Assignment 2

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Subject : Probability & Statistics - 1

Sol^n ① $E(X_1) = 800$ $\quad E(X_2) = 1200$

$V(X_1) = (100)^2$ $\quad V(X_2) = (300)^2$

ATQ

$Y \sim N(800, 100^2)$

$Y \sim N(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P[(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3 \times 1200 - 4100), 300^2(3) + 4..$

$\sim N(400, 310000)$

$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$

$P[Z > 0.71842)$

$= 1 - P(Z - 0.72)$

$= 1 - 0.76424$

$= \boxed{0.23576}$

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Sol^n (2) N ~ Neg Binomial (k, p)

$$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$$
$$= {}^{(n+2)-1}C_{n-1} (0.9)^3 (0.1)^n$$

$k = 3,\ P = 0.9,$

$$M_y(t) = M_n(\log M_x t)$$

$X \sim$ Gamma (6, 2)
$Y \sim$ Total Claim amount.

(i) MGF of Y

$$M_y(t) = M_n[\log M_x t]$$

$$M_x(t) = \left[1 - \frac{t}{2}\right]^{-\alpha}$$

$$M_y(t) = M_n\left[\log\left[1 - \frac{t}{2}\right]^{-6}\right]$$

$$M_n(t) = \left[\frac{pe^t}{1 - qe^t}\right]^k = \left[\frac{p(1-\frac{t}{2})^{-6}}{1 - q[1-\frac{t}{2}]^{-6}}\right]^3 = \left[\frac{0.9(1-\frac{t}{2})^{-6}}{1 - 0.1(1-\frac{t}{2})^{-6}}\right]^3$$

$$V(Y) = E(N) \cdot V(X) + [E(X)]^2 V(N)$$
$$= \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left[\frac{6}{2}\right]^2 \times \frac{3(0.1)}{(0.9)^2}$$
$$= 8.3347$$

$sd = 2.887$

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Sol^n ⑤ $f(x) = \lambda e^{-\lambda(x)} = \lambda e^{\lambda(x-\lambda)}$

Sol^n ②

$$MGF = \int_{\alpha}^{\infty} e^{tx} \lambda e^{\lambda(x-\alpha)}$$

$$M_X(t) = \lambda e^{\lambda\alpha} \int_{0}^{\alpha} e^{-(\lambda-t)x} \quad x = \frac{e^{\alpha\lambda}\lambda}{(t-\lambda)} \left[e^{(\lambda-t)x} \right]_{\alpha}^{\infty}$$

$$= e^{t\alpha} \left[\frac{\lambda}{\lambda - t} \right]$$

(ii)

$$M_X(t) = \frac{\lambda e^{t\alpha}}{\lambda - t} \Rightarrow \lambda e^{t\alpha} (\lambda - t)^{-1}$$

$$M'_X(t) = \lambda \left[e^{t\alpha} (\lambda-t)^{-2}(+1) + (\lambda-t)^{-1} e^{t\alpha}(\alpha) \right]$$

$$= \lambda e^{t\alpha} (\lambda-t)^{-2} (\alpha(\lambda-t)+1)$$

at $t=0$

$$M'_X(0) = \lambda(\lambda)^{-2} [\alpha(\lambda)+1]$$

$$= \frac{\lambda\alpha+1}{\lambda}$$

$$M''_X(t) = \lambda [\alpha e^{t\alpha}(\lambda-t)^{-2} + \alpha^2(\lambda-t)^{-1}e^{t\alpha} + e^{t\alpha}(+2)(\lambda-t)^{-3} + (\lambda-t)^{-2}e^{t\alpha})$$

at $t=0$

$$M''_X(0) = \lambda \left[\alpha(\lambda)^{-2} + \alpha^2(\lambda)^{-1} + 2(\lambda)^{-3} + \alpha(\lambda)^{-2} \right]$$

$$E(X^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda} \right]^2$$

λ^2

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Solⁿ (4) $G_Y(t) = \left(\frac{p}{1-qt}\right)^k \cdot \Sigma t^y \times P(Y=y)$

$P(Y=4) = \frac{\sqrt{k+4}}{\sqrt{5}\sqrt{k}} p^k q^k$

$= \frac{(k+3)!}{(k-1)!\,4!} p^k (1-p)^k$

Solⁿ (5) $f(x,y) = ax(x+y)$ $\quad 0<x<5,\ 10<y<20$

(i) $P\left(Y > \frac{1}{3}X + 10\right)$

$= a\int_{10}^{20}\int_{0}^{5} (x^2+xy)\,dx\,dy$

$= a\int_{10}^{20}\left[\frac{x^3}{3} + \frac{x^2}{2}y\right]_0^5 dy$

$= a\int_{10}^{20}\left(\frac{125}{3} + \frac{25}{2}y\right)dy$

$= \left[\frac{125}{3}y + \frac{25}{4}y^2\right]_{10}^{20} = \frac{1}{9}$

$= \frac{\cancel{625}}{\cancel{8}} = \frac{1}{9}$

$= 833.33 + 2500 - (416.667 + 625) = \frac{1}{9}$

$\boxed{a = 0.0004363}$

Sol^n (6)

Sol^n (4)

(ii) $P\left(Y > \frac{1}{3}x + 10\right)$

$$f(y) = \frac{3}{6875}\int_0^5 (x^2 + xy)\,dx$$

$$= \frac{3}{6875}\left[\frac{125}{3} + \frac{25}{2}y\right]$$

Sol^n (5)

(i)

$$\therefore = \frac{3}{6875}\int_{\frac{1}{3}x+10}^{20}\left(\frac{125}{3} + \frac{25}{2}y\right)$$

$$= \frac{3}{6875}\left[\frac{125}{3}y + \frac{25}{4}y^2\right]_{\frac{1}{3}x+10}^{20}$$

$$= \frac{3}{6875}\left[\left(\frac{125}{3}(20) + \frac{25}{4}(400)\right) - \left[\frac{125}{3}\left(\frac{1}{3}x+10\right) - \frac{25}{4}\left(\frac{1}{3}+10\right)^2\right]\right]$$

$$= \frac{3}{6875}\left[\frac{2500}{3} + 2500 - \frac{125}{9}x\left(-\frac{1250}{3}\right) - 625 - \frac{25}{4}x\right]$$

$$= \frac{3}{6875}\left[\frac{1250}{3} + 18\right]$$

(iii) $E(Y/X) = ?$

$$f_{y/x} = \frac{f(x,y)}{f(x)} \to$$

$$f(x) = \frac{3}{6875}\int_{10}^{20}(x^2 + xy)\,dy = \frac{3}{6875}\left[x^2y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$= \frac{3}{6875}\left[20x^2 + x(200) - 10x^2 - 50x\right]$$

$$= \frac{3}{6875}\left[10x^2 + 150x\right] = \frac{30x}{6875}\left[x + 15\right]$$

$$E(y/x) = \int_{10}^{20} y\left(\frac{x+y}{10(x+15)}\right) dy$$

$$= \frac{1}{10(x+5)} \int_{10}^{20} (xy+y^2)\, dy$$

$$= \frac{1}{10(x+15)} \left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}$$

$$= \frac{1}{10(x+15)} \left[200x + \frac{8000}{3} - 50x - \frac{1000}{3}\right]$$

$$\boxed{E(y/x) = \frac{1}{10(x+5)} \left[15x + \frac{7000}{3}\right]}$$

Soln ⑥

$X \sim Poiso(\lambda)$, $\quad Y \sim Pois(\mu)$

$M_X(t) = e^{\lambda[e^t - 1]}$ $\quad M_Y t = e^{\mu(e^t-1)}$

$Z = X - y$

$$M_X(t) = e[e^{t\cdot}$$
$$= E[e^{t(X-\mu)}$$
$$= \frac{E[e^{tX}]}{E[\cdot]}$$
$$= \frac{e^{\lambda(e^t-1)}}{e^{\mu[e^t-1)}}$$
$$= e^{(\lambda-\mu)(e^t-1)}$$

$Z = X + y$

by uniqueness property

$Z \sim Pois(\lambda\ \mu)$

$Z \sim Pois(\lambda + \mu)$.

Sol^n ⑦

(i) $v(X/Y=2) = E[X^2/Y=2] - [E(X/Y=2)]^2$

$E(X^2/Y=2) = \sum_x x^2 \frac{P[X=x, Y=2]}{P(Y=2)}$

$= 8.8333$

$E(X/Y=2) = \sum_x x \frac{P(X=x, Y=2)}{P(Y=2)} = 0.807$

$v(X/Y=2) = 8.8333 - 0.807^2$
$= 8.8333 - 0.651$
$v(X/Y=2) = 8.182$

(ii) $f(u,v) = \frac{48}{67}(2uv - v^2)$

$E(U/V=v) = \frac{f(u,v)}{f(v)}$

$f(v) = \frac{48}{67}\int_0^1 (2uv - v^2)\,du$

$= \frac{48}{67}\left[v - \frac{1}{3}\right] = \frac{16}{67}\left[3v - 1\right]$

$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}}{\cancel{67}} \frac{(2uv^2 - v^2) \times \cancel{67}}{\cancel{16}(3v-1)} = 2u^2v^2 - v^3$

$E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - v^2)\,du$

$= \frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{v^4}{4}\right]_0^1 = \frac{3}{3v-1}\left[\frac{8v^2-3}{12 \times 4}\right] = \frac{8v^2-3}{4(3v-1)}$

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Sol^n

8) $X \sim gamma(3,2)$

$E(X) = 3/2 \quad V(X) = 3/4$

$E(Y/X=x) = 3x+1 \quad , Var(Y/X=x) = 2x^2+5$

$E(Y) = E[E(Y|X=x)] = 3x+1$

$= 3E(X)+1$

$= 3(3/2)+1 = 9/2+1 = 11/2$

$V(Y) = E[V(X|Y)] + V[E(X|Y)]$

$= E[2x^2+5] + V[3x+1]$

$= 2E(x^2)+5 + 9V(x)$

$= \cancel{Ed}$

$V(X) = E(X^2) - E(X))^2$

$3/4 + 9/4 = E(X)$

$E(X^2) = 3$

$V(Y) = 2(3)+5+9(3/4)$

$= 11 + 27/4 = 71/4.$

Sol^n (9)

$E(X) = 3$

$V(X) = 4$

$E(Y) = 4$

$V(Y) = 1$

$Z = X + Y$

$Cov(X,Y) = -0.6$

$Corr(X,Y) = -0.3\sqrt{4}$

(i) $Cov(X, X+Y) = Cov(X,X) + Cov(X,Y)$

$= V(X) + Cov(X,Y)$

$= 4 + (-0.6)$

$= 3.4$

(ii) $Var(Z) = Cov(X+Y, X+Y)$

$= Cov(X,X) + 2Cov(X,Y) + Cov(Y,Y)$

$= V(X) + 2Cov(X,Y) + V(Y)$

$= 4 + 2(0.6) + 1$

$Var(Z) = 3.8$

Sol^n (10) $f(x,y) = Kx^{-\alpha} e^{-y/\beta}$

$1 < x < \infty, \quad 1 < y < \infty$

$\alpha > 1, \quad \beta > 0$

$\therefore K \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta}\, dx\, dy = 1$

$K \int_1^{\infty} e^{-y/\beta} \left[\frac{x^{-\alpha+1}}{-\alpha+1} \right]_1^{\infty} dy$

$= \frac{K}{-\alpha+1} \int e^{-y/\beta}\, dy = 1$

$\frac{K}{\alpha-1} (\beta e^{-1/\beta}) = 1$

$\boxed{\therefore K = \frac{\alpha - 1}{\beta(e^{-1/\beta})}}$