

Homework 1

* Lagrange's Interpolation

Q1)	x	7	8	9	10
	$f(x)$	3	1	1	9

Find $f(9.5)$.

$$\begin{aligned}
 f(9.5) &= \frac{(9.5-8)(9.5-9)(9.5-10)}{(7-8)(7-9)(7-10)} \cdot 3 + \\
 &+ \frac{(9.5-7)(9.5-9)(9.5-10)}{(8-7)(8-9)(8-10)} \cdot 1 + \\
 &+ \frac{(9.5-7)(9.5-8)(9.5-10)}{(9-7)(9-8)(9-10)} \cdot 1 + \\
 &+ \frac{(9.5-7)(9.5-8)(9.5-9)}{(10-7)(10-8)(10-9)} \cdot 9
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{3}{16} - \frac{5}{16} + \frac{5}{16} + \frac{45}{16} \\
 &= 3.625.
 \end{aligned}$$

Q2)	x	3	7	9	10	$f(x) = ?$
	$f(x)$	108	120	72	63	

$$\begin{aligned}
 f(6) &= \frac{(6-7)(6-9)(6-10)(108)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)} + \\
 &+ \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)}
 \end{aligned}$$

$$\begin{aligned}
 &= 12 + 180 + 27 - 72 \\
 &= 147.
 \end{aligned}$$

Q3)

n	1	2	7	8	$f(6) = ?$
$f(n)$	4	5	5	4	

$$\begin{aligned}
 f(6) &= \frac{(6-2)(6-7)(6-8)4}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)5}{(2-1)(2-7)(2-8)} \\
 &+ \frac{(6-1)(6-2)(6-8)5}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)4}{(8-1)(8-2)(8-7)} \\
 &= \frac{-16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{40}{21}
 \end{aligned}$$

$$= 5.6667.$$

Q4)

n	1	2	3	4	7	$f(5) = ?$
$f(n)$	2	4	8	16	128	

$$\begin{aligned}
 f(5) &= \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \\
 &+ \frac{(5-1)(5-2)(5-4)(5-7)8}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \\
 &+ \frac{(5-1)(5-2)(5-3)(5-4)16}{(7-1)(7-2)(7-3)(7-4)}
 \end{aligned}$$

$$= \frac{-2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{15}$$

$$= 32.9334.$$

Q5.

x	10	20	14	16	$f(15) = ?$
$f(x)$	0.25	0.20	0.15	0.10	

$$f(15) = \frac{(15-20)(15-14)(15-16)0.25}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)0.20}{(20-10)(20-14)(20-16)}$$

$$+ \frac{(15-10)(15-20)(15-16)0.15}{(24-10)(24-20)(14-16)} + \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)}$$

$$= \frac{-1}{112} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96}$$

$$= 0.1208$$

2]

Q1 $x^3 + x - 1 = 0$.

$$y = x^3 + x - 1$$

x	0.5	0.6	0.7	$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$
$f(x)$	-0.315	-0.184	0.043	

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.184 \quad y_2 = 0.043$$

$$x_n = \frac{(0.043)(0.6) + (0.7)(0.184)}{0.043 + 0.184} = 0.6810$$

$$n = -0.003042$$

$$x_1 = 0.6810$$

$$x_2 = 0.7$$

$$y_1 = -0.003042$$

$$y_2 = 0.043$$

$$x_n = \frac{(0.6810)(0.043) + (0.7)(0.003042)}{0.043 + 0.003042}$$

$$= 0.6823$$

$$y_n = -0.000045$$

Root $y'x^3 + x - 1 \approx 0$ is 0.6823

q2) $x^3 + 2x + 0.5 \approx 0$

$$y = x^3 + 2x + 0.5$$

x	0	0.1	0.2	0.3
y	0.5	0.301	0.108	-0.073

$$x_1 = 0.2$$

$$x_2 = 0.3$$

$$y_1 = 0.108$$

$$y_2 = -0.073$$

$$x_n = \frac{(-0.073)(0.2) - (0.3)(0.108)}{-0.073 - 0.108}$$

$$= 0.259669$$

$$y_n = -0.001829$$

$$x_1 = 0.2$$

$$x_2 = 0.259669$$

$$y_1 = 0.108$$

$$y_2 = -0.001829$$

$$x_n = \frac{(-0.001829) - (0.259669)(0.108)}{-0.001829 - 0.108}$$

$$= 0.255675$$

$$y_n = -0.000041$$

Root of " $x^3 + 2x + 0.5$ " is $= 0.255675$

$$Q3. \quad x^3 + 2x - 20 = 10$$

$$y = x^3 + 2x - 10$$

$$\begin{array}{ccc} x & 2 & 2.5 \\ y & -8 & 0.625 \end{array}$$

$$\begin{aligned} x_n &= \frac{2(0.625) - 8(2.5)}{0.625 - (-8)} \\ &= \frac{1.25 + 20}{8.625} = \frac{21.25}{8.625} \\ &= 2.4638 \end{aligned}$$

$$y = -0.1164$$

$$\begin{array}{ccc} x & 2.4638 & 2.5 \\ y & -0.1164 & 0.625 \end{array}$$

$$x_n = \frac{2.4638(0.625) - 2.5(-0.1164)}{0.625 - (-0.1164)}$$

$$= \frac{1.539875 + 0.291}{0.7414} = 2.4695$$

$$y_n = -0.000926497625$$

$\therefore x = 2.4695$ is the root of $x^3 + 2x - 20 = 0$.

$$Q4) \quad 3x - \cos x - 1 = 0$$

$$\begin{array}{ccc} x & 0 & 1 \\ y & -2 & 1.0002 \end{array}$$

$$x_n = 0(1.0002) - (-2)(1) = 0.6666$$

$$y_n = -0.00006$$

$$\therefore x \approx 0.6666$$

Q6] $x \log_{10} x - 1.2 = 0$

x	2	3
y	-0.5979	0.2314

$$x_n = \frac{2(0.2314) - (-0.5979)(3)}{0.2314 - (-0.5979)}$$

$$= 2.7210$$

$$y_n = -0.0171$$

x	2.7210	3
y	-0.0171	0.2314

$$x_n = \frac{2.7210(0.2314) - (-0.0171)(3)}{0.2314 - (-0.0171)}$$

$$= 2.7402$$

$$y_n = -0.000389$$

$$\therefore x \approx 2.7402$$

Q7] $y = x^3 - 4x + 1 = 0$

x	0	1
y	1	-2

$$x_n = \frac{0(-2) - 1(1)}{-2 - 1}$$

$$= \frac{-1}{-3}$$

$$= 0.3333$$

$$y_n = -0.2962$$

$$\begin{array}{ccc} x & 0 & 0.3333 \\ y & 1 & -0.2962 \end{array}$$

$$x_n = \frac{-0.3333}{-1.2962} = 0.2571$$

$$y_n = -0.01146$$

$$\begin{array}{ccc} x & 0 & 0.2571 \\ y & 1 & -0.0114 \end{array}$$

$$x_n = \frac{6(-0.0114) - 1(0.2571)}{-0.0114 - 1} = 0.2542$$

$$y_n = -0.00037$$

$$\therefore x \approx 0.2542$$

Q8] $y = x^3 + 2x^2 - 8 = 0$

$$\begin{array}{ccc} x & 0 & 1 & 2 \\ y & -8 & -5 & 8 \end{array}$$

$$x_n = \frac{1(8) - (-5)(2)}{8 + 5} = 1.3846$$

$$y_n = -1.5113$$

$$\begin{array}{ccc} x & 1.3846 & 2 \\ y & -1.5113 & 8 \end{array}$$

$$x_n = \frac{(1.3846)8 - 2(-1.5113)}{8 + 1.5113} = 1.4824$$

$$y_n = -0.3474$$

$$x \quad 1.4824 \quad 2$$

$$y \quad -0.3474 \quad 8.$$

$$x_n = \frac{1.4824(8) - 2(-0.3474)}{8 + 0.3474}$$

$$= 1.5039$$

$$y_n = -0.0752$$

$$x \quad 1.5039 \quad 2$$

$$y \quad -0.0752 \quad 8.$$

$$x_n = \frac{1.5039(8) - (-0.0752)(2)}{8 + 0.0752}$$

$$= 1.5085$$

$$y_n = (1.5085)^3 + 2(1.5085)^2 - 8$$

$$= -0.0162$$

$$x \quad 1.5085 \quad 2$$

$$y \quad -0.0752 \quad 8.$$

$$x_n = \frac{1.5085(8) - (-0.0162)(2)}{8 + 0.0162}$$

$$= 1.5095$$

$$y_n = -0.0032$$

$$\therefore x \approx 1.5095$$

Q9] $x^3 - 9x + 1 = 0$

x	0	1
y	1	-7

$$x_n = \frac{0(-7) - (1)(1)}{-7 - 1}$$

$$= 0.125$$

$$y_n = -0.1230$$

x	0	0.125
y	1	-0.1230

$$x_n = \frac{0(-0.1230) - 1(0.125)}{-0.1230 - 1}$$

$$= 0.1113$$

$$y_n = -0.0003$$

$$\therefore x \approx 0.1113$$

Q10] $x^3 - x - 4 = 0$

x	0	1	2
y	-4	-4	2

$$x_n = \frac{1(2) - (-4)(2)}{2 + 4}$$

$$= 1.6667$$

$$y_n = -1.0368$$

x	1.6667	2
y	-1.0368	2

$$x_n = \frac{1.6667(2) - (-1.0368)(2)}{2 + 1.0368}$$

$$= 1.7805$$

$$y_n = -0.1360$$

$$x \quad 1.7805 \quad 2$$

$$y \quad -0.1360 \quad 2$$

$$x_n = \frac{1.7805(2) - (-0.1360)(2)}{2 + 0.1360}$$

$$= 1.7945$$

$$y_n = -0.0158$$

$$x \quad 1.7945 \quad 2$$

$$y \quad -0.0158 \quad 2$$

$$x_n = \frac{1.7945(2) - (-0.0158)(2)}{2 + 0.0158}$$

$$= 1.7961$$

$$y_n = -0.0019$$

$$x \approx 1.7961$$