

HW-1

Saathi

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Q1)

x	7	8	9	10
f(x)	3	1	1	9

$$f(9.5) = ?$$

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)(3)}{(7-8)(7-9)(7-10)} + \frac{(9.5-7)(9.5-9)(9.5-10)(1)}{(8-7)(8-9)(8-10)} \\ + \frac{(9.5-7)(9.5-8)(9.5-10)(1)}{(9-7)(9-8)(9-10)} + \frac{(9.5-7)(9.5-8)(9.5-9)(9)}{(10-7)(10-8)(10-9)} \\ = \frac{3}{16} - \frac{5}{16} + \frac{15}{16} + \frac{45}{16} = 3.625$$

Q2)

x	3	7	9	10
f(x)	108	120	72	63

$$f(6)$$

$$f(6) = \frac{(6-7)(6-9)(6-10)(108)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)} \\ + \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)} \\ = 12 + 180 - 72 + 27 = 147$$

Q3)

x	1	2	7	8
f(x)	4	5	5	4

$$f(6) = ?$$

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)} \\ + \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)} \\ = \frac{-16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{40}{21} = 5.667$$

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Q4)

x	1	2	3	4	7
f(x)	2	4	8	14	128

f(15)

$$\begin{aligned}
 f(15) &= \frac{(15-2)(15-3)(15-4)(15-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(15-1)(15-3)(15-4)(15-7)}{(2-1)(2-3)(2-4)(2-7)} \\
 &+ \frac{(15-1)(15-2)(15-4)(15-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(15-1)(15-2)(15-3)(15-7)}{(4-1)(4-2)(4-3)(4-7)} \\
 &+ \frac{(15-1)(15-2)(15-3)(15-4)(16)}{(7-1)(7-2)(7-3)(7-4)}
 \end{aligned}$$

$$= -\frac{2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{15}$$

$$= 32.9334$$

Q5)

x	10	20	14	16
F(x)	0.45	0.2	0.15	0.10

f(15) = ?

$$\begin{aligned}
 f(15) &= \frac{(15-20)(15-14)(15-16)(0.45)}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \\
 &+ \frac{(15-10)(15-20)(15-16)(0.15)}{(14-10)(14-20)(14-16)} + \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)}
 \end{aligned}$$



$$= \frac{-1}{112} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96}$$

$$= 0.1208$$

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Q1)

$$x^3 + x - 1 = 0 \quad \text{b/w } 0.5 \text{ and } 1$$

$$\therefore y = x^3 + x - 1$$

x	y
0.5	-0.375
0.6	-0.184
0.7	0.043

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.184$$

$$y_2 = 0.043$$

$$x_n = \frac{(0.043)(0.6) + (0.7)(0.184)}{0.043 + 0.184} = \frac{0.1309}{0.227} = 0.5766$$

$$y_n = -0.003042$$

$$x_1 = 0.5766$$

$$x_2 = 0.7$$

$$y_1 = -0.003042$$

$$y_2 = 0.043$$

$$x_n = \frac{(0.043)(0.5766) + (0.7)(0.003042)}{0.043 + 0.003042} = \frac{0.0251}{0.046042} = 0.5432$$

$$y_n = -0.000045$$

$$\therefore \text{Root of } (x^3 + x - 1 = 0) \text{ is } 0.5432$$

Q2)

$$x^3 + 2x + 0.5 = 0 \quad \text{b/w } 0 \text{ and } 1$$

$$\therefore y = x^3 + 2x + 0.5$$

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073

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$$x_1 = 0.2 \quad x_2 = 0.3$$

$$y_1 = 0.108 \quad y_2 = -0.073$$

$$x_n = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{(-0.073)(0.2) - (0.3)(0.108)}{-0.073 - 0.108}$$

$$= 0.259669$$

$$y_n = -0.001829$$

$$x_1 = 0.2 \quad x_2 = 0.259669$$

$$y_1 = 0.108 \quad y_2 = -0.001829$$

$$x_n = \frac{(-0.001829)(0.2) - (0.259669)(0.108)}{-0.001829 - 0.108}$$

$$= 0.258675$$

$$y_n = -0.000041$$

$\therefore$ Root of $x^3 - 2x + 0.5$ is $= 0.258675$

Q3) $2x^3 + 2x - 20 = 0$ b/w 2 and 2.5

$\therefore y = x^3 + 2x - 20$

x	y
2.0	-8
2.1	-6.539
2.2	-4.952
2.3	-3.233
2.4	-1.376
2.5	0.625

$$x_1 = 2.4$$

$$y_1 = -1.376$$

$$x_2 = 2.5$$

$$y_2 = 0.625$$

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$$x_n = \frac{(0.625)(2.4) + (1.376)(2.5)}{0.625 + 1.376} = 2.46876 \quad y_n = -0.01582$$

$$x_1 = 2.46876 \quad x_2 = 2.5$$

$$y_1 = -0.01582 \quad y_2 = 0.625$$

$$x_n = \frac{(0.625)(2.46876) + (0.01582)(2.5)}{0.625 + 0.01582} = 2.46954 \quad y_n = -0.000176$$

$\therefore$ Root of $x^3 + 2x - 20$ is $= 2.46954$

Q4) $3x - \cos x - 1 = 0$ b/w 0 and 1

$\therefore y = 3x - \cos x - 1$

x	y
0	-2
0.1	-1.69398
0.2	-1.39994
0.3	-1.09998
0.4	-0.7999
0.5	-0.4999
0.6	-0.1999
0.7	0.10007

$$x_1 = 0.6 \quad x_2 = 0.7$$

$$y_1 = -0.1999 \quad y_2 = +0.10007$$

$$x_n = \frac{(0.10007)(0.6) + (0.7)(0.1999)}{0.10007 + 0.1999} = 0.6666 \quad y_n = -0.0000003$$

$\therefore$ Root of $3x - \cos x - 1$ is $= 0.6666$

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Q5) $x^3 - 3x - 5 = 0$ b/w 2 and 2.5

$\therefore y = x^3 - 3x - 5$

x	y
2.0	-3
2.1	-2.039
2.2	-0.952
2.3	0.267
2.4	1.824

$x_1 = 2.2$

$x_2 = 2.3$

$y_1 = -0.952$

$y_2 = 0.267$

$x_n = \frac{(0.267)(2.2) + (-0.952)(2.3)}{0.267 + 0.952} = 2.27809$

$y_n = -0.01159$

$x_1 = 2.27809$

$x_2 = 2.3$

$y_1 = -0.01159$

$y_2 = 0.267$

$x'_n = \frac{(0.267)(2.27809) + (2.3)(0.01159)}{0.267 + 0.01159} = 2.2794$

$y'_n = -0.001$

$\therefore$ Root of $x^3 - 3x - 5 = 0$ is $\hat{x} = 2.2794$

Q6)

$x \log_{10} x - 1.2 = 0$ b/w 2 and 3

$\therefore y = x \log_{10} x - 1.2$

x	y
2.0	-0.59794
2.1	-0.52339
2.2	-0.447667
2.3	-0.368026
2.4	-0.28749
2.5	-0.20514
2.6	-0.12106
2.7	-0.0353
2.8	+0.0512

$$x_1 = 2.7$$

$$x_2 = 2.8$$

$$y_1 = 0.0353$$

$$y_2 = 0.05205$$

$$x_n = \frac{(0.05205)(2.7) + (2.8)(0.0353)}{(0.05205) + (0.0353)} = 2.7404$$

$$y_n = -0.00019$$

$\therefore$ Root of $x \log_{10} x - 1.2$ is $= 2.7407$

Q7)

$$x^3 - 4x + 1 = 0$$

$$\therefore y = x^3 - 4x + 1$$

x	y
0	1
0.1	0.601
0.2	0.208
0.3	-0.173

$$x_1 = 0.2$$

$$x_2 = 0.3$$

$$y_1 = 0.208$$

$$y_2 = -0.173$$

$$x_n = \frac{(-0.173)(0.2) - (0.3)(0.208)}{-0.173 - 0.208} = 0.2546$$

$$y_n = -0.00187$$

$$x_1 = 0.2$$

$$x_2 = 0.2546$$

$$y_1 = 0.208$$

$$y_2 = -0.00187$$

$$x_n' = \frac{(-0.00187)(0.2) - (0.208)(0.2546)}{-0.00187 - 0.208} = 0.2541$$

$$y_n' = -0.00002$$

$\therefore$ Root of ' $x^3 - 4x + 1 = 0$ ' is $= 0.2541$

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Q8)

$$x^3 + 2x^2 - 8 = 0$$

$$\therefore y = x^3 + 2x^2 - 8$$

x	y
0	-8
1	-5
2	8

$$x_1 = 1$$

$$y_1 = -5$$

$$x_2 = 2$$

$$y_2 = 8$$

$$x_n = \frac{(8)(1) + (5)(2)}{8 + 5} = 1.3846$$

$$y_n = -1.5712$$

$$x_1 = 1.3846$$

$$x_2 = 2$$

$$y_1 = -1.5712$$

$$y_2 = 8$$

$$x'_n = \frac{(8)(1.3846) + (2)(1.5712)}{8 + 1.5712} = 1.4824$$

$$y'_n = -0.3475$$

$$x_1 = 1.4824$$

$$x_2 = 2$$

$$y_1 = -0.3475$$

$$y_2 = 8$$

$$x''_n = \frac{(8)(1.4824) + (0.3475)(2)}{(8) + 0.3475} = 1.5039$$

$$y''_n = 0.0747$$

$$x_1 = 1.5039$$

$$x_2 = 2$$

$$y_1 = -0.0747$$

$$y_2 = 8$$

$$x'''_n = \frac{(8)(1.5039) + (0.0747)(2)}{8 + 0.0747} = 1.5085$$

$$y'''_n = -0.0158$$

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$$x_1 = 1.5085 \quad x_2 = 2$$

$$y_1 = -0.0158 \quad y_2 = 0$$

$$x_n = \frac{(8)(1.5085) + (0.0158)(2)}{8 + 0.0158} = 1.5095$$

$$y_n = -0.0033$$

$$x_1 = 1.5095 \quad x_2 = 2$$

$$y_1 = -0.0033 \quad y_2 = 0$$

$$x_n = \frac{(8)(1.5095) + (0.0033)(2)}{8 + 0.0033} = 1.5097$$

$$y_n = -0.0007$$

$\therefore$ Root of $x^3 + 20x^2 - 8$ is $\boxed{1.5097}$

(9) $x^3 - 9x + 1 = 0$

$$\therefore y = x^3 - 9x + 1$$

x	y
0	1
0.1	0.101
0.2	-0.792

$$x_1 = 0.1$$

$$x_2 = 0.2$$

$$y_1 = 0.101$$

$$y_2 = -0.792$$

$$x_n = \frac{(-0.792)(0.1) - (0.2)(0.101)}{-0.792 - 0.101} = 0.1113$$

$$y_n = 0.00041$$

$\therefore$ Root of $x^3 - 9x + 1$ is $\boxed{0.1113}$

Q10)

$$x^3 - x - 4 = 0$$

$$\therefore y = x^3 - x - 4$$

x	y
0	-4
1	-4
2	2

$$x_1 = 1$$

$$x_2 = 2$$

$$y_1 = -4$$

$$y_2 = 2$$

$$x_n = \frac{(2)(1) + 4(2)}{2 + 4} = 1.3334$$

$$y_n = -2.9629$$

$$x_1 = 1.3334$$

$$x_2 = 2$$

$$y_1 = -2.9629$$

$$y_2 = 2$$

$$x_n = \frac{(2)(1.3334) + (2)(-2.9629)}{2 + 2.9629} = 1.7313$$

$$y_n = -0.5415$$

$$x_1 = 1.7313$$

$$x_2 = 2$$

$$y_1 = -0.5415$$

$$y_2 = 2$$

$$x_n = \frac{(2)(1.7313) + (2)(-0.5415)}{2 + 0.5415} = 1.7885$$

$$y_n = 0.6680$$

$$x_1 = 1.7885$$

$$x_2 = 2$$

$$y_1 = -0.6680$$

$$y_2 = 2$$

$$x_n = \frac{(2)(1.7885) + (2)(-0.6680)}{2 + 0.6680} = 1.7954$$

$$y_n = -0.0078$$

$$x_1 = 1.7984$$

$$x_2 = 2$$

$$y_1 = -0.0078$$

$$y_2 = 2$$

$$x_n = \frac{(2)(1.7984) + (2)(0.0078)}{(2) + 0.0078} = 1.7962$$

$$y_n = 0.00091$$

$$\therefore \text{Root of } |x^3 - 3x - 4| \text{ is } = 1.7962$$