Koshan Mehta

NMA - assigment

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Q1) $r = 12.65$ cm
$r(\text{actual}) = 12.5$ cm

① Absolute error

$\rightarrow |\text{Actual value} - \text{Measured value}|$
$= |12.5 - 12.65|$
$= 0.15$

② Relative error

$\rightarrow \left|\dfrac{\text{measured} - \text{Actual}}{\text{Actual}}\right|$

$= 0.012$

③ % error $\rightarrow$ 1.2%

Q2) given $f(x) = \log(x)$

4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

a) $\log(5)$ Let $\log(5)$ be y

$y = 0.60206 + \frac{1}{2}(0.7781513 - 0.60206)$

ii) $\log(5) \Rightarrow$ let $\log(5)$ be z

$$z = 0.6532125 + \frac{1}{2}\left(0.7703627 - 0.653125\right)$$

$$\log(5) = 0.6967876$$

iii) root of the eq $\Rightarrow x^4 - x - 10 = 0$

upto 5 iterations

$a = 1.5$

$b = 2$

x	y
1.5	-6.4375
2	4

① $x_1 = \frac{1.5+2}{2} = \frac{7}{4}$

$y_1 = -2.37109375$

② $x_2 = \frac{1.75+2}{2} = 1.875$

$y_2 = 0.486191$

(iii) $x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

$y_3 = -1020248413$

(4) $x_4 = \frac{1.8125 + 1.875}{2} = 1.84375$

$y_4 = -0.2877340$

(5) $x_5 = \frac{1.84375 + 1.875}{2} = 1.859375$

$y_5 = 0.0933781266 2$

$\therefore$ root of the eqⁿ $\simeq 1.859375$

Q4) $f(x) = x^3 - x - 1$

using Newton Raphson

x	$f(x)$
0	-1
1	-1
2	5

$f(x) = 3x^2 - 1$

$x_0 = 1$

$f(x_0) = 1$

$f(x_0) = 2$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 1 - (-1/2)$

$= 1.5$

$y_1 = 0.875$

$f'(x_1) = 5.75$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.5 - \frac{0.875}{5}$

$= 1.347826$

$y_2 = 0.100681$

$f'(x_2) = 4.449$

$x_3 = x_2 - f(x_2 / f'x_2)$

$= 1.325200$

$y_3 = 0.0020566$

$f'(x_3) = 4.268465$

$x_4 = x_3 - f(x_3 / f'(x_3))$

$= 1.3247181$

$y_4 = 0.000000\,60879$

Q6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$ To find $\Rightarrow$ Inverse of A

$|A| = 1(-1(0) - 6(1)) + 1(-4-0) + 8$

$= -6 - 4 + 8$

$= -2$

Since $|A| \neq 0$ A^{-1} exists

$\Rightarrow \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 1 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \dfrac{\text{adj } A}{|A|} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$

Ans 7 Given $A = 8i - 9j$
$B = 7i - 9j$

$|\vec{a}| = \sqrt{64 + 72} = 2\sqrt{34}$
$|\vec{b}| = \sqrt{49 + 81} = \sqrt{130}$

$\cos\theta = \cancel{84 - 93}\ \dfrac{(8i - 9j)\cdot(7i - 9j)}{2\sqrt{34} \times \sqrt{130}}$

$= \dfrac{56 + 81}{2\sqrt{34 \times 130}} = \dfrac{137}{2\sqrt{34 \times 130}}$

$\theta = \cos^{-1} \dfrac{137}{2\sqrt{4420}}$

Ans 11. $f(x) = (4+3x)^5$

Using the binomial theorem

$$\Rightarrow {}^5C_0 (3x)^5 + {}^5C_1 (3x)^4 (4) + {}^5C_2 (3x)^3 (4)^2 + {}^5C_3 (3x)^2 (4)^3$$
$$+ {}^5C_4 (3x)(4)^4 + {}^5C_5 (4)^5$$

$$\Rightarrow 1 \times 243x^5 + 5(81)x^4(4) + 10(27)x^3(16) + 10(3)x^2(64)$$
$$+ 5(3)x(256) + 1(1024)$$

$$\Rightarrow 243x^5 + \cancel{2x}\ 1620x^4 + 4320x^3 + 7560x^2 + 3840x$$
$$+ 1024$$

Ans 12. $f(x) = x^3 - 7x^2 + 8x - 3$

if $x_0 = 5$

$x_0 = 5$ $\qquad f'(x) = 3x^2 - 14x + 8$

$f(x_0) = -13$

$f'(x_0) = 13$

$x_1 = 5 - \left(\frac{-13}{13}\right)$

$= 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$x_2 = 6 - \frac{9}{32}$

$= 5.71875$

Ans 14. Expand $\Rightarrow (1 - x + x^2)^4$

Let $1 - x = y$

the $(y + x^2)^4$

Using binomial theorem

$\Rightarrow {}^4C_0 y^4 + {}^4C_1 y^3 (x^2) + {}^4C_2 y^2 (x^2)^2 + {}^4C_3 y^1 (x^2)^3 + {}^4C_4 (x^2)^4$

$\Rightarrow y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

$\Rightarrow (1-x)^4 + 4(1-x)^3 x^2 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$\Rightarrow (1-x)^2(1-x)^2 + 4x^2(1 - x^3 - 3x + 3x) + 6x^4(1 + x^2 - 2x) + 4x^6 - 4x^7 + x^8$

$\Rightarrow 1 - 4x + 10x^4 - 16x^3 + 14x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$

Camlin

Ans 15. $e^{-x} = 3\log(x)$

for value of x
upto 4 iterations.

$e^{-x} - 3\log(x) = 0$

x	y
0.5	1.50962
1	0.36787
1.5	-0.305142

(i) $x_1 = \frac{1+1.5}{2} = 1.25$

$y_1 = -0.0042252$

(ii) $x_2 = \frac{(1+1.25)}{2} = 1.125$

$y_2 = 0.1711944$

(iii) $x_3 = \frac{1.125+1.25}{2} = 1.1875$

$y_3 = 0.0810819$

(iv) $x_4 = \frac{1.1875+1.25}{2} = 1.21875$

$y_4 = 0.0278555$

Therefore after 4 iterations
the value of $x = 1.21875$