FM-Assignment 1

1) $d^{(4)} = 8\%$.

(i)
$$1-d = \left(1-\frac{d^{(4)}}{4}\right)^4$$

$$\cancel{1-d} \quad 1-\left(1-\frac{d^{(4)}}{4}\right)^4 = d$$

$$d = 1-\left(1-\frac{8}{100\times4}\right)^4$$

$$d = 7.7631840\%$$

(i) $\delta = -\ln(1-d)$
$$= -\ln(1-7.763184)$$
$$= 8.081083\%$$

(ii) $i = \dfrac{d}{1-d}$
$$= \frac{0.07763184}{1-0.07763184}$$
$$= 8.416578\%.$$

(iii) $d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$
$$= 12\left[1(1-0.07763184)^{1/12}\right]$$
$$= 8.053937\%.$$

Page No.
Date

2) i) $\delta(t) = 0.004t + 0.0002t^2$

Amt. due = ₹1000

$PV_0 = C \times v(t)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2}$

$= 1000 \times e^{-\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= 1000 \times e^{-0.266666667}$

$= ₹765.93$

(ii) $(1+i)^{10} = e^{0.266666667}$

$i = 2.7025403\%$

3) i) The equation $d = iv$, has an interesting verbal interpertation. The interest charged on an investment paid at the beginning of the time period is d and the one at the end of the period is i. If we discount i from the end of the period with the accumulation factor v, we obtain d.

Proof:

We know that

$A(0,t) = \frac{1}{v(0,t)}$

$(1+i) = \frac{1}{(1-d)}$ — ①

$1-d = \frac{1}{1+i}$

$d = 1 - \frac{1}{1+i}$

$d = \frac{i}{1+i}$

$d = \cancel{i}$

$d = i \cdot \frac{1}{1+i}$

$\therefore d = iv$ — from eqn. 1

(ii) a) $\frac{d^{(4)}}{4} = 2.25\%$

$d = 1 - (1 - 2.25\%)^4$

$d = 8.7007806\%$

$d^{(2)} = 2(1 - (1-d)^{1/2})$

$d^{(2)} = 2(1 - (1 - 8.7007806\%)^{1/2})$

$d^{(2)} = 8.89875\%$

b) $d^{(1/2)} = 5\%$

$d = 1 - \left(1 - \frac{d^{(12)}}{1/2}\right)^{1/2}$

$d = 1 - (1 - 10\%)^{1/2}$

$d = 5.1316702\%$

$d^{(2)} = 2(1 - (1-d)^{1/2})$

$d^{(2)} = 5.1992507\%$

(iii) $v\left(0, \frac{91}{365}\right) = \frac{1}{(1+i)^{91/365}} = \left(1 - \frac{91}{365}d\right)$

$\frac{1}{(1.05)^{91/365}} = \left(1 - \frac{91}{365}d\right)$

$d = \frac{365}{91}\left(1 - \frac{1}{(1.05)^{91/365}}\right)$

$d = 4.849462\%$

Q4) i) Same as 3) i)

(ii) $i = 0.08$

(a) $d^{(12)}$

$$d = \frac{i}{1+i}$$

$$d = \frac{0.08}{1+0.08}$$

$$= 7.407407407\%$$

$$d^{(12)} = 12\left[1-(1-d)^{1/12}\right]$$

$$d^{(12)} = 7.671478\%$$

(b) $i^{(365)} = 365\left((1+i)^{1/365}-1\right)$

$$= 7.696915\%$$

(c) $\delta = \ln(1+i)$

$$= \ln(1+0.08)$$

$$= 7.696104\%$$

d) $i^{(0.5)} = \frac{1}{2}\left((1+i)^{1/\frac{1}{2}}-1\right)$

$$= \frac{1}{2}\left((1+i)^2-1\right)$$

$$= 8.32\%$$

Q5) $d^{(4)} = 6\% = 0.06$

i) a) $\left(1+\frac{i^{(2)}}{2}\right)^{2} = \left(1-\frac{d^{(4)}}{4}\right)^{-4}$

$\left(1+\frac{i^{(2)}}{2}\right)^{2} = \left(1-\frac{d^{4}}{4}\right)^{-4}$

$i^{(2)} = \left[\left(1-\frac{d^{(4)}}{4}\right)^{-2} - 1\right] \times 2$

$i^{(2)} = 6.13775\%$

b) $\left(1+\frac{d^{(12)}}{12}\right)^{12} = \left(1-\frac{d^{(4)}}{4}\right)^{4}$

$d^{(12)} = \left[1-\left(1-\frac{d^{(4)}}{4}\right)^{1/3}\right] \times 12$

$d^{(12)} = 6.03025\%$

ii) $\frac{22\cancel{0000}}{20\cancel{0000}} = 1+i$

$i = \frac{22-20}{20}$

$i = \frac{\cancel{2}^{1}}{\cancel{20}\,10}$

$i = 0.1$

$i = 10\%$

Time period between 15th April, 2005 to 17th July, 2005 is 3 months

Date

Sum paid on 17^{th} July, 2005 $= \left[(1+i)^{3/12}\right] \times 200,000$

$= [1.1]^{3/12} \times 200,000$

$= 204822.7378$

6)

i) $X = R(1-0.3) - (\text{cash})$ $\quad R = \text{Retail Price}$

$Y = R(1-0.25)^2 - (\text{Six month credit})$

$\frac{X}{Y} = \frac{R(\cancel{1-0.3}\ 1-0.3)}{R\ (1-0.25)^2} = 1-d$

$\frac{0.7}{(0.75)} = 1-d$

$d = \frac{0.75-0.7}{0.75}$

$d = \frac{0.05}{0.75}$

$= \frac{5}{75}$

$= 6.6667\%$

(ii) $d^{(12)} = \left[1-(1-d)^{1/12}\right] \times 12$

$d^{(12)} = 6.8798\%$

$i^{(2)} = \left[(1-d)^{-1/2} - 1\right] \times 2$

$i^{(2)} = 7.02003\%$

$\delta = -\ln(1-d)$

$\delta = 6.8996\%$

Page No.
Date

7) $C = 20,000$

$AV = 26,000$

$n = 17$ months

$= 1 + \frac{5}{12}$ years

$= 1.416666667$

$AV = C(1+i)^n$

$26000 = 20,000(1+i)^{1.416666667}$

$i = 20.34570672\%$

(i) $i^{(4)} = 4((1+i)^{1/4} - 1)$

$i^{(4)} = 18.955255\%$

(ii) $d = \frac{i}{1+i} = 16.90605114\%$

$d^{(2)} = 2(1-(1-d)^{1/2})$

$d^{(2)} = 17.688235\%$

(iii) $26000 = 20000(1 + 1.416666667 \times i)$

$\frac{26}{20} = 1 + 1.416666667 \times i$

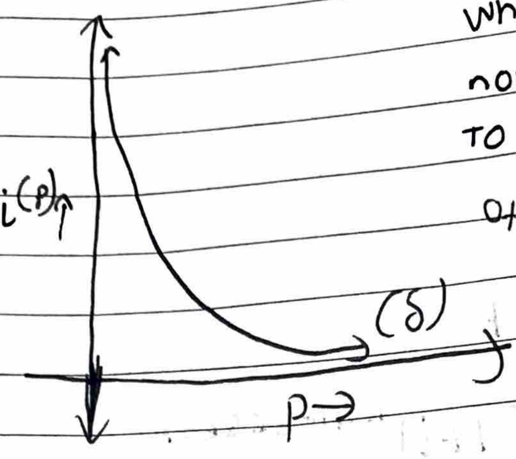
$i = 21.1764706\%$

Simple interest rate = 21.1764706%

(iv) Rate of simple interest > i > $i^{(4)}$ > $d^{(2)}$ > d

In terms of compounding, is the greatest numerical value and d is the lowest.

8)



When we increase p, the nominal rate of interest tends to come closer to the value of force of interest.

9) i) Force of interest refers to a nominal interest rate or a discount rate compounded infinite number of times per time period.

(ii) $i^{(2)} = 0.0775$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1$$

$$i = \left(1 + \frac{0.0775}{2}\right)^2 - 1$$

$$i = 7.90015\%$$

$$\delta = \ln(1+i)$$

$$\delta = \ln(1.0790015)$$

$$\delta = 7.60361\%$$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$$

$$d^{(12)} = \left[1 - (1+i)^{-1/12}\right] \times 12$$

$$d^{(12)} = \left[1 - \left(1 + \frac{7.90015}{100}\right)^{1/12}\right] \times 12$$

$$d^{(12)} = 7.579568\%$$

Page No.
Date

$$\left(1+2i^{(1/2)}\right)^{1/2} = 1+i$$

$$1+2i^{(1/2)} = (1+i)^2$$
$$2i^{(1/2)} = (1+0.0790015)^2$$
$$i^{(1/2)} = \frac{(1+0.0790015)^2}{2}$$

$$i^{(1/2)} = 8.21221\%$$

10) $$\delta(t) = \begin{cases} 0.08 & , 0<t\leq 5 \\ 0.06 & , 5\leq t\leq 10 \\ 0.04 & , t\geq 10 \end{cases}$$

- when $0\leq t\leq 5$

$$v(t) = e^{-\int_0^t 0.08\,ds}$$

$$= e^{-[0.08s]_0^t}$$
$$= e^{-0.08t}$$

- when $5\leq t\leq 10$

$$v(t) = e^{-0.08(5)} \cdot e^{-\int_5^t 0.06\,ds}$$
$$= e^{-0.4}\left(e^{-[0.06s]_5^t}\right)$$
$$= e^{-0.4}\left(e^{-0.06t+0.3}\right)$$
$$v(t) = e^{-0.1-0.06t}$$

- When $t\geq 10$

$$v(t) = e^{-0.1-0.06(10)} \cdot e^{-\int_{10}^t 0.04\,ds}$$
$$= e^{-0.1-0.6} \cdot e^{-[0.04s]_{10}^t}$$
$$= e^{-0.7} \cdot e^{-0.04t+0.4}$$
$$v(t) = e^{-0.3-0.04t}$$

Page No.
Date

11) a) $T = 9$ month loan

$C = 50000$

$d = 0.18$ p.a.

$$PV_0 = C(1-d)^{9/12}$$

$$PV_0 = 50000(1-0.18)^{9/12}$$

$$= ₹43085.42623$$

12)

b) $\delta = 6\%$

(i) $$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$$

$$d^{(12)} = \left[1 - e^{-\delta/12}\right] \times 12$$

$$d^{(12)} = 5.985023\%$$

(ii) $d^{(4)} = 6\%$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$$

$$1 + \frac{i^{(12)}}{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{-1/3}$$

$$i^{(12)} = \left[\left(1 - \frac{d^{(4)}}{4}\right)^{-1/3} - 1\right] \times 12$$

$$i^{(12)} = 6.060708\%$$

c) $d < \delta < i^{(4)} < i$

d) Same as 3) i)

Page No.
Date

12) $C = 7049$
~~AV = 7049~~
$i^{(12)} = 6\%$ $t = 0, 2$
$i_s = 1.5\%$ $t = 2, 4\frac{1}{2}$ $= 10$ quarters
$d^{(12)} = 6\%$ $t = 4\frac{1}{2}, 6$

$$AV_6 = 7049\left(1 + \frac{i^{(12)}}{12}\right)^{24}\left(1 + (0.015) \times 10\right)\left(1 - \frac{d^{(12)}}{12}\right)^{-18}$$

$$AV_6 = 7049(1.127159)(1.15)(1.094421)$$
$$AV_6 = 9999.883762$$

13) i) $C = x$
$AV = 2x$
$AV = C(1+0.1)^t$
$2x = x(1.1)^t$
$\ln(2) = t \ln(1.1)$
$t = \frac{\ln(2)}{\ln(1.1)}$
$t = 7.2725$ years

(ii) $\left(1 - \frac{0.1}{12}\right)^{-12t} = 2$

$$\ln 2 = -12t \ln\left(1 - \frac{0.1}{12}\right)$$

$$t = \frac{\ln(2)}{-12\ln\left(1 - \frac{0.1}{12}\right)}$$
$$= 6.90255$$