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1) Given

$S_0 = £65$

$\sigma = 25\%$ p.a

$r = 2\%$ p.a

$K = £55$

$T = 6$ months (0.5)

(i) $C_t = S_0 \Phi(d_1) - Ke^{-rt} \Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d_1 = \frac{\ln\left(\frac{65}{55}\right) + \left(2\% + \frac{(20\%)^2}{2}\right)0.5}{(20\%)\sqrt{0.5}}$$

$d_1 = 1.08996$

$d_2 = d_1 - \sigma\sqrt{T}$

$d_2 = 0.91318$

$\therefore C_t = 68\,\Phi(1.09) - 55e^{-0.2\times0.5}\,\Phi(0.92)$

$C_t = 11.464$

(ii) Delta of a call option

$$= \frac{\text{change in price of call option}}{\text{change in price of underlying}}$$

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(iii) $\Delta_c = \Phi(d_1)$

$\Delta_p = -\Phi(-d_1)$

$\Delta_c = 0.86214$

(iv) $\Delta_p = -0.13786$

2)

(i) Delta : its rate of change of option price with respect to change in price of underlying asset.

$$\Delta = \frac{\text{change in price of derivative}}{\text{change in price of underlying asset}}$$

Vega : It is the rate of change of option price with respect to change in volatility of underlying asset

$$\vartheta = \frac{\text{change in price of option}}{\text{change in volatility of underlying asset}}$$

(ii) $c - p = S - Ke^{-rt}$

$$\frac{d}{d\sigma}(c-p) = \frac{d}{d\sigma}(S - Ke^{-rt})$$

$\vartheta_c - \vartheta_p = 0$

Since K & S are constants

$\therefore \; \vartheta = \vartheta_p$

(iii) Given :

$\sigma = 25\%$ p/a

$r = 5\%$

$S_0 = \$\,55$

$X = \$\,50$

$T = 1$

$$a_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{55}{50}\right) + \left(5\% + \frac{1}{2} \times (25\%)^2\right)1}{25\%\,(\sqrt{T})}$$

$a_1 = 0.70624$

$a_2 = a_1 - \sigma\sqrt{T}$

$a_2 = 0.70624 - 25\%\,(\sqrt{T})$

$a_2 = 0.4562$

$C_t = S_0\,\Phi(a_1) - Ke^{-rt}\,\Phi(a_2)$

$= 55\,(0.70624) - 50e^{-0.05}(0.4562)$

$C_t = \$\,9.65$

$c - p = S - Ke^{-rt}$

$$9.65 - p = 55 - 50e^{-0.03}$$

$$p = \$2.21$$

(iv) Vega Hedging:

It is a portfolio for which the vega is zero ($v = 0$). This type of portfolio is also known as vega neutral. This type of portfolio is safe from small changes in the assumed level of volatility.

Delta Hedging:

It is a type of portfolio for which the delta is zero ($\Delta = 0$). This type of portfolio is also known as delta neutral. This type of portfolio is safe from change in the price of underlying asset.

(v) We assume

a units of call option

b units of put option

c units of forwards

	CF @ PV	Delta	Vega
Call	$c = 9.65$	Δ_c	v_c
Put	$p = 2.211$	Δ_p	v_p
forward	-	1	0

~~Δ of forward = 1~~

~~v of forward = 0~~

Vega Hedging

$$a v_c + b v_p + c v_f = 0$$

$a\,\vartheta_c + b\,\vartheta_p = 0 \quad (\because \vartheta_p = 0)$

$\vartheta_c\,(a+b) = 0 \quad (\because \vartheta_c = \vartheta_p$

$(a+b) = 0$

$a = -b \quad | \quad b = -a \quad (i)$

Delta Hedging

$a\,\Delta_c + b\,\Delta_p + c\,\Delta_F = 0$

$a\,\Delta_c + b\,\Delta_p + c = 0 \quad (\because \Delta_F = 1)$

$a\,\Delta_c + (-a)(\Delta_c - 1) + c = 0$

$(\because$ from (i) & $\Delta_p = \Delta_c - 1)$

$a\cancel{\Delta_c} + a - a\cancel{\Delta_c} + c = 0$

$a = -c \quad - (ii)$

from (i) & (ii)

$\therefore \quad a = -b = -c$

Portfolio = 1000

$a.C + b.P + c.F = 1000$

$aC + bP = 1000 \quad (\because F = 0)$

$9.6528a + 2.2190b = 1000$

$9.6528a - 2.2190a = 1000 \quad (\because b = -a)$

$7.4385a = 1000$

$\therefore a = 134.43$

$b = c = -134.43$

$\therefore$ Portfolio

LP on 134 call options
SP on 134 put options
SP on 134 forwards

3) (i) $C_t = E\left(e^{-r(T-t)} C_T / F_t\right)$

$F_t \Rightarrow$ filtration @ $t > 0$
$C_T \Rightarrow$ payoff under derivative at T
C_t = derivation value at time t

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(ii) $r = 5\%$ p/a

$\sigma = 25\%$ p/a

$T = 6$ month $= 0.5$

$S_0 = £50$

$X = £49$

$C_t = S_0\Phi(a_1) - Ke^{-rt}\Phi(a_2)$

$$a_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$a_1 = \frac{\ln\left(\frac{50}{49}\right) + \left(5\% + \frac{1}{2}(25\%)^2\right)0.5}{(25\%)(\sqrt{0.5})}$$

$= 0.399$

$a_2 = a_1 - 25\%(\sqrt{0.5})$

$a_2 = 0.1673$

$\Phi(a_1) = 0.6346$

$\Phi(a_2) = 0.5669$

$C_T = S_0\Phi(a_1) - Ke^{-rT}\Phi(a_2)$

$= 50 \times 0.6346 - 49e^{-0.050}(0.5664)$

$C_T = 4.66$

(iii) 4.66

(iv) $S_t + p = C + Ke^{-rT}$

$p = 4.66 + 49e^{-0.03 \times 2.5} - 50$

$p = 2.45$

(v) The price of underlying would fall if the stock pays a dividend. It would make a rise in price of european put as it would lead to fall in price of european call.

Also the american call would be more expensive than european call due to the potential early ex opportunity.

5)

(i) $\Delta = \Phi(a_1)$

$= \frac{\text{change in price of underlying asset}}{\text{change in price of call option}}$

(ii) $a_1 = \frac{\ln\left(\frac{S_0}{K}\right) + r\left(\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$

$0.3 = \frac{\ln\left(\frac{40}{45.91}\right) + (0.02 + 0.5 \times \sigma^2)5}{\sigma\sqrt{5}}$

$0.3(\sqrt{5}) = \frac{(-1.378) + (0.1 + 2.5\sigma^2)}{\sigma}$

$\sigma - 2.5\sigma^2 = 0.05634$

$2.5\sigma^2 - 0.67083\sigma - 0.0378.0$

$\sigma = \frac{0.67083 \pm \sqrt{0.67083^2 - 4 \times 2.5 (0.384)}}{2 \times 2.5}$

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$\sigma = 32\%$

(iii) $R_0 = \$30$

$\sigma = \sqrt{16}$ p/a

Option Payoff = c at time T if,

$\frac{S_1}{S_0} > K_S$, $\frac{R_1}{R_0} < K_R$

payoff = $ce^{-\lambda T}\Phi\left(\frac{S_1}{S_0} < K_S\right)\Phi\left(\frac{R_1}{R_0} < K_R\right)$

(iv) Perfect correlation =

$\frac{R_1}{R_0} = \frac{S_1}{S_0}$

$ce^{-\lambda T}\Phi(R_1 \times R_0 < K_S) = ce^{-\lambda T}\Phi(S_1/S_0 < K_S)$

$\therefore$ price $= ce^{-\lambda T}\Phi\left(\frac{R_1}{R_0} < \min(K_S, K_R)\right)$

$= ce^{-\lambda T}\Phi\left(\frac{S_1}{S_0} < \min(K_S, K_R)\right)$

(v) Price

$ce^{-\lambda T}\Phi\left(\frac{S_1}{S_0} < K_S\right).\Phi\left(\frac{R_1}{R_0} < K_R\right)$

$= 50\, e^{-0.20 \times 1T}.\Phi(S < 32).\Phi(R_1 < 18)$

$= \$1.61$

6)

(i)

a) $\Delta = -\Phi(-a_1)$

$= \Phi(a_1) = 1$

b) $\Delta = 0$

$100{,}000\,\Delta + 24830 = 0$

$\Delta = \dfrac{-24830}{100000}$

$\Delta = -0.2483$

(ii) $\Phi(-a_1) = 0.2483$

$1 - \Phi(a_1) = 0.2483$

$\Phi(a_1) = 0.7517$

$a_1 = 0.68$

$0.68 = \dfrac{\ln\left(\dfrac{0.64}{0.63}\right) + \left(0.03 + \dfrac{1}{2}\sigma^2\right)}{\sigma}$

$\sigma = 7.1\%$

(iii)

a) $x e^{-rT}\,\Phi(-a_2) - S_0\left[\Phi(-a_1)\right] = 0.2983$

$= 0.63 \times e^{-0.03} \times \left[1 - \Phi(0.609)\right]$

$- 0.64 \times 0.2483$

$= 6.894$

b) Cash Holding in Hedging Portfolio

$= 24830 \times 0.64 + 100{,}000 \times 0.6894$

$= £\,165{,}806$

Q7

(i) $\Delta = \frac{da}{db} = \frac{da}{db}(t, b_t)$

a ⇒ derivative

b = underlying asset

$\gamma = \frac{d^2 a}{d b^2}$

$\nu = \frac{da}{d\sigma_b}$

(ii) $r = 3\%$ p/a

$v = \$29$

$x = 80$

$c = 17.91$

$S_0 = 60$

$\sigma = 25\%$ p/a

$T = 3$ years

(iii) 0.801 short

C − Δ × S_0 short cash = 30.15 short cash i??

(iv) $a(b, \sigma+\delta) = a(b,\sigma) + \delta \frac{da}{d\sigma}$

$= 17.91 + 29 \times 0.02$

$= 18.49$

8) (i) $\Delta = \frac{\delta a}{\delta b}$

(ii) $S_0 = \$\,100$

$r = 3\%$

$x = 109.42$

$T = 1$

$\Delta = 0.42079$

$\Phi(a_1) = 0.42074$

$a_1 = -0.2$

$$a_1 = \frac{\ln\left(\frac{S_0}{x}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$-0.2 = \frac{\ln\left(\frac{100}{109.42}\right) + \left(0.03 + \frac{\sigma^2}{2}\right)}{\sigma}$$

$= (-0.2\sigma) + 0.09 - 0.03 - \frac{\sigma^2}{2} = 0$

$-\frac{\sigma^2}{2} - 0.2\sigma + 0.06 = 0$

$\sigma = 0.2$

a)

(i) Partial Differentiation Equation

$$\frac{dg}{dt} + (r-q) S_T \frac{dg}{dS_T} + \frac{1}{2}\sigma^2 S_t^2 \frac{d^2 g}{dS_t^2} = rg$$

Boundary condition

$D_T = g(T, S_T) = f(S_T)$

(ii) Price of alternative @ time t & value for μ

$D_T = g(t, S_t) = \frac{S_t^n \cdot e^{\mu(T-t)}}{S_0^{n-1}} \quad (n > 1)$

Then, $D_T = g(T, S_T)$

$= f(S_T) = \frac{S_T^n}{S_0^{n-1}}$

$$\frac{dg}{dt} = -\mu \frac{S_t^n}{S_0^{n-1}} e^{\mu(T-t)} = -\mu g$$

$$\frac{dg}{dS_t} = \frac{n S_t^{n-1}}{S_0^{n-1}} e^{\mu(T-t)} = \frac{n}{S_t} g$$

$$\frac{n(n-1)\, S_t^{\,n-2}\, e^{\mu(T-t)}}{} = \frac{n(n-1)}{S_t^2}\, y$$

$$\frac{d^2 y}{d S_t^2} = \frac{n(n-1)\, S_t}{S_t^{\,n-1}}$$

$$= -\mu y + (r-q) S_t \frac{n}{S_t}\, y + \frac{1}{2}\sigma^2 S_t^2 \frac{(n(n-1))}{S_t^2} y = ry$$

$$-\mu + (r-q)n + \frac{1}{2}\sigma^2 n(n-1) = r$$

$$\mu = (r-q)n - r + \frac{1}{2}\sigma^2 n(n-1)$$

If $q = 0$

$$\mu = rn - r + \frac{1}{2}\sigma^2 n(n-1)$$

$$= r(n-1) + \frac{1}{2}\sigma^2 n(n-1)$$

$$\left(r + \frac{1}{2}\sigma^2 n\right)(n-1)$$

10)

(i) $C_t + Ke^{-r(T-t)} = S_t + P_t$

(ii) $X = 120$

$c = 10.09$

$r = 2\%$ pa

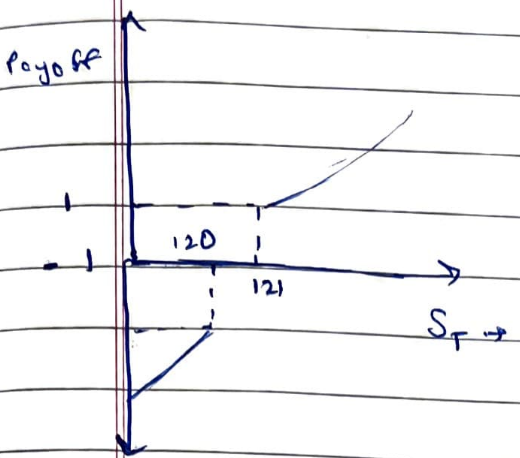
$S_0 = 110$

$T = 1$

$$C = S_0\, \Phi(a_1) - Ke^{-rt}\, \Phi(a_2)$$

$$a_2 = a_1 - \sigma$$

(iii)



(iv) $S_1 - 121 \le D \le S_1 - 120$

$$S_0 - 121 e^{-\lambda} \le V \le S_0 - 120 e^{-\lambda}$$

4) (i) $Z_t \sim$ SBM under P

Y_t = permisible proces

$\therefore$ There exists a measure Q equivalent to P where,

$$\tilde{Z} = Z_t + \int_0^t \gamma_s ds$$

$\tilde{Z}_t \rightarrow$ SBM under Q

If Z_t is a SBM under P & if Q is equivalent to P then there exists a permissible process γ_t such that

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s ds$$

$\tilde{Z}_t \rightarrow$ SBM under Q

(ii) The discounted value of asset prices are martingales