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Assignment - 2.

1. $$I = \int_{1/50}^{2} \frac{e^{2/w}}{w^2} \cdot dw \quad -(1)$$

Put $t = \frac{2}{w} \quad -(2)$

$\therefore \frac{dt}{dw} = -2w^{-2}$

$\frac{dt}{-2} = w^{-2} \cdot dw \quad -(3)$

subs. (2) & (3) in (1)

$$I = \int_{100}^{1} \frac{e^t}{-2} \cdot dt$$

when $w = 1/50$
$t = \frac{2}{1/50} = 100$

$$= -\frac{1}{2} \int_{100}^{1} e^t \cdot dt$$

when $w = 2$
$t = \frac{2}{2} = 1$
— from (2)

$$= -\frac{1}{2} \left[e^t \right]_{100}^{1}$$

$$= \frac{-1}{2} \left[\frac{e^1}{1} - e^{100} \right]$$

$$= -\frac{1}{2} \times \left(2.718 - 2.68117 \times 10^{43}\right)$$

$$= \cancel{1.34405857 \times 10^{41}} = 1.30585 \times 10^{43}$$

2. $I = \int \frac{2t^3+1}{(t^4+2t)^3} \cdot dt$ —(1)

Put $u = t^4 + 2t$ —(a)

$\frac{du}{dt} = 4t^3 + 2$

$= 2(2t^3+1)$

$\frac{du}{2} = 2t^3+1 \cdot dt$ —(2)

~~when~~ from (1) and (2)

$I = \frac{1}{2} \int \frac{1}{u^3} \cdot du$

$= \frac{1}{2} \int u^{-3} \cdot du$

$= \frac{1}{2} \cdot \frac{u^{-2}}{-2} + c$

$= -\frac{1}{4} u^{-2} + c$

$=$ from a, $-\frac{1}{4} (t^4+2t)^{-2} + c$

3. $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)$

$= \int x^4 + 3x - 9 \cdot dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

$$f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$$

4. $$\int_{-2}^{3} f(x) \cdot dx$$

$$= \int_{-2}^{1} 3x^2 \cdot dx + \int_{1}^{3} 6 \cdot dx$$

$$= \left(\frac{3(x^3)}{3} \right)_{-2}^{1} + (6x)_{1}^{3}$$

$$= 1 + \frac{8}{3} + 18 - 6$$

$$= \frac{3+8}{3} + 12 = \frac{11+36}{3} = \frac{47}{3}$$

6. $$I = \int 3(8y-1) \cdot e^{4y^2-y} \cdot dy \quad — ①$$

put $t = 4y^2 - y \quad — ②$

$$\frac{dt}{dy} = 8y - 1$$

$$dt = (8y-1) \cdot dt \quad - ③$$

$$I = 3 \int 1 \cdot e^t \cdot dt$$

$$= 3 \cdot [e^t] + c$$

$$I = 3 \cdot [e^{4y^2-y}] + c$$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6, \quad f(1) = \frac{-5}{4}, \quad f(4)$

$f'(x) = \int f''(x).dx = \int 15x^{1/2} + 5x^3 + 6$

$= \left[\frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x\right] + c_1$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + c_1$

$f(x) = \int f'(x).dx = \int\left[10x^{3/2} + \frac{5}{4}x^4 + 6x + c_1\right]dx$

$= \frac{10x^{5/2}}{5/2} + \frac{5}{4}\cdot\frac{x^5}{5} + \frac{6x^2}{2} + c_1x + c_2$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c_1x + c_2$

Put $x=1$,

$\cancel{f(1) = -\frac{5}{4} = 10 + \frac{5}{4} + 6 + c_1}$

$\cancel{0 = -\frac{10}{4} + 10 + 6 + c_1}$

$\cancel{\frac{10}{4} - 16 = c_1}$

$f(1) = 4 + \frac{1}{4} + 3 + c_1 + c_2 = -\frac{5}{4}$

$16 + 1 + 12 + 4c_1 + 4c_2 = -5$

$29 + 5 + 4(c_1 + c_2) = 0$

$4(c_1 + c_2) = -34$

$c_1 + c_2 = -17/2$

$c_2 = -\frac{17}{2} - c_1 \quad -(1)$

$f(4) = 404$

$$404 = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + c_1(4) + c_2$$

$$128 + 256 + 48 + c_1(4) + c_2 = 404$$

$$c_1(4) + c_2 = -28$$

$$4c_1 + \left(\frac{-17}{2} - c_1\right) = -28$$

$$4c_1 - \frac{17}{2} - 2c_1 = -28$$

$$8c_1 - 17 - 2c_1 = -56$$

$$6c_1 = -39$$

$$\boxed{c_1 = -6.5}$$

$$c_2 = \frac{-17}{2} - (-6.5)$$

$$\boxed{c_2 = -8.5 + 6.5 = -2}$$

from ①, $f(x) =$

$$\boxed{4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2}$$

//

7. Formulate PDE from $f(x+iy) + g(x-iy)$

Let $u(x,y) = f(x+iy) + g(x-iy)$

Let $z = x+iy$, $\bar{z} = x - iy$ where $i = \sqrt{-1}$

$\therefore u = f(z) + g(\bar{z})$, ∂ = del

Partially diff. 'u' wrt 'x'

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial f(z)}{\partial x} + \frac{\partial}{\partial x} g(\bar{z})$$

$$= f'(z) \cdot \frac{\partial z}{\partial x} + g'(\bar{z}) \frac{\partial}{\partial x}(\bar{z})$$

$$= f'(z)\frac{\partial z}{\partial x} + g'(\bar{z})\frac{\partial}{\partial x}(\bar{z})$$

$$= f'(x+iy) + \frac{\partial}{\partial x}(x+iy) + g'(x-iy)\cdot\frac{\partial}{\partial x}(x-iy)$$

$$= f'(x+iy)(1+0) + g'(x-iy)(1-0)$$

$$\frac{\partial u}{\partial x} = f'(x+iy) + g'(x-iy)$$

Similarly, P.D 'u' wrt 'y'

$$\frac{\partial u}{dy} = f'(z)\frac{\partial z}{\partial y} + g'(\bar{z})\frac{\partial z}{\partial y}$$

$$= f'(x+iy)\cdot(0+1.i) + g'(x-iy)(0-i.1)$$

$$\frac{\partial u}{dy} = i[f'(x+iy) - g'(x-iy)]$$

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r(1) = 2$

$$\frac{dr}{r^2} = \frac{1}{\theta}\cdot d\theta$$

on $\int$ on b.s.

$$\int \frac{1}{r^2}\cdot dr = \int \frac{1}{\theta}\cdot d\theta$$

$$\frac{r^{-2+1}}{-2+1} = \log\theta$$

$$\frac{r^{-1}}{-1} + c = \log\theta$$

$$c = \log\theta + r^{-1} \quad \text{—} (1)$$

$r(1) = 2 = \cancel{\log\theta + (1)^{-1}}$

$\cancel{2-1}$ Put $\theta = 1$ & $r = 2$

$c = \log 1 + 1/2$

$= 0 + 1/2$

$c = 1$

$\left| \frac{1}{\ } - \log_e \frac{1}{\ } \right|$

$$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\frac{dv}{9.8 - 0.196v} = 1 \cdot dt$$

$$\frac{dv}{1 - 0.02v} = 9.8\, dt$$

on $\int$

$$\int \frac{1}{1-0.02v} \cdot dv = 9.8 \int 1 \cdot dt$$

$$\frac{\log(1-0.02v)}{-0.02} = 9.8t + c$$

Initial value Prob. (IVP).

$$ty' + 2y = t^2 - t + 1 \qquad y(1) = \tfrac{1}{2}$$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$\div$ by t, $\frac{dy}{dt} + \frac{2}{t}y = \frac{t^2}{t} - \frac{t}{t} + \frac{1}{t}$

Type: $\frac{dy}{dt} + Py = Q$ where $P = 2/t$

$Q = t - 1 + 1/t$

P & Q are functions of only 't'

$\therefore$ Given ODE is LINEAR

Integrating factor: (I.F.) $= e^{\int P dt} = e^{\int \frac{2}{t} dt}$

$= e^{2\int \frac{1}{t} dt} = e^{2\log t} = e^{\log t^2}$

$= t^2$

General Sol^n is

$$y\ I.F = \int Q \cdot IF\,dt + C$$

$$y \cdot t^2 = \int \left(t - 1 + \frac{1}{t}\right) \times t^2\,dt + C$$

$$= \int (t^3 - t^2 + t)\,dt + C$$

$$= \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + C \quad -(1)$$

IVP → Put $t = 1$ & $y = \frac{1}{2}$

$$\frac{1}{2} \cdot 1^2 = \frac{1}{4} - \frac{1}{3} + \frac{\cancel{1}}{\cancel{2}} + C$$

$$C = \frac{1}{3} - \frac{1}{4}$$

$$C = \frac{1}{12} \quad -(2)$$

From (1) & (2), ~~General~~ Particular sol^n is

$$y t^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}$$

Q11.
$$y' = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1 \quad -(1)$$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\frac{1}{y^3} \cdot dy = \frac{x}{(1+x^2)^{1/2}} \cdot dx$$

$\int$ on both sides.

$$\int \frac{1}{y^3}\,dy = \int \frac{x}{(1+x^2)^{1/2}} \cdot dx \qquad \frac{d}{dx}(1+x^2)$$

$$\int y^{-3}\,dy = \int x \cdot (1+x^2)^{-1/2} \cdot dx \qquad \frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}}\,dx$$

$$\frac{\cancel{y^{-2}}}{\cancel{2}}$$

$$\frac{y^{-2}}{2} = \frac{1}{2} \times 2\sqrt{1+x^2} + c \quad \rightarrow \int \frac{f'(x)}{\sqrt{f(x)}} = 2\sqrt{f(x)} + c$$

$$\frac{y^{-2}}{2} = \frac{1}{2y^2} = \sqrt{1+x^2} + c$$

put $x = 0$, $y = -1$ from ①

$$\frac{(-1)^{-2}}{2} = \sqrt{1+0^2} + c$$

$$\frac{1}{2} = 1 + c$$

$$\boxed{c = -1/2}$$

Particular soln:-

$$\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{1}{2}$$

$$\therefore \frac{1}{2y^2} = \frac{2\sqrt{1+x^2} - 1}{2}$$

$$\therefore y^2 = \frac{1}{2\sqrt{1+x^2} - 1}$$

$$y = \pm \frac{1}{\sqrt{2\sqrt{1+x^2} - 1}}$$

Above ~~eqn~~ DE is valid when

$1 + x^2 \geq 0$, which is true for all real values of x.

$\cancel{x^2 \geq -1}$

and,

and,

$$2\sqrt{1+x^2} - 1 > 0$$

$$2\sqrt{1+x^2} > 1$$

$$\sqrt{1+x^2} > 1/2$$

$$\cancel{\sqrt{1+}}\ 1+x^2 > \frac{1}{4}$$

$$x^2 > \frac{-3}{4}$$, which is not possible for real x.

The ~~Soln to D.E~~ ~~D.E.~~ $y = \pm \frac{1}{\sqrt{2\sqrt{1+x^2} - 1}}$ is defined for all real values of x.

Taylor,

12. $f(x) = x^3 - 10x^2 + 6 = -57$ abt. $x = 3$

$f'(x) = 3x^2 - 10(2x) + 0 = 3x^2 - 20x = -33$

$f''(x) = 6x - 20 = -2$

$f'''(x) = 6$

$x = a = 3$

$f(x) = f(a) + (x-a)\frac{f'(x)}{1!} + \frac{f''(x)}{2!}\cdot(x-a)^2 + \frac{f'''(x)}{3!}(x-a)^3$

$= -57 + (x-3)\cdot -33 + (x-3)^2\cdot\frac{-2}{2} + (x-3)^3\cdot\frac{6}{3\times 2}$

$= -57 - 33(x-3) - (x-3)^2 + (x-3)^3 + \ldots$

13. Maclaurin series

$f(x) = (1+x)^u$

$f'(x) = u(1+x)^{u-1}$

$f''(x) = u(u-1)(1+x)^{u-2}$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3}$

let $x = a = 0$

$f(x) = 1^u = 1$

$f'(x) = u$

$f''(x) = u(u-1)$

$f'''(x) = u(u-1)(u-2)$

$\therefore f(x) = (1+x)^u = 1 + u\cdot x + \frac{u(u-1)}{2!}x^2 + \frac{u(u-1)(u-2)}{3!}\cdot x^3 + \ldots$

14. $f(x) = \sqrt{1+x} = (1+x)^{1/2} = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2} = \frac{1}{2}$

$f''(x) = \frac{1}{2}\times\frac{-1}{2}(1+x)^{-3/2} = \frac{-1}{4}(1+x)^{-3/2} = -\frac{1}{4}$

$f'''(x) = \frac{-1}{4}\cdot\frac{-3}{2}(1+x)^{-5/2} = \frac{3}{8}(1+x)^{-5/2} = \frac{3}{8}$

$$f(x) = 1 + \frac{1}{2}(x-0)^1 + \frac{(x-0)^2}{2!} \cdot -\frac{1}{4} + \frac{3}{8}\frac{(x)^3}{3!} + \ldots$$

$$f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots$$

→ Maclaurin series.

15. $f(x) = e^{-6x}$ about $x = -4$

$f'(x) = e^{-6x}(-6) = -6e^{-6x}$

$f''(x) = 36e^{-6x}$

$f'''(x) = -216e^{-6x}$

Put $x = a = -4$

$f(a) = e^{24}$

$f'(a) = -6e^{24}$

$f''(a) = 36e^{24}$

$f'''(a) = -216e^{24}$

By Taylor Series,

$$e^{-6x} = e^{24} + (x+4)\cdot\left(\frac{-6e^{24}}{1}\right) + (x+4)^2\cdot\left(\frac{36e^{24}}{2}\right) + (x+4)^3\left(\frac{-216e^{24}}{6}\right)$$

$$e^{-6x} = e^{24}\left(1 - 6(x+4) + 18(x+4)^2 - 36(x+4)^3 + \ldots\right)$$

16. Taylor S.

$f(x) = \ln(3+4x)$ about $x = 0$

$f'(x) = \frac{1}{3+4x}(4)$

$f''(x) = (3+4x)^{-1} \cdot 4$

$\cancel{(3+4x)^{-1} \cdot 0} + -4 \cdot (3+4x)^{-2}$

$f''(x) = -4(3+4x)^{-2} \cdot 4 = -16(3+4x)^{-2}$

$f'''(x) = +8 \times 4 (3+4x)^{-3} \cdot 4 = 128(3+4x)^{-3}$

Put $x = a = 0$

$f(a) = \ln(3)$

$f'(a) = \frac{4}{3} = \cancel{1.34}$

$\cancel{f''(a) = 8(3)^{-2} = 0.888 = 0.89}$ $\quad f''(a) = -16/9$

$\cancel{f'''(a) = 8(3)^{-3} = 0.296}$ $\quad f'''(a) = \frac{128}{27}$

$f(x) = f(a) + (x-a) \cdot \frac{f'(a)}{1!} + (x-a)^2 \cdot \frac{f''(a)}{2!} + (x-a)^3 \cdot \frac{f'''(a)}{3!}$

$= \ln 3 + \frac{4}{3}x - \frac{\cancel{16}^{8}}{9} \cdot \frac{x^2}{\cancel{2}} + \frac{\cancel{128}^{64}}{27} \cdot \frac{x^3}{3 \times \cancel{2} \times 1} + \cdots$

$= \ln 3 + \frac{4}{3}x - \frac{8}{9}x^2 + \frac{64}{81}x^3 + \cdots$