Probability Assignment

Q1) Mean $= \dfrac{4+7+9+11+15+18+18+18+25}{10} = 13.4$

Median $= 4, 7, 9, 9, 11, 15, 18, 18, 18, 25$

$= \dfrac{10+1}{2} = \cancel{50}\,5.5^{th}$ Term.

$\Rightarrow \cancel{\dfrac{15+18}{2}} = \dfrac{11+15}{2} = 13$

Mode $= 18$

ii) $\sum f_i x_i = 0\times5 + 1\times11 + 2\times26 + 3\times15 + 4\times3$

$= 11 + 52 + 45 + 12 = 120$

$\sum x_i = 0 + 1 + 2 + 3 + 4 = 10$

$\sum f_i = 60$

Mean $= \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{120}{60} = 2$

Median $= \dfrac{5+1}{2} = 3rd$ term $= 2$

Mode $= 2$

Q2) i) Poisson $= \dfrac{e^{-0.5} \cdot \lambda^0}{0!} = \dfrac{e^{-0.5} \times 1}{1} = 0.606530$

$n = 0$

Q3) i) Mean $= \dfrac{\alpha}{\lambda} = 2/\tfrac{1}{2} = 4$

Standard deviation $= \sqrt{\dfrac{\alpha}{\lambda^2}} = \sqrt{\dfrac{2}{(1/2)^2}} = \sqrt{8}$

ii) $CDF = \int_0^{\alpha} \frac{(\frac{1}{2})^2}{(2-1)!} x^{1} e^{-\frac{1}{2}x} = \frac{1}{4}\int x e^{-\frac{1}{2}x} dx$

3.4

$= \frac{1}{4}\left(x(e^{-1/2})\cdot(-2) - \int e^{-\frac{1}{2}x}(-2)\, dx\right.$

$= \frac{1}{4}\left(-2x^{-\frac{1}{2}x} - (-2)\cdot e^{-\frac{1}{2}x}\cdot(-2)\right)$

$= \frac{1}{4}\left(-2xe^{+\frac{1}{2}x} - 4e^{-\frac{1}{2}x}\right)_0^{\alpha}$

$= \frac{1}{4}(0 - (-4)) = 1$

= 13

Q4) No. of male $= 10$
No. of female $= 8$

Fully qualified $= 6$ males
$= 7$ females

+4x3

20

Team of two $= {}^{10}C_6 \times {}^{8}C_7$

Q5) 40% via level 1 $= P(1) = 40\%$
$P(2) = 50\%$

2

Prob of package arriving late $= 20\%, 40\%, 20\%$

Q6) $X \sim U(1, 2)$ and $Y = \frac{3}{6-X}$ $\quad y \sim U\left(\frac{3}{5}, \frac{3}{4}\right)$

2

Q7) i) $F(3) = 0.05 + 0.15 + 0.35 = 0.55$

ii) $E(X) = 0.05 + 0.3 + 1.05 + 1.6 + 0.25 = 3.25$

5065 30

iii) $E(X)^2 = 1^2 \times 0.05 + 2^2 \times 0.15 + 3^2 \times 0.35 + 4^2 \times 0.47 + 5^2 \times 0.05$

$= 0.05 + 0.6 + 3.15 + 7.52 + 1.25$

$= 12.52$

$V(X) = 12.52 - 10.56$

$= 1.9575$

8) P (exceeding 1000) = 0.2

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= {}^{15}C_0 (0.2)^0 (0.8)^{15} + {}^{15}C_1 (0.2)(0.8)^{14}$

$+ {}^{15}C_2 (0.2)^2 (0.8)^{13}$

$= (0.8)^{15} + 150.13194139$

$= 0.3980$

~~9) P(winning) = 0.01 K=3~~

~~$P(X=7) = {}^{7-1}C_{3-1} (0.01)^3 (0.99)^4$~~

~~$= {}^6C_2 (0.01)^3 (0.99)^4$~~

~~$= 0.00014409$~~

11) ~~P(in a week) = 2~~

$P(F) = 0.5 = \frac{1}{2}$

$P(3 \text{ next boys}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 0.125$

13) $f_X(x) = \frac{1}{b-a} = \frac{1}{120-75} = \frac{1}{45}$

$P(80 < x < 100) = \int_{80}^{100} \frac{1}{45} dx = \frac{1}{45} [x]_{80}^{100} = \frac{1}{45} (100-80)$

$= \frac{20}{45} = \frac{4}{9}$

14) $X \sim \Gamma(5, 0.4)$ $\alpha = 5$

$\lambda = 0.4$ $P(X < 22.87)$

$2\lambda X \sim \chi^2_{10}$

$P(X < 22.8) = P(2\lambda X < 2\lambda (22.87)$

$= P(\chi^2_{10} < 18.\cancel{272}\ 32)$

$P(X<18) = 0.9450$ $P(X<18.5) = 0.9529$

$0.5 = 0.0079$ 1 0.0158 0.132 0.0020856

$P(\chi^2_{10} < 18.132) = 0.9470856$