

Assignment 1 PRL

$$1. (i) (Ia)_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i}$$

PV of annuity:

$$a_{\overline{n}|i} + 1 \sum_{i=1}^{n-1} v^i a_{\overline{n-1}|i}$$

$$a_{\overline{n}|i} + 1 \sum_{i=1}^{n-1} v^i \frac{1-v^{n-i}}{i}$$

$$a_{\overline{n}|i} + 1 \sum_{i=1}^{n-1} v^i - (n-1)v^n$$

$$a_{\overline{n}|i} + 1 \sum_{i=1}^{n-1} v^i - nv^n$$

$$a_{\overline{n}|i} + \frac{a_{\overline{n}|i} - nv^n}{i}$$

$$\frac{Ia_{\overline{n}|i}}{i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i}$$

EPV of benefit:

$$\frac{(200000 - X) \bar{a}}{8.107} + X \frac{(I \bar{a})}{8.107} =$$

$$3000000 (1.06)^{10}$$

2. EPV of benefit:

$$100000 \int_0^{20} e^{-\delta t} \times {}_{40}P_{40+t}^{HH} \mu_{40+t} dt +$$

$$100000 \int_0^{20} e^{-\delta t} \times {}_{40}P_{40+t}^{HH} \sigma_{40+t} dt$$

$$100000 \int_0^{20} e^{-(1.05)t} \times \int_0^t (P_{40+t} + \sigma_{40+t}) d\epsilon$$

$${}_{40}P_{40+t}^{HH} = e^{-\int_0^t (0.004 + 0.002) dt}$$

$$e^{-\int_0^t 0.006 dt}$$

$$e^{-0.006t}$$

$$= e^{-0.006t}$$

EP v of benefit

$$= 100000 \int_0^{20} e^{-\ln(1.05)t} \times e^{-0.06t} \times (0.004 + 0.002) dt$$

$$100000 \int_0^{20} e^{-\ln(1.05)t} e^{-0.06t} (0.004) +$$

$$100000 \int_0^{20} e^{-\ln(1.05)t} e^{-0.06t} (0.002) dt$$

$$100000 \int_0^{20} e^{-0.10879t} (0.006) dt$$

$$600 \left[-\frac{e^{-0.10879t}}{0.10879} \right]_0^{20}$$

$$\frac{600}{0.10879} [-0.1135173 - -1]$$

$$4889.14$$

3. EPV of premium:

$$12 P \ddot{a}_{\overline{12}|} = P \ddot{a}_{\overline{40:20}|} - \frac{14}{24} \left(1 - \frac{P_{60}}{P_{40}} \right)$$

$$= P(13.66576)$$

EPV of benefit:

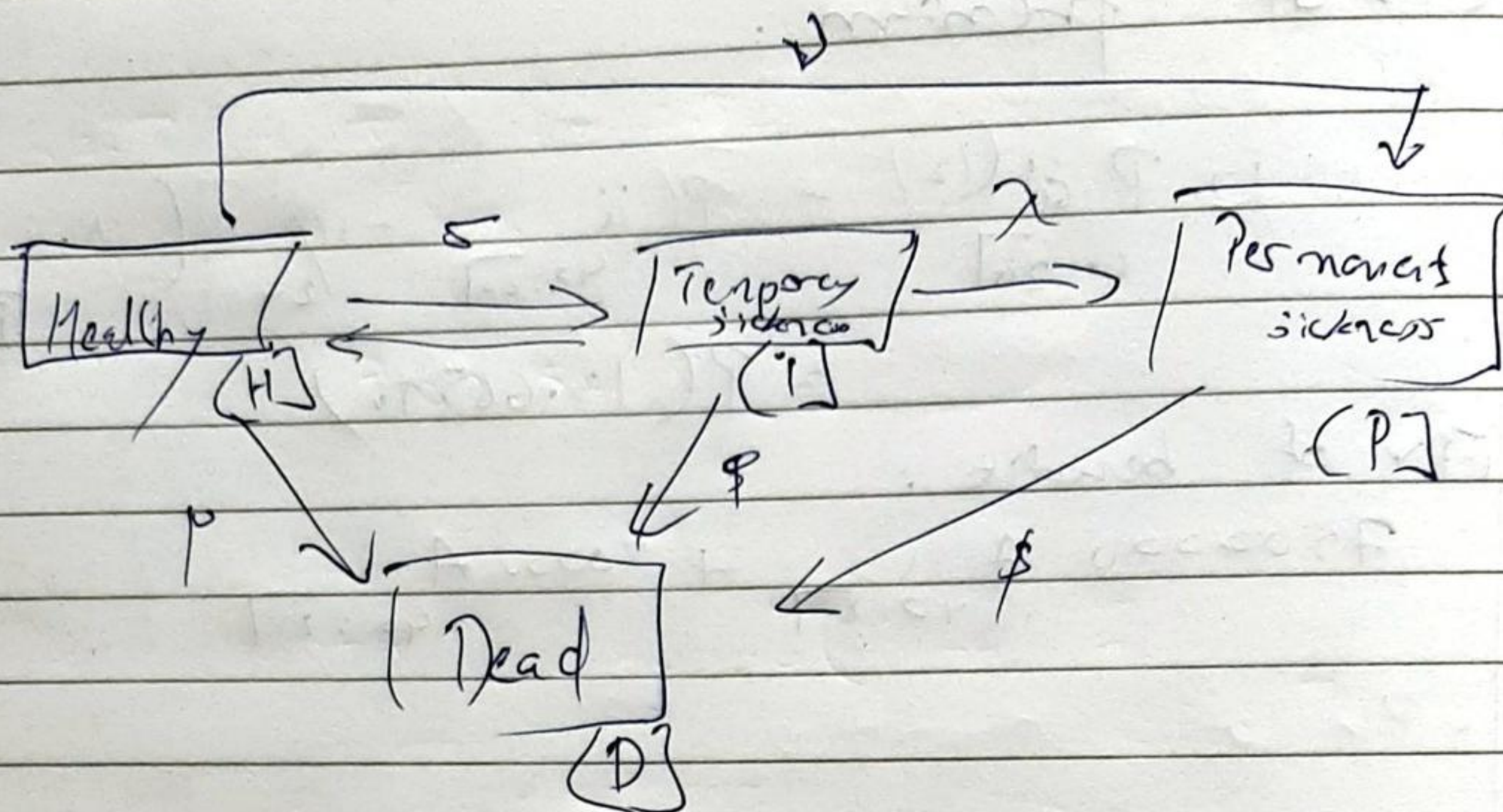
$$7500000 A_{\overline{1}|} + 50000 A_{\overline{1}|}$$

EPV of expense:

$$0.35 \times 12 P a + 0.05 \times 12 P \ddot{a}_{\overline{40:20}|} @ 4\% +$$

$$500 + 0.10 \times 12 P \ddot{a}_{\overline{40:20}|}$$

4.



EPV of benefit: (permanent sickness)

$$20000 \int_0^{15} e^{-\delta t} \times \left(\frac{tP^{HI}}{50} \frac{v}{50+t} + \frac{tP^{HI}}{50} \lambda \right) dt$$

EPV of sickness

$$10000 \int_0^{15} e^{-\delta t} \times \frac{EP^{HI}}{50} dt$$

EPV of death:

$$100000 \int_0^{15} e^{-\delta t} \left(\frac{EP^{HI}}{50} p \frac{1}{50+t} + \frac{EP^{HI}}{50} \frac{\delta}{50+t} + \frac{tP^{HP}}{50+t} \right) dt$$

EPV of premium:

$$? \bar{a}_{50:65}$$

5. EPV of premium.

$P \ddot{a}$

$(40]:25]$

$$P \times (15.887)$$

EPV of benefit:

$$\frac{200000}{1.04} \bar{A}_{(40]:25]} @ i' = 19230.769$$

$$i' = \left(\frac{1 + 0.04}{1 + 0.04} - 1 \right) = 0$$

EPV of expense:

$$1200 + 0.02 \times P \left(\ddot{a}_{(40]:25]} - 1 \right) + 500 \bar{A}_{(40]:25]} @ 4\%$$

$$1200 + 0.02 P (14.887) + 500 (1.04)^{0.5} (0.38896)$$

$$8250.822240 + 0.29777P$$

EPV of

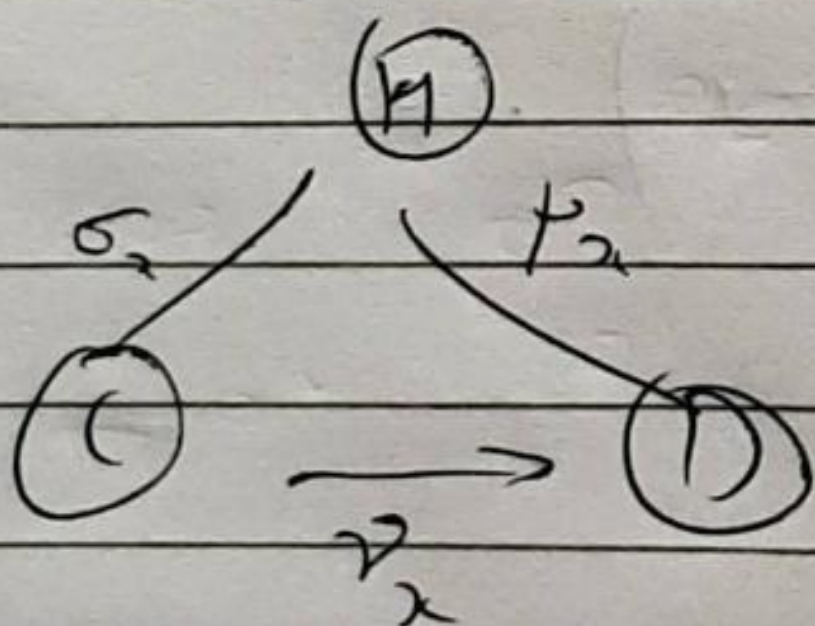
EPV of benefit + EPV of expense = EPV of premium

$$19230.769 + 8250.82224 + 0.2977P = 15.877P$$

$$P = 1762.8495$$

(ii) EPV of benefit: $20000(1-q) \bar{A}_{(40):25} + 20000(q) \bar{A}_{(47)}$

6.



EPV of benefit: $100000 \int_0^{\infty} e^{-\delta t} {}_t p_{45}^{HH} \left(\mu_{45+t} + \frac{\delta}{45+t} \right) dt$

$= 100000 \int_0^{\infty} e^{-0.04819t} \times {}_t p_{45}^{HH} (0.008) dt$

$\rightarrow 0.008$

$= 800 \times \left(\frac{-e^{-0.05671 \times 10} + 1}{0.05671} \right)$

9562.7307

f. EPV of benefit:

$$1000000 A$$

$$(45):20$$

$$= 1000000 (0.32711) = 327110$$

EPV of premium:

$$30000 \ddot{a}$$

$$(45):20$$

$$= 30000 (11.88) = 356400$$

EPV of expense:

$$5000 + 2.5\% \times (30000) + \frac{500}{1.019231} IA @ 4\% (47) + 0.02 (30000) \left(\ddot{a}_{(45):20} - 1 \right) + IC \text{ (initial commission)}$$

$$5000 + 750 + 4145.286 + 6532.8 + IC$$

$$\text{Net cash flow} = 327110 + 750 + 5000 + 4145.286 + 6532.8 + IC$$

$$= 343538.026 + IC - 356400$$

$$IC - 13101.914$$

$$\frac{IC - 13101.914}{30000} = 0.10$$

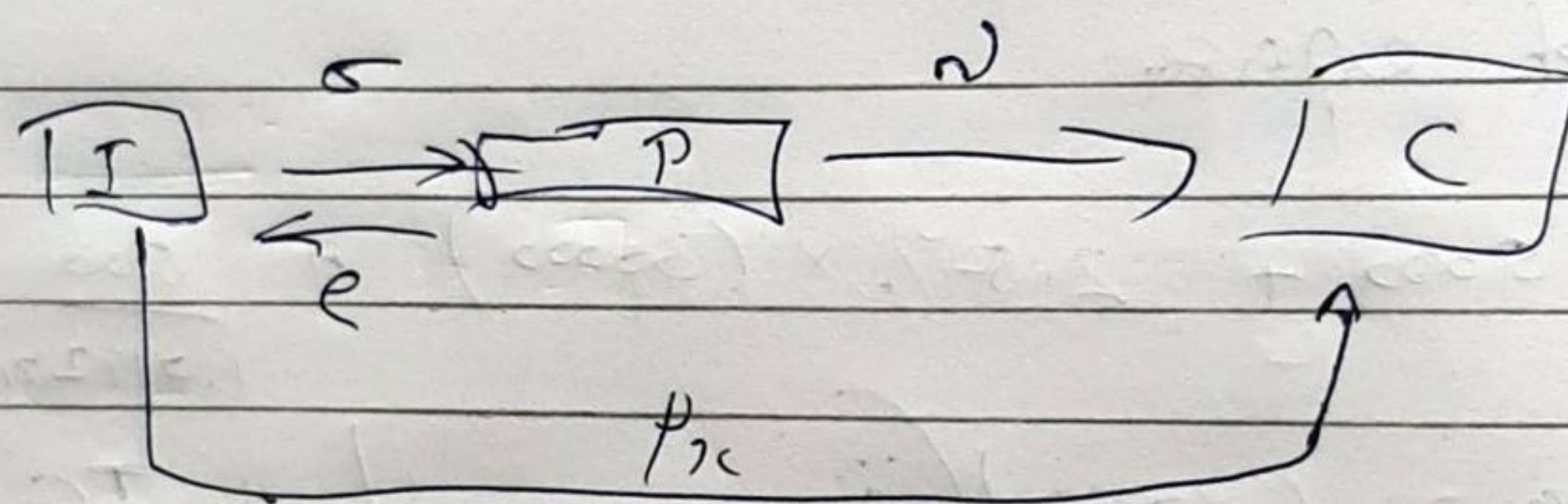
$$IC = 16101.914$$

8. (i) Equal forces in the multiple and single decrement

uniform distribution of all decrement across year of age.

(ii)

(iii)



$$(a) EPV = 1000 \int_0^5 tP_{45}^{AC} v dt$$

$$(b) EPV = 250 \int_0^5 \left[tP_{45}^{IP} 5 \right] v dt$$

$$(c) EPV = 150 \int_0^5 tP_{45}^{I\cancel{P}} dt$$

$$(d) EPV = 150 \int_0^5 tP_{45}^{PC} v dt$$

9. (i) To defer eventual surplus distribution
To allow smoothing of payout.

(ii) Investment surplus, surrender surplus

(iii) Terminal bonus are added by allocated as a percentage of the basic sum assured and the bonus allocated prior to distribution. The percentage rate will vary with the term of the policy at the date of payment.

10. EPV of benefit:
 $\frac{1000000}{1.0192308} A @ 4\%$
 $50 : 15$

$$= \frac{1000000}{1.0192308} (11.253) = 11040627$$

EPV of expense:

$$20\% \times 12P + 2500 + 0.35 \times 12P + 200P + 0.2 \times 12P \left(\frac{\ddot{a}_{50:15} - 1}{50:15} \right) + 400 \left(\frac{\ddot{a}_{50:15} - 1}{50:15} \right) +$$

$$0.015 \times 12P \left(\frac{\ddot{a}_{50:15} - 1}{50:15} \right)$$

$$6.6P + 25000 +$$

EPV of premium:

$$12 P \ddot{a}_{50:\overline{12}|} (1.06)^{12} = 89.7648 P$$

$$\ddot{a}_{50:\overline{12}|} = \ddot{a}_{50:\overline{12}|} - \frac{11}{24} (1 - 12P_{50} v^{12}) = 7.4804$$

11.

EPV of benefit:

$$400000 (1 + 20(0.04)) A_{30:\overline{20}|}$$

$$400000 (1 + 0.8) = 720000$$

EPV of premium:

$$P \ddot{a}_{30:\overline{20}|}$$

$$P (\ddot{a}_{30} - n P_{30} v^n \ddot{a}_{50})$$

$$P (12.08105)$$

$$A_{30:\overline{20}|} = A_{30:\overline{20}|} + A_{30:\overline{20}|} = 0.315817$$

$$A_{30:\overline{20}|} = v^{20} 20P_{30}$$

$$A_{30:\overline{20}|} = A_{30} - v^{20} P_{30} A_{50}$$

$$P = \frac{400000 (0.315817)}{12.08755}$$

$$= 10451.42$$

Bonus:

$$7 \times 4\% \times 400000 = 112000$$

$$SA = 400000 + 112000$$

$$= 512000$$

$$PV = 512000 A_{\overline{37}|7.5\%} - 10451.42 \ddot{a}_{\overline{37}|13\%}$$

$$= 51200 (0.47163) - 10451.42 (9.338)$$

$$= 143926.89$$