Calculus "    " Assignment - 2

① $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} \, dw$ ..... $\frac{2}{w} = t$

$\frac{dt}{dw} = \frac{-2}{w^2}$

$\frac{-dt}{2} = \frac{dw}{w^2}$

$= \int_{1/50}^{2} \frac{e^t}{-2} \, dt$

$= -\frac{1}{2} \left[ e^t \right]_{1/50}^{2}$

$= -\frac{1}{2} \left[ e^{2/w} \right]_{1/50}^{2} = -\frac{1}{2} \left[ e - e^{100} \right]$

$= \frac{e^{100} - e}{2} =$

② $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} \, dt$

$t^4 + 2t = u$

$\frac{du}{dt} = 4t^3 + 2$

$du = (2t^3 + 1)(2) \, dt$

$\frac{du}{2} = (2t^3 + 1) \, dt$

$= \int \frac{1}{2(u)^3} \, du$

$= \frac{1}{2} \int \frac{1}{u^3} \, du$

$= \frac{1}{2} \int u^{-3} \, du$

$= \frac{1}{2} \left[ \frac{u^{-2}}{-2} \right] \cancel{du} + C$

$= \frac{-1}{4u^2} + C = \frac{-1}{4(t^4 + 2t)^2} + C$

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(3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)\, dx$

$= \int x^4 + 3x - 9 \, dx$

$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

(4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$\int_{-2}^{3} f(x)\, dx = \int_{-2}^{1} f(x) dx + \int_{1}^{3} f(x)\, dx$

$= \int_{-2}^{1} 3x^2 dx + \int_{1}^{3} 6 \, dx$

$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= 1 + 8 + 18 - 6$

$\int_{-2}^{3} f(x)\, dx = 21$

(5) $\int 3(8y-1)\, e^{4y^2-y}\, dy$

$= \int 3 e^t \, dt \quad \ldots\ldots \quad 4y^2 - y = t$

$\frac{dt}{dy} = 8y - 1$

$= 3[e^t] + C$

$dt = (8y-1)\,dy$

$= \underline{3(e^{4y^2-y}) + C}$

(6) $f''(x) = 15x^{1/2} + 5x^3 + 6$

$f'(x) = \int f''(x)\,dx = \int 15x^{1/2} + 5x^3 + 6 \, dx$

$= \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + C$

$f(x) = \int f'(x)\,dx = \int \cancel{30}\ 10x^{3/2} + \frac{5x^4}{4} + 6x + C \, dx$

$= \frac{10x^{5/2}}{5/2} + \frac{5x^5}{4(5)} + \frac{6x^2}{2} + Cx + D$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + D$

$f(1) = -\frac{5}{4}$ (given) $\Rightarrow -\frac{5}{4} = 4 + \frac{1}{4} + 3 + C + D$

$-\frac{5}{4} = \frac{29}{4} + C + D$

$C + D = -\frac{34}{4} = -\frac{17}{2}$ —— (1)

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$f(4) = 404$ (given) $\Rightarrow f(4) = 4(4)^{5/2} + \frac{(4)^5}{2} + 3(4)^2 + C(4)$

$404 = 128 + \frac{512}{2} + 48 + 4C + D$

$404 = \cancel{688}\ 432 + 4C + D$

$\cancel{4C + D} = -284$ — ② $\quad 4C + D = -28$

② − ①

$4C + D = -28\cancel{4}$

$C + D = -\frac{17}{2}$

$3C = -28\cancel{4} + \frac{17}{2}$

$3C = \frac{-56\cancel{8} + 17}{2}$

$\cancel{3C = \frac{-551}{2} \Rightarrow C =}$

$3C = -\frac{39}{2} \Rightarrow C = -\frac{13}{2}$

$C + D = -\frac{17}{2}$

$D = -\frac{17}{2} + \frac{13}{2} = -2$ $\quad \boxed{(C, D) = \left(-\frac{13}{2}, -2\right)}$

$\Rightarrow f(x) = 4x^{5/2} + \frac{x^5}{2} + 3x^2 - \frac{13}{2}x - 2$

7 $z = f(x + iy) + g(x - iy)$

$p = \frac{dz}{dx} = f'(x + iy) + g'(x - iy)$

$q = \frac{dz}{dy} = if'(x+iy) + ig'(x-iy)$

$r = \frac{d^2z}{dx^2} = f''(x+iy) + g''(x+iy)$

$t = \frac{d^2z}{dy^2} = i^2 f''(x+iy) + i^2 g''(x-iy)$

$= -g''(x-iy) + f''(x+iy)$

$r+t = 0$ is the p.d.e.

(8) $\frac{dx}{d\theta} = \frac{x^2}{\theta}$ , $x(1) = \theta$

$\Rightarrow$ $\cancel{x^2 dx =}$

$\int x^{-2} dx = \int \theta^{-1} d\theta$

$-x^{-1} = \ln(\theta) + C$

$-\frac{1}{x} = \ln\theta + C \Rightarrow \frac{1}{x} = -\ln\theta + C$

$\cancel{x = \frac{-1}{\ln\theta} - C}$ $\qquad x = \frac{1}{-\ln\theta + C}$

$x(\theta) = \frac{1}{-\ln\theta + C}$ ... $(x(1) = 2)$

$x(1) = 2 = \frac{1}{-\ln 1 + C}$ $\Rightarrow$ $\cancel{x}$ $-\frac{1}{x} = \ln\theta - \frac{1}{2}$

$2 = \frac{1}{-C}$ $\qquad \frac{1}{x} = \frac{1}{2} - \ln\theta$

$\boxed{C = -\frac{1}{2}}$ $\qquad \boxed{x = \frac{2}{1 - 2\ln\theta}}$

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(9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\Rightarrow \frac{dv}{9.8 - 0.196v} = dt$

$\Rightarrow \frac{dv}{dt} + 0.196v = 9.8 \quad \ldots \left(\frac{dv}{dt} + Pv = Q\right)$

$IF = e^{\int 0.196\, dt} = e^{0.196t}$

$v = \frac{1}{IF}\left[\int IF(Q)\,dt + c\right]$

$v = \frac{1}{e^{0.196t}}\left[\int e^{0.196t}(9.8)\,dt + c\right]$

$v = \frac{1}{e^{0.196t}}\left[\frac{9.8e^{0.196t}}{0.196} + c\right]$

$v = \frac{9.8}{0.196} + ce^{-0.196t}$

$v = 50 + ce^{-0.196t}$

(10) $ty' + 2y = t^2 + 1 - t \qquad y(1) = \frac{1}{2}$

$t\left(\frac{dy}{dt}\right) + 2y = t^2 - t + 1$

$\frac{dy}{dt} = -\frac{2y}{t} + t - 1 + \frac{1}{t}$

$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$

$$IF = e^{\int \frac{2}{t} dt}$$

$$= e^{2 \ln t}$$

$$= t^2$$

$$Y = \frac{1}{t^2}\left[\int t^2\left(t - 1 + \frac{1}{t}\right) dt + c\right]$$

$$Y = \frac{1}{t^2}\left[\frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c\right]$$

$$y(1) = \frac{1}{2}$$

$$\therefore \quad \frac{1}{2} = 1\left[\frac{1}{4} - \frac{1}{3} + \frac{1}{2} + c\right]$$

$$0 = \frac{3-4}{12} + c$$

$$\frac{1}{12} = c$$

$$\therefore \quad \boxed{t^2 y = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + \frac{1}{12}}$$

(11) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\qquad$ $y(0) = k$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$y^{-3} dy = \frac{x}{\sqrt{1+x^2}} dx$$

.... $1+x^2 = t$

$2x\,dx = dt$

$\frac{dt}{2} = x\,dx$

$$y^{-3} dy = \frac{1}{\sqrt{t}} \frac{dt}{2}$$

$$\int y^{-3}dy = \frac{1}{2}\int t^{-1/2}\,dt$$

$$\frac{y^{-2}}{-2} = \frac{1}{\cancel{2}}\left[\cancel{2}t^{1/2}\right] + c$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$$

$$y(0) = -1$$

$$y = \sqrt{-2(\sqrt{1+x^2}+c)}$$

$$-1 = \sqrt{-2\sqrt{1+x^2} - 2c}$$

$$1 = -2\sqrt{1+x^2} - 2c$$

$$-\frac{1}{2} = \sqrt{1+(0)} + c$$

$$-\frac{1}{2} = 1 + c$$

$$c = -\frac{3}{2}$$

$$\Rightarrow \boxed{\frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}}$$

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$x=3$

(12) $f(x) = x^3 - 10x^2 + 6$ $\qquad f(3) = (3)^3 - 10(3)^2 + 6 = -57$

$f'(x) = 3x^2 - 20x$ $\qquad f'(3) = 3(3)^2 - 20(3) = -33$

$f''(x) = 6x - 20$ $\qquad f''(3) = 6(3) - 20 = -2$

$f'''(x) = 6$ $\qquad f'''(3) = 6$

$f''''(x) = 0$

Taylor Series: $f(x) = f(3) + \frac{f'(3)(x-3)}{1!} + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(3)(x-3)^3}{3!}$

$$f(x) = -57 + \frac{(-33)(x-3)}{1!} + \frac{(-2)(x-3)^2}{2!} + \frac{6(x-3)^3}{3!}$$

$\Rightarrow$ ~~$f(x) = -57 - (33)(x-3) - (2)(x-3)^2$~~

$\Rightarrow f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3$

~~(13)~~

(14) $f(x) = \sqrt{1+x} = (1+x)^{1/2}$ $\qquad f(0) = 1$

~~$f'(x) =$~~ P.T.O

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $\qquad f''(0) = \frac{-1}{4}$

$f'''(x) = +\frac{3}{8}(1+x)^{-5/2}$ $\qquad f'''(0) = \frac{3}{8}$

Maclaurin Series

$$: f(x) = f(0) + \frac{x f'(0)}{1!} + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \dots$$

$$\cancel{f(x) = \sqrt{1+x} = \frac{1}{2} - \frac{1}{4}x +}$$

$$f(x) = \sqrt{1+x} = 1 + \frac{1(x)}{2} - \frac{1}{8}(x)^2 + \frac{1}{16}(x^3) + \dots$$

$$\Rightarrow f(x) = \sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \dots$$

(15) $f(x) = e^{-6x}$ $\qquad f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\qquad f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\qquad f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ $\qquad f'''(-4) = -216e^{24}$

Taylor Series

:

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots$$

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$$f(x) = e^{-6x} = f(-4) + \frac{f'(-4)(x+4)}{1!} + \frac{f''(-4)(x+4)^2}{2!} + \frac{f'''(-4)(x+4)^3}{3!} + \ldots$$

$$f(x) = e^{-6x} = e^{24} + e^{24}(-6)^{(x+4)} + \frac{e^{24}(36)^{(x+4)^2}}{2} + \frac{e^{24}(-216)^{(x+4)^3}}{6} + \ldots$$

$$\Rightarrow f(x) = e^{-6x} = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3 + \ldots$$

(16) $f(x) = \ln(3+4x)$ $\qquad f(0) = \ln 3$

$f'(x) = \frac{4}{3+4x} = 4(3+4x)^{-1}$ $\qquad f'(0) = \frac{4}{3}$

$f''(x) = (-1)(4)(3+4x)^{-2}(4)$ $\qquad f''(0) = -\frac{16}{9}$

$f'''(x) = (-1)(-2)(16)(3+4x)^{-3}(4)$ $\qquad f'''(0) = \frac{128}{27}$

Taylor Series: $f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \ldots$

$$f(x) = \ln(3+4x) = \ln 3 + (x-a)\left(\frac{4}{3}\right) + \frac{(x-a)^2}{2}\left(\frac{-16}{9}\right) + \frac{(x-a)^3}{6}\left(\frac{128}{27}\right) + \ldots$$

$$\Rightarrow f(x) = \ln(3+4x) = \ln 3 + \frac{4(x-a)}{3} + \frac{8}{9}(x-a)^2 + \frac{128}{162}(x-a)^3 + \ldots$$