NMA assignment

1. Observed radius = 12.65 cm
Actual radius = 12.5 cm

Observed area $= \pi r^2$

$A_0 = \pi \times (12.65)^2$

$\therefore A_0 = 502.725 \text{ cm}^2$

Actual Area $= \pi r^2$

$A_1 = \pi \times (12.5)^2$

$\therefore A_1 = 490.874 \text{ cm}^2$

(i) Absolute error = observed value − actual value
$= 502.725 - 490.874$
$= 11.851 \text{ cm}^2$

~~(ii) Relative error ~~Area~~ = Relative error x100~~
~~= 2.414%~~

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~~2(i)~~

$= \dfrac{11.851}{490.874}$

$= 0.2414$

(iii) Percentage error = Relative error x 100
$= 2.414\%$

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$\therefore A_o = 502.725 \text{ cm}^2$

Actual Area $= \pi r^2$
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= 502.725 − 490.874
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= 2.414%

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2 (i) $x_1 = 4$ $y_1 = 0.60206$

$x = 5$ $y = ?$

$x_2 = 6$ $y_2 = 0.7781513$

$$\therefore y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{(x_2 - x_1)}$$

$$= \frac{0.7781513(1) + 0.60206(1)}{2}$$

$$\therefore y = 0.6901$$

(ii) $x_1 = 4.5$ $y_1 = 0.6532125$

$x = 5$ $y = ?$

$x_2 = 5.5$ $y_2 = 0.7403627$

$$\therefore y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{x_2 - x_1}$$

$$= \frac{0.7403627(0.5) + 0.6532125(0}{1}$$

$$\therefore y = 0.6901$$

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A

$\ldots + A)^3$: To find

$\ldots (I + A)^3 = 7A - (I^3 + A^3 + 3A^2I + 3AI^2)$

$= 7A - I^3 - A^3 - 3A^2I - 3AI^2$

$= 7A - I^3 - A^2 \cdot A - 3A^2 - 3A$

$= 7A - I - A \cdot A - 3A - 3A$

$= 7A - I - A - 6A$

$= \underline{7A - 7A - I}$

$- I$

$+ 2 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$

-2

$\ldots 125 / (0.5)$

0

3rd iteration

$x_3 = \frac{1.\ldots}{2}$

$f(x_3) = f(1.8125) = \ldots$

$= -1.0\ldots$

4th iteration

$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$

$f(x_4) = 1.84375^2 - 1.84375 - 10 = -0.2877$

5th iteration

$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$

$f(x_6) = 1.859375^4 - 1.859375 - 10$

$= 0.093$

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3. $x^4 - x - 10 = 0$

$a = 1.5 \qquad b = 2$

$\therefore f(a) = f(1.5) = (1.5)^4 - (1.5) - 10$

$= -0.4375$

$f(b) - f(2) = 2^4 - 2 - 10$

$= 4$

$\therefore$ <u>1st iteration</u>

$x_1 = \frac{2+1.5}{2} = 1.75$

$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = -2.3711$

<u>2nd iteration</u> :

$x_2 = \frac{1.75+2}{2} = 1.875$

$f(x_2) = 1.875^4 - 1.875 - 10 = 0.4846$

<u>3rd iteration</u>

$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$

$f(x_3) = f(1.8125) = 1.8125^4 - 1.8125 - 10$

$= -1.0202$

<u>4th iteration</u>

$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$

$f(x_4) = 1.84375^2 - 1.84375 - 10 = -0.2877$

<u>5th iteration</u>

$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$

$f(x_6) = 1.859375^4 - 1.859375 - 10$

$= 0.093$

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$\therefore$ root of $x^4 - x - 10 = 0$ is 1.859375
which is approx 1.86.

4. $f(x) = x^3 - x - 1$
$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$\therefore x_0 = \frac{1+2}{2} = 1.5$

$f(x_0) = 0.875$ $\quad f'(x_0) = 5.75$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1.5 - \frac{0.875}{5.75} = 1.34783$

$f(x_1) = 0.10068$ $\quad f'(x_1) = 4.44991$

$x_2 = 1.34783 - \frac{0.10068}{4.44991}$

$= 1.3252$

$\therefore f(x_2) = 0.00205$ $\quad f'(x_2) = 4.2684$

$x_3 = 1.3252 - \frac{0.00205}{4.2684}$

$\therefore x_3 = 1.3247$

$f(x_3) = 0.0000007543$ $\quad f'(x_3) = 4.2645$

$\therefore x_4 = 1.3247 - \frac{0}{4.2645}$

$\therefore x_4 = 1.3247$

$\therefore$ Root of $x^3 - x - 1.0$ is 1.3247

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5. $A^2 = A$

$7A - (1+A)^3$ ∴ To find

$$\therefore 7A - (1+A)^3 = 7A - (I^3 + A^3 + 3A^2I + 3AI^2)$$
$$= 7A - I^3 - A^3 - 3A^2I - 3AI^2$$
$$= 7A - I^3 - A^2.A - 3A^2 - 3A$$
$$= 7A - I - A.A - 3A - 3A$$
$$= 7A - I - A - 6A$$
$$= 7A - 7A - I$$
$$\therefore 7A(I+A)^3 = -I$$

6. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & 1 \end{bmatrix}$

$$\therefore |A| = 1\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$$
$$= 1(-6) + 1(-4) + 2(4) = -2$$

$|A| \neq 0$ ∴ inverse exists

∴ Co-factor matrix $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|} \times \text{Adjoint}$$

$$= \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$$

$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ 2 & -1/2 & -2 \end{bmatrix}$

9. The smallest 3 digit no. that leaves remainder of 2 is 101 and the largest is 998

$\therefore a = 101 \qquad l = 998$

$d = 3$

$\therefore t_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$\frac{998 - 101}{3} = n - 1$

$\therefore n = 300$

$S_n = \frac{n}{2}(a + l) = \frac{300}{2}(101 + 998)$

$= 164850$

$\therefore$ sum of all numbers that leave a remainder of 2 when divided by 2 is 164850.

10. $t_6 = 32$

$t_8 = 128$

$\therefore t_6 = ar^5$

$32 = ar^5 \rightarrow (1)$

$t_8 = ar^7$

$128 = ar^7 \; — \; (2)$

Dividing (2) and (1)

$\frac{\cancel{a}r^{7\,2}}{\cancel{a}r^{\cancel{5}}} = \frac{128}{32} \qquad r^2 = 4$

$\boxed{r = 2}$

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11. expand $(4+3x)^5 = (3x+4)^5$

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3$$

$$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4 4 + \frac{5 \times \cancel{4}^{2}}{\cancel{2}}(3x)^3 4^2 + \frac{5\times4\times3}{3\times2}(3x)^2 4^3 + \frac{5\times4\times3\times2}{4\times3\times2}(3x)4^4 + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\therefore \lambda^3 - (0)\lambda^2 + (-13)\lambda - (-30+18+0) = 0$

$\therefore \lambda^3 - 13\lambda + 12 = 0$

$\therefore \lambda = 1$ satisfied

$$\therefore \begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & \downarrow & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$\therefore \lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$(\lambda+4)(\lambda-3) = 0$

$\lambda = -4$ or $\lambda = 3$

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13. $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$\therefore f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = -13$

$f'(x) = 3x^2 - 14x + 8$

$f'(5) = 13$

$\therefore x_1 = 5 - \left(\frac{-\cancel{13}^{1}}{\cancel{13}}\right)$

$= 6.$

$f(6) = 9$

$f'(6) = 32$

$x_2 = 6 - \left(\frac{9}{32}\right)$

$\boxed{\therefore x_2 = 5.71875}$

14. $(1 - x + x^2)^4$

$\therefore [x^2 + (1-x)]^4$

let $1 - x = y$

$\therefore [x^2 + y]^4$

using binomial Theoram

$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^2 \dots + b^2$

$\therefore [x^2 + y]^4 = (x^2)^4 + 4(x^2)^3 y + \frac{4 \times 3}{2} (x^2)^2 y^2 + \frac{4 \times 3 \times 2}{3 \times 2} (x^2) y^3$

$= x^8 + 4x^6 y + 6x^4 y^2 + 4x^2 y^3 + y^4$

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$$\therefore [x^2+(1-x)]^4 = x^8+4x^6(1-x)+6x^4(1-x)^2+ 4x^2(1-x)^3+(1-x)^4.$$

15. $e^{-x} = 3\log(x)$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1 \quad b = 2$

$\therefore f(a) = -0.367879$

$f(b) = 1.944106$

$\therefore \frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$

$f(x_1) = 0.993265$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = 3\log(1.25) - e^{-1.25}$
$= 0.382925$

$x_3 = \frac{1+1.25}{2} = 1.125$

$f(x_3) = 3\log(1.125) - e^{-1.125}$
$= 0.02869664$

$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = 3\log(1.0625) - e^{-1.0625}$
$= -0.16372$