$

$$x_2 = 2$$

$$y_1 = -0.1343$$

$$y_2 = 2$$

$$x_n = \frac{(1.78)(2) - (2)(-0.1343)}{2 + 0.1343}$$

$$= 1.793843415$$

$$y_n = -0.0214$$

$$x_1 = 1.793$$

$$x_2 = 2$$

$$y_1 = -0.0214$$

$$y_2 = 2$$

$$x_n = \frac{(1.793)(2) - (2)(-0.0214)}{2 + 0.0214}$$

$$= 1.795191451$$

$$y_n = -0.009805795$$

$$x_1 = 1.795$$

$$x_2 = 2$$

$$y_1 = -0.0098$$

$$y_2 = 2$$

$$x_n = 1.795$$

$$y_n = -0.0027$$