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Assignment 1 - Numerical Methods

Algebra

$$x = \sqrt{7 + \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots}}}}$$

On squaring both sides,

$$x^2 = 7 + \sqrt{7 + \sqrt{7 + \dots}}$$

$$\Rightarrow x^2 - 7 + x = 0 \quad \text{Putting the value of } x$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{49 - 4}}{2}$$

$$x = \frac{7 + \sqrt{45}}{2} \quad \text{OR} \quad \frac{7 - \sqrt{45}}{2}$$

$$\Rightarrow x = \underline{\underline{6.8541}} \quad \text{OR} \quad x = \underline{\underline{0.1458}}$$

Assignment 1 — Numerical Metho

2.

$$7x^2 - 6y^2 = -11x^2 - 4y^2$$

Putting the x^2 terms in L.H.S and y^2 terms in R.H.S
we get,

$$7x^2 + 11x^2 = 6y^2 - 4y^2$$

$$18x^2 = 2y^2$$

$$9x^2 = y^2$$

$$\frac{x^2}{y^2} = \frac{1}{9}$$

$$\left(\frac{x}{y}\right)^2 = \frac{1}{9}$$

$$\Rightarrow \boxed{\frac{x}{y} = \pm \sqrt{\frac{1}{9}}}$$

$$3. \quad \begin{aligned} x+y &= 4 & \text{--- (1)} \\ x^2+y^2+4y &= 12 & \text{--- (2)} \end{aligned}$$

--- from eqⁿ (1) in (2)

$$x = 4 - y$$

$$(4-y)^2 + y^2 + 4y = 12$$

$$\begin{aligned} 16 + y^2 - 8y + y^2 + 4y &= 12 \\ 16 + 2y^2 - 4y &= 12 \\ y^2 - 6y + 8 &= 0 \\ y^2 - 6y + 2 &= 0 \end{aligned}$$

$$y = \frac{6 \pm \sqrt{36-8}}{2}$$

$$= \frac{6 \pm \sqrt{28}}{2}$$

$$= 3 \pm \sqrt{7}$$

$$\begin{aligned} x_1 &= 4 - y_1 \\ &= 4 - 3 + \sqrt{7} \\ &= 1 + \sqrt{7} \end{aligned}$$

$$\begin{aligned} x_2 &= 4 - y_2 \\ &= 4 - 3 - \sqrt{7} \\ &= 1 - \sqrt{7} \end{aligned}$$

The ordered pairs are

$$(1+\sqrt{7}, 3-\sqrt{7}) \text{ and } (1-\sqrt{7}, 3+\sqrt{7})$$

4. $d = 2x^2 - 16x + 34 \rightarrow$ density

i) $x = 2.5$

$d = 2.5$

mill particles / St^3

ii) $d - 2x^2 - 16x + 34 = 0$

differentiate w.r.t. x

$4x - 16 = 0$

①

$x = 4$

(400 RPM)

differentiation of ①

4

Minima:

$d = 2(4)^2 - 16(4) + 34$

$d = 2$

$\therefore$ The no of revolution for minimum smoke - 4

iii) $d = 100$

$100 = 2x^2 - 16x + 34$

$50 = x^2 - 8x + 17$

$x =$

$0 = x^2 - 8x + (-33)$

$x = (x-11)(x+3)$

$\therefore x = 11$

The speed must be 11 revolutions per minute

$$5. \quad 1. \quad 21500 + 25(\text{km})$$

$$2. \quad 2100 + 22(\text{km})$$

i) Cost to rent car from Co. 1

$$C_1 = 1500 + 25n$$

ii) Cost to rent car from Co. 2

$$C_2 = 2100 + 22n$$

$$\text{iii) } C_2 < C_1$$

$$1500 + 25n > 2100 + 22n$$

It is less expensive to take C_2 when the no of km is more than 200

$$x^3 - 7x^2 + 14x - 6 = 0$$

$$a] \quad a=0, \quad b=1$$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.625	0.9844	0.2508

6 b]

$$a=1$$

$$b=3.2$$

$$f(x) = x^3 - 7x^2 + 16x - 6$$

a	b	$a+b/2$	$f(a)$	$f(b)$	$f\left(\frac{a+b}{2}\right)$
1	3.2	2.1	2	-0.42	1.791
2.1	3.2	2.65	1.791	-0.112	0.9521
2.65	3.2	2.925	0.5521	-0.112	0.0833

$$\underline{\underline{x = 2.925}}$$

Q. C) [3.2, 4]

a	b	$a + \frac{b}{2}$	$F(a)$	$F(b)$	$F\left(\frac{a+b}{2}\right)$
3.2	4	3.6	-0.112	2	0.326
3.2	3.6	3.4	-0.112	0.836	-0.016
3.4	3.6	3.5	-0.040	0.836	0.125
3.4	3.5	3.45	-0.016	0.125	0.4612
3.4	3.45	3.435	-0.016	0.0461	0.0120

$n = 3.425$

7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$$f'(x) = 920x^3 + 54x^2 + 18x - 221$$

(a)	i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - f(x_i)/f'(x_i)$
	1	-1	156.1398	-0.1105	-0.60814
	2	-0.6031	43.994	-418.896	-0.2354
	3	-0.23941	1.537	-234.246	-0.04737
	4	-0.04739	0.0004	-221.704	-0.47666
	5	-0.04066	~0	-221.70443	-0.4666
	6	-0.04066	~0	-221.70443	-0.4666

$$\underline{x = -0.04060}$$

8. let no. scored in fifth exam be x

$$\text{average} = \frac{70+86+81+83+x}{5} = 64$$

8. let no. scored in fifth exam be x

$$\text{average} = \frac{70+86+81+83+x}{5} = \frac{320+x}{5}$$

according to the question : ① $\frac{320+x}{5} \geq 80$

$$320+x \geq \cancel{160} 400$$

$$x \geq 80$$

$$\textcircled{2} \frac{320+x}{5} < 90$$

$$x < 450 - 320$$

$$x < 130$$

$$\text{Thus, } 80 \leq x < 130$$

Since maximum
marks
= 100

$$80 \leq x \leq 100$$

9. $x_1 = 0.04$

$x_2 = 0.2$

$x = 0$

$f(x_1) = 8.54$

$f(x_2) = -11.81$

$f(x) = ?$

$$\frac{x - x_1}{x_2 - x_1}$$

$$= \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)}$$

$$\frac{0.04}{0.16}$$

$$= \frac{f(x) - 8.54}{-20.35}$$

$$f(x) = 13.6275$$

10. Profit : $R(x) - C(x)$

$$\text{Profit} : (32x - 0.21x^2) - (195 + 12x)$$

$$\text{Profit} : -0.21x^2 + 20x - 195$$

$$\text{set profit} = 0$$

$$0 = -195 + 20x^2 - 0.21x$$

Solving using the quadratic formula; where $a = 195$, $b = 20$ and $c = 0.21$

$$x = \frac{-20 \pm \sqrt{(20)^2 - 4(195)(0.21)}}{2(-0.21)}$$

$$= \frac{-20 \pm \sqrt{400 - 164}}{-0.42}$$

$$x = \frac{-20 \pm 15}{-0.42}$$

$$x_1 = \frac{-35}{-0.42} = 84$$

$$x_2 = \frac{-5}{-0.42} = 12$$

To be profitable (if $n = 1000$), the app has to ~~sell~~ ^{have} either 12000 or 84000 orders.