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Assignment 1

1. $$\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{2(9+h^2-6h) - 18}{h}.$$

$$\Rightarrow \lim_{h\to 0} \frac{\cancel{18}+2h^2-12h-\cancel{18}}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{2\cancel{h}(h-6)}{\cancel{h}} \Rightarrow \lim_{h\to 0} 2(h-6).$$

$$\Rightarrow 2(-6+0) \Rightarrow -12 \text{ (ans)}.$$

2. $$g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

a) $x = 4$, b) $x = 6$.

a) $2(4) = 8$.
b) $x - 1 \Rightarrow 6 - 1 \Rightarrow 5$.

3. $$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\Rightarrow \lim_{x\to 9} \frac{-(9-x)}{3-\sqrt{x}} \Rightarrow \lim_{x\to 9} -\frac{(3+\sqrt{x})\cancel{(3-\sqrt{x})}}{\cancel{(3-\sqrt{x})}}$$

$$\Rightarrow \lim_{x\to 9} -(3+\sqrt{x}) \Rightarrow -(3+3) \Rightarrow -6.$$

4. $$f(x) = \frac{4x+5}{9-3x}$$

a) $\frac{4(-1)+5}{9-3(-1)} \Rightarrow \frac{-4+5}{9+3} \Rightarrow \frac{1}{12}$

b) $\frac{4(0)+5}{9-3(0)} \Rightarrow \frac{5}{9}$

c) $\frac{4(3)+5}{9-3(3)} \Rightarrow \frac{17}{0}$

Thus $f(x)$ is continuous at $x=-1, 0$ and is disiontinuous at $x=3$

5. $\lim_{x\to\infty} \frac{6e^{4x}-e^{-2x}}{8e^{4x}-e^{2x}+3e^{-2x}}$

$\Rightarrow \lim_{x\to\infty} \frac{\cancel{e^{4x}}(6-e^{-6x})}{\cancel{e^{4x}}(8-e^{-2x}+3e^{-5x})}$

$= \dfrac{6-\frac{1}{e^{6(\infty)}}}{8-\frac{1}{e^{2(\infty)}}+\frac{3\cdot 1}{e^{\infty}}}$

$= \frac{6-1/\infty}{8-1/\infty+3/\infty}$

$= \frac{3}{4}$

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6) $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

a) $\lim_{y \to 6} g(y) \quad 1-3(6)$

$\Rightarrow 1-18 \Rightarrow -17$

b) $\lim_{y \to -2} g(y) \quad 1-3(-2)$

$= 1+6 \Rightarrow 7$

7) $f(t) = (4t^2-t)(t^3-8t^2+12)$

$\Rightarrow \cancel{d}(4t^2-t)\frac{d}{dt}(t^3-8t^2+12) +$

$(t^3-8t^2+12)\frac{d}{dt}(4t^2-t)$

$\Rightarrow (4t^2-t)(\cancel{t}\,3t^2-16t) + (t^3-8t^2+12)(8t-1)$

$\Rightarrow 12t^4 - 64t^3 - 3t^3 + 16t^2 + \cancel{2}t^4 - \cancel{64}t^3$

$\overset{+96}{\cancel{+24}}t - t^3 + 8t^2 - 12$

$\Rightarrow \cancel{12t}^4 \; \cancel{14t^4 - 84t^3 + 24t^2 + 24t - 12}$

$\Rightarrow 20t^4 - 132t^3 + 24t^2 + 96t - 12$

$\Rightarrow 4(5t^4 - 33t^3 + \cancel{8}\,6t^2 + 24t - 3)$

8) $R(w) = \frac{3w+w^4}{2w^2+1}$

$\Rightarrow \frac{(2w^2+1)\frac{d}{dw}(3w+w^4) - (3w+w^4)\frac{d}{dw}(2w^2+1)}{(2w^2+1)^2}$

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$$\frac{(2w^2+1)(3+4w^3)-(3w+w^4)(4w)}{4w^4+1+4w^2}$$

$$\Rightarrow \frac{6w^2+8w^5+3+4w^3-12w^2-4w^5}{4w^4+1+4w^2}$$

$$\Rightarrow \frac{4w^5+4w^3-6w^2+3}{4w^4+4w^2+1}$$

9 $f(x) = 2e^x - 8^x.$

$= 2e^x - 8^x \log 8.$

10) $y = z^5 - e^2 \ln(z)$

$= 5z^4 - \left[e^2 \frac{d}{dz} \ln(z) + \ln(z) \frac{d}{dx} e^2 \right]$

$= 5z^4 - \left[\frac{e^2}{z} + e^2 \ln(z)' \right]$

$= 5z^4 - \frac{e^2}{z} + e^2 \ln(z)$

11 a) $f(x) = (6x^2+7x)^4$

$= 4(6x^2+7x)(12x+7).$

b) $f(t) = 5 + e^{4t+t^7}$

$= e^{4t+t^7}(4+7t^6)$

12a) $z = \ln(7-x^3)$

$z' = \frac{1}{7-x^3}(-3x^2) = \frac{-3x^2}{7-x^3}$

$$= \frac{-3x^2}{7-x^3}$$

$$\frac{d^2z}{dz^2} = \frac{(7-x^3)\frac{d}{dx}(-3x^2) - (-3x^2)\frac{d}{dx}(7-x^3)}{(7-x^3)^2}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)(-3x^2)}{(7-x^3)^2}$$

$$= \frac{-42x + 6x^4 - 9x^4}{(7-x^3)}$$

$$= \frac{-42x - 3x^4}{(7-x^3)^2}$$

b) $g(v) = \frac{2}{(6+2v-v^2)^4}$

$= 2(6+2v-v^2)^{-4}$

$= 2(-4)(6+2v-v^2)^{-5} \cdot (2-2v)$

$= -8(6+2v-v^2)^{-5} \cdot (2-2v)$

$g''(v) = -8(6+2v-v^2)^{-5}\frac{d}{dx}(2-2v) +$

$(2-2v)\frac{d}{dx} -8(6+2v-v^2)^{-5}$

$= 40(6+2v-v^2)^{-6}(2-2v) +$
$-8(6+2v-v^2)^{-5}(-2)$

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c) $f(t) = \ln(1+t^2)$

$f'(t) = \frac{1}{1+t^2} \cdot \frac{d}{dt}(2t)$

$= \frac{2t}{1+t^2}.$

$f''(t) = \frac{1+t^2 \frac{d}{dt} 2t - 2t \frac{d}{dt} 1+t^2}{(1+t^2)^2}$

$= \frac{(1+t^2)2 - 2t(2t)}{(1+t^2)^2}.$

$= \frac{2+2t^2-4t^2}{(1+t^2)^2}$

$= \frac{2-2t^2}{(1+t^2)^2}$ or $\frac{2(1-t^2)}{(1+t^2)^2}.$

13) $f(x) = x^3 - 3x^2 - 9x + 12.$

$f'(x) = 3x^2 - 6x - 9$

Putting $f'(x) = 0$

$3x^2 - 6x - 9 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x-3) + 1(x-3) = 0$

$\Rightarrow (x+1)(x-3) = 0$

$x = -1$ or $3.$

$f''(x) = 6x - 6$

Putting $x = 3$ in $f''(x)$

$f''(3) = 6(3) - 6$

$= 18 - 6$

$= 12$.

$\therefore$ $f(x)$ is minimum at $x = 3$.

~~Put~~

Putting ~~$f(x)$~~ $x = -1$ in $f''(x)$

$f''(-1) = 6(-1) - 6$

$= -6 - 6$

$= -12$.

$\therefore$ $f(x)$ is maximum at $x = -1$

Put $x = 3$ in $f(x)$

$\therefore$ Minimum value $= (3)^3 - 3(3)^2 - 9(3) + 12$

$= 27 - 27 - 27 + 12$

$= -15$

Put $x = 1$ in $f(x)$

$\therefore$ Maximum value $= (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$.

14) $4x^3 - 18x^2 + 24x - 7 = f(x)$

$f'(x) = 12x^2 - 36x + 24$

Putting $f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$12(x^2 - 3x + 2) = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x-2) - 1(x-2) = 0$

$\Rightarrow (x-1)(x-2) = 0.$

$x - 1 = 0$ or $x - 2 = 0$

$x = 1$ $\quad$ $x = 2$

$f''(x) = 24x - 36$

Putting $x = 1$ in $f''(x)$

$24(1) - 36 = -12$

$\therefore f(x)$ is maximum at $x = 1$

$f''(x) = 24x - 36$

Putting $x = 2$ in $f''(x)$

$24(2) - 36 = 48 - 36$

$= 12$

$\therefore f(x)$ is minimum at $f(x) = 2$

Put $x = 2$ in $f(x)$

$4(2)^3 - 18(2)^2 + 24(2) - 7$

$= 32 - 72 + 48 - 7 = 1$

$\therefore$ Minimum value $= 1$

Put $x = 1$ in $f(x)$

$4(1)^3 - 18(1)^2 + 24(1) - 7$

$\Rightarrow 4 - 18 + 24 - 7$

3

$\therefore$ Maximum value $= 3$

15) $f(x) = x^2(x+3)$

$f'(x) = 2x(x+3) + x^2$

$= 2x^2 + 6x + x^2$

$= 3x^2 + 6x$

Put $f'(x) = 0$

$3x^2 + 6x = 0$

$3(x^2 + 2x) = 0$

$\Rightarrow x^2 + 2x = 0$

$\Rightarrow x(x+2) = 0$

$\therefore x = 0$ or $x = -2$

$f''(x) = 6x + 6$

Put $x = 0$ in $f''(x)$ $\therefore f''(0) = 6$ $f''(0) > 0$

Put $x = -2$ in $f''(x)$ $\therefore f''(-2) = -6$ $f''(0) < 0$

$\therefore$ $f(x)$ is maximum at $x = -2$ and minimum at $x = 0$

Put $x = -1$ in $f'(x)$

$f'(\cancel{*}1) = 3(-1)^2 + 6(-1)$

$= 3 - 6 = -3$

$\therefore$ The given function is decreasing between -2 & 0

Put $x = 0$ in $f'(x)$

Minimum value $= f(0) = 0(0+3) = 0$

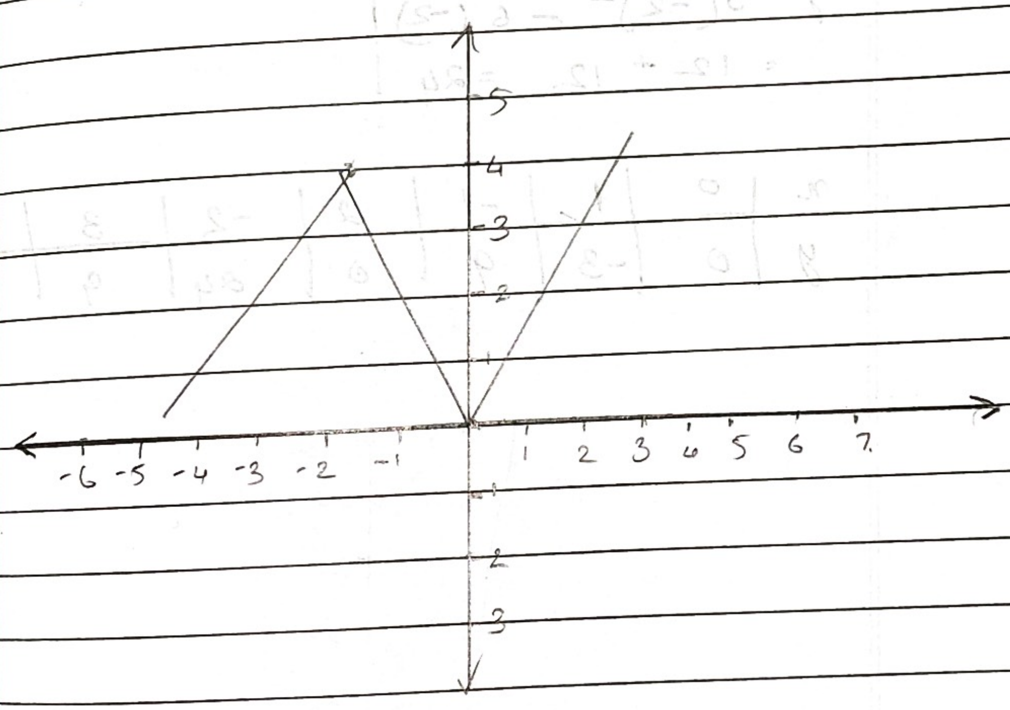
$\therefore x = 0, y = 0$

Put $x = -2$ in $f'(x)$

Maximum value $= f(-2) = (-2)^2(-2+3)$

$= 4(1) = 4$

$x = -2, y = 4$



16) $f(x) = 3x^2 - 6x$

i.e $y = 3x^2 - 6x$.

Put $x = 0$ in $f(x)$

$y = 3(0)^2 - 6(0) = 0$

Put $x = 1$ in $f(x)$

$y = 3(1)^2 - 6(1) = 3 - 6 = -3$

Put $x = -1$ is $f(x)$

$y = 3(-1)^2 - 6(-1)$

$= 3 + 6 = 9$

Put $x = 3$ is $f(x)$

$y = 3(3)^2 - 6(3)$

$= 27 - 18$

$= 9$

Put $x = 2$ is $f(x)$

$y = 3(2)^2 - 6(2)$

$= 12 - 12 = 0$

Put $x = -3$ is $f(x)$

$y = 3(-3)^2 - 6(-3)$

$= 27 + 18$

$= 45$

Put $x = -2$ is $f(x)$

$y = 3(-2)^2 - 6(-2)$

$= 12 + 12 = 24$

x	0	1	-1	2	-2	3	-3
y	0	-3	9	0	24	9	45