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Assignment 2 , FE

q. i) Consider the portfolio which is long one call plus cash of $Ke^{-r(T-t)}$ and short one put.

The portfolio has a payoff at the time of expiry of S_T.

Since this is the value of the stock at time T, the stock price should be the value at any time $t < T$: that is

$$C_t + Ke^{-r(T-t)} - P_t = S_t$$

iii) The Black scholes formula gives us that $S_0 \Phi(d_1)$ $Ke^{-rT} \Phi(d_2)$,

$S_0 = 110$, $K = 120$, $r = 0.02$, $T = 1$

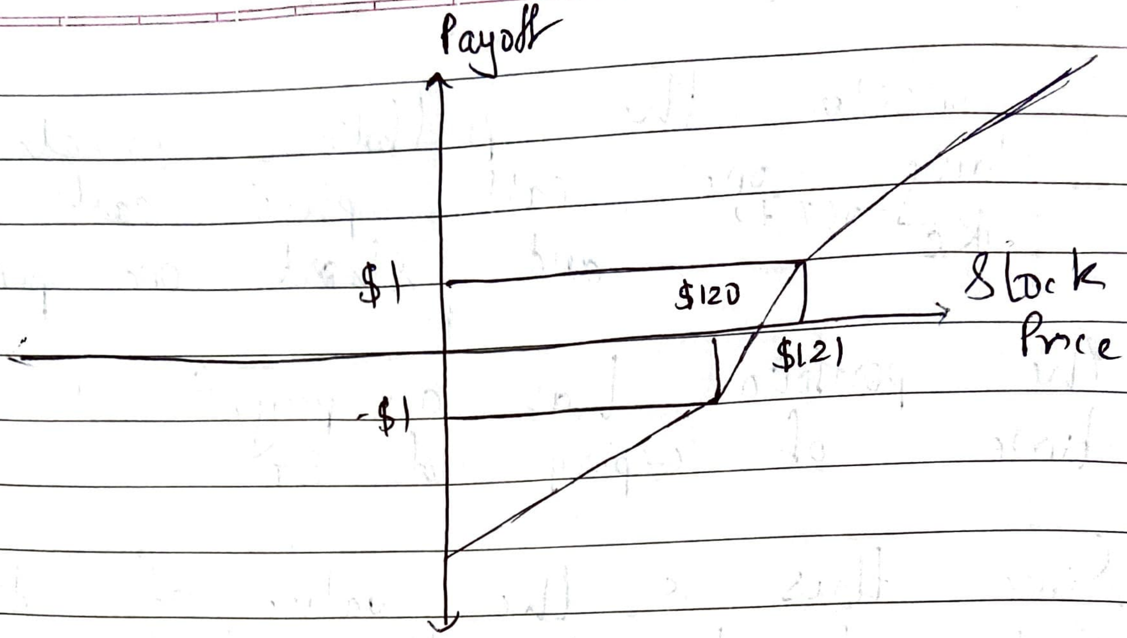
$d_1 = (\log(S_0/K) + r + \frac{1}{2}\sigma^2 T)\, /$

$\sigma\sqrt{T} = (\log(11/12) + 0.02 + \frac{1}{2}\sigma^2)/\sigma$,

$d_2 = d_1 - \sigma$

$\therefore$ $\sigma = 30\%$ by interpolation.

iii)



iv) a) The payoff from the portfolio, P, satisfies

$$S_1 - 121 \leq P \leq S_1 - 120$$

It follows that the initial price, V, of portfolio should satisfy

$$S_0 - 121\, e^{-r} \leq V \leq S_0 - 120\, e^{-r}$$

i.e. $-8.604 \leq V \leq -7.624$

v) Black scholes price is $18.35 by looking at the formula book.

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7. i) Δ is the first partial derivative of the option price with respect to the underlying asset price.

ii) Using the formula for the Δ we see that $\phi(d_1) = 0.42074$ and $d_1 = -0.2$.

$\therefore -0.2\sigma = -0.0600 + \frac{1}{2}\sigma^2$ or
$\frac{1}{2}\sigma^2 + 0.2\sigma - 0.06 = 0$.

$\therefore$ Solving the equation we get $\sigma = -60\%$ and 20%.

we consider only the positive value hence 20%.

1. i) for a derivative whose price at time t is $f(t, S_t)$ where S_t is the price of the underlying asset.

- Delta is the rate of change of its price with respect to a change in S_t : $\Delta = \frac{\partial f}{\partial S_t}$.

- Vega is the rate of change of its price with respect to a change in assumed level of volatility of S_t : $\nu = \frac{\partial f}{\partial \sigma}$

ii) Put call parity states that :
$c + K \times e^{(-r\tau)} = p + S$ where
c and p are the price of a European call and put option respectively with strike K and time to expiry τ and S is the current stock price.

iii) $$d_1 = \frac{\log S/K + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}$$

$\therefore$ $d_1 = 0.7062$

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$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$\therefore d_2 = 0.4562$$

$$c = S\Phi(d_1) - Ke^{-r\tau}\Phi(d_2)$$

$$\therefore c = 9.652$$

$$p = c + Ke^{-r\tau} - S$$

$$\therefore p = 2.214$$

iv) A portfolio for which the overall delta is equal to zero is described as delta-hedged or delta neutral. Such a portfolio is immune to small changes in the price of the underlying asset.

v) Let the required portfolio consist of x call options, y put options and z forwards.

The delta and vega for a forward are 1 and 0 respectively and there are no current cashflows.

Thus for a single unit of each of them, we have

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	Present V / Cashflow	Delta	Vega
Call option	$c = 9.652$	Δ_c	V_c
Put option	$p = 2.214$	Δ_p	V_p
Forward	-	1	-

Vega neutrality - The vega of a forward is 0. For the portfolio must be vega neutral, we must have:

$$x \times V_c + y \times V_p = 0 ,$$

Delta neutrality :

We know that Δ of a forward is 1. For the portfolio to be delta neutral, we need:

$$x \times \Delta_c + y \times \Delta_p + z = 0$$

Overall portfolio

$$x \times c + y \times p + z \times 0 = 1000$$

$$x \times 9.652 - x \times 2.214 = 10000$$

$\therefore$ $x = 134.4$

$y = z = -135.4$.

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2. i) $C_t = E\left(e^{-r(T-t)} C_T \mid F_t\right)$
where F_t denotes the filtration at time $t > 0$
C_T is the payoff under the derivative at maturity time T

C_t is the derivative value at t and the expectation is taken under the risk neutral martingale measure

ii) $S = 50$, $K = 49$, $r = 5\%$
$\sigma = 25\%$, $T = 0.5$

Using the Black scholes ~~ro~~ formula
$d_1 = 0.3441$
$d_2 = 0.1673$
$N(d_1) = 0.6346$
$N(d_2) = 0.5664$

$Call = 50 \times 0.6346 - 49e^{-0.05 \times 0.50} \times 0.5664$
$= 4.66$

iii) 4.66
iv) Using put-call parity

$p_t = c_t + Ke^{-r(T-t)} - S_t$

Hence $p_t = 2.45$

v) If the stock is dividend paying, the payment of the dividends would cause the value of the underlying asset to fall. which follows from the no arbitrage principle.

3. i) Suppose that Z_t is a standard Brownian motion under P.

Furthermore, suppose that γ_t is a previsible process.

Then there exists a measure Q equivalent to P.

where $\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$ is a standard Brownian motion under Q.

5. i) a) Let f denote the price of a put option

then $d_1 = \left(\ln(S_0/K) + (r + \tfrac{1}{2}\sigma^2)T\right) / \sigma\sqrt{T}$

and $\Delta = -\Phi(-d_1) = \Phi(d_1) - 1$

b) In this case we must have $100{,}000\Delta = -24{,}830$ and so $\Delta = -0.25$

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ii) $\Delta = -0.2483$
It follows and so $d_1 = 0.68$
that $(0.01578 + 0.03 + 0.5 \sigma^2) = 0.68\,\sigma$

Solving the quadratic equation we obtain $\sigma = 0.68 \pm \sqrt{0.3709} = 0.07090$
$= 7.1\%$.

iii) $Ke^{-rT}\,\Phi(-d_2) = e^{-r}\,\Phi(-d_1 + \sigma\sqrt{T})$

$= 630\, e^{-0.03}\,\Phi(-0.609)\, p$
$= 630\, e^{-0.03} \times 0.2712 = 165.806\, p.$

So the option price is
$165.806 - 24830 \times 640/100{,}000 = 6.894\, p.$

6. i) Denote the individual derivative by f and assume this is written on an underlying security S.

Delta $= \partial f / \partial S$
Gamma $= \partial^2 f / \partial S^2$
Vega $= \partial f / \partial \sigma$

ii) Delta $= 0.801$
iii) The hedge is delta $= 0.801$ shares short in cash $= 17.91 - 0.801 \times 60 = \$30.15$

iv) Using the approximation $f(S, \sigma + \delta) \approx f(S, \sigma) + \delta\, df/d\sigma$ we obtain an option price $\approx 17.91 + 29.00 \times 0.02$

$= \$18.49$

8. i) The PDE is the Black Scholes PDE:

$$\tfrac{1}{2}\sigma^2 x^2 g_{xx} + (r-q) x g_x - rg + g_t = 0$$

with boundary condition as above: $g(T, x) = f(x)$.

ii) The proposed solution implies that for this derivative the function g is given by $g(t, x) = (x^n / S_0^{n-1}) e^{\mu(T-t)}$ where n is an integer great than 1.

This gives $x g_x$, $x^2 g_{xx} = n(n-1) g$ and $g_t = -\mu g$.

Thus to solve the PDE we need $\mu' = \tfrac{1}{2}\sigma^2 n(n-1) + (n-1) r - nq$