classmate
Date
Page

1 The radius of a circular plate is measured at 12.65 cm instead of actual length 12.5 cm find following in calculating the area of the circular plate:-

i) Absolute error ii) Relative error iii) % error.

i) absolute error = |actual ~~error~~ value − measured value|

$= |12.5 - 12.65| = |-0.15|$

$= \underline{0.15}$

ii) Relative error = $\frac{\text{Absolute error}}{\text{Actual } \cancel{\text{Error}} \text{ value}} \times 100 = \frac{0.15}{12.5} \times 100 = \frac{\cancel{15}^{3}}{\cancel{1250}_{\cancel{25}\,5}} \times \cancel{100}^{2} = \frac{6}{5} = 1.2\%$

iii) Percentage error = $\frac{0.15}{12.5} \times 100 = \underline{1.2\%}$

Q2 The following logarithmic table is given

x	$f(x) = \log(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.778153

a) Interpolate log(5) using the points $x=4$ and $x=6$.

b) Interpolate log(5) using the points $x=4.5$ and $x=5.5$.

x	y
4	0.60206
6	0.778153.

$\cancel{\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{-2.260678}{2} = -1.130339} - \frac{0.49978}{2} = \cancel{-0.249874}$

$\cancel{\bar{y} = \log(x) =}$

$\left[\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{4(0.778153) - 6(0.60206)}{0.778153 - 0.60206} \right]$

$\left[x_0 = 4 \quad f(4) = 0.60206 \right.$

$\left. f'(x) = \frac{1}{x} = \frac{1}{0.60206} = 1.660964024 \right]$

a)

x	$f(x) = \log(x)$
4	0.602026
6	0.7781513

$$f(x_k) = \frac{(x_k - x_1)}{(x_0 - x_1)} f(x_0) + \frac{(x_k - x_0)}{(x_1 - x_0)} f(x_1)$$

$$= \frac{(5-6)}{(4-6)} 0.602026 + \frac{(5-4)}{(6-4)} 0.7781513$$

$$= \frac{+1}{+2} \times 0.602026 + \frac{1}{2} 0.7781513$$

$$= \underline{\underline{0.69010565}}$$

b)

x	$f(x) = \log(x)$
4.5	0.6532125
5.5	0.7403627

$$f(x_k) = \frac{(x_k - x_1)}{(x_0 - x_1)} f(x_0) + \frac{(x_k - x_0)}{(x_1 - x_0)} f(x_1)$$

$$= \frac{5 - 5.5}{4.5 - 5.5} \times 0.6532125 + \frac{5 - 4.5}{5.5 - 4.5} \times 0.7403627$$

$$= \frac{+0.5}{+1} \times 0.6532125 + \frac{0.5}{1} \times 0.7403627$$

$$= \underline{\underline{0.69678}}76$$

3. Find the root of $x^4 - x - 10 = 0$ approximately upto 5 interations using Bisection Method let $a = 1.5$ and $b = 2$.

$f(x) = x^4 - x - 10.$

(x)	$f(x)$
1.5	-6.4375
2	4

$x_1 = 1.5$

$x_2 = 2$

$x_3 = \frac{1.5+2}{2} = \frac{3.5}{2} = 1.75$

$y_3 = (1.75)^4 - 1.75 - 10 = -2.3710$

x	$f(x)$
1.75	-2.3710
2	4

$x_3 = 1.75$

$x_2 = 2$

$x_4 = \frac{1.75+2}{2} = 1.875$

$y_4 = (1.875)^4 - 1.875 - 10$

$= 0.484619$

x	$f(x)$
1.75	-2.3710.
1.875	0.48461

$x_3 = 1.75$

$x_4 = 1.875$

$x_5 = \frac{1.75 + 1.875}{2} = 1.8125$

$y_5 = (1.8125)^4 - 1.8125 - 10.$

$= -1.020248.$

x	$f(x)$
1.8125	-1.002048.
1.875	0.48461

$x_5 = 1.8125$

$x_4 = 1.875$

$x_6 = \frac{1.8125 + 1.875}{2} = 1.84375$

$y_6 = (1.84375)^4 - 1.84375 - 10$

$= -0.28773.$

x	$f(x)$
1.84375	- 0.28773
1.875	0.4861

$x_6 = 1.84375$

$x_4 = 1.875$

$x_7 = \frac{1.84375 + 1.875}{2} =$

$1.859375.$

$y_7 = (1.859375)^4 - 1.859375 - 10$

$= \underline{\underline{0.093378}}.$

$\therefore x \approx 1.859375$

4 Find the root of an equation $f(x) = x^3 - x - 1$ using Newton Raphson Method.

$\cancel{y = \sqrt{x^3 - x - 1}}$ $\quad f(x) = x^3 - x - 1$

~~Squaring~~ $\quad f'(x) = 3x^2 - 1$

$x_1 = 1$

$f(x_1) = (1)^3 - 1 - 1 = 1 - 1 - 1 = -1$

$f'(x_1) = 3(1)^2 - 1 = 3 - 1 = 2$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1 - \left(\frac{-1}{2}\right)$

$= 1 + \frac{1}{2}$

$= 3/2 = 1.5$

$f(x_2) = (1.5)^3 - 1.5 - 1 = 0.875$

$f'(x_2) = 3(1.5)^2 - 1 = 5.75$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.5 - \frac{0.875}{5.75} = 1.3478$

$f'(x) = 3(1.3478)^2 - 1$

$= 4.449905$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.5 - \frac{0.875}{5.75}$$

$$x_3 = 1.34783$$

$$f(x_3) = \cancel{f}\cancel{f}(1.34783)^3 - 1.34783 - 1$$

$$= 0.100699$$

$$f'(x_3) = 3(1.34783)^2 - 1.$$

$$= 4.449937$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_4)}$$

$$= 1.34783 - \frac{0.100699}{4.449937}$$

$$= 1.325200692.$$

$$f(x_4) = 0.00205610697$$

$$f'(x_4) = 4.268470622.$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$= 1.325200692 - \frac{0.00205610697}{4.268470622}$$

$$= 1.324718175$$

$$f(x_5) = 0.0000009$$

$$\boxed{\therefore x \approx 1.324718175}$$

Q5 If A is a square matrix such that $A^2 = A$ then find the value of $7A - (I+A)^3$, here I is an identity matrix.

$= 7A - (I^3 + A^3 + 3A^2 I + 3A I^2)$

$= 7A - (I + A \cdot A^2 + 3A^2 + 3A)$

$= 7A - (I + A \times A + 3A + 3A) \quad \because A^2 = A.$

$= 7A - (I + A^2 + 6A) \qquad A^2 = A$

$= 7A - [I + 7A)$

$= 7A - I - 7A$

$= \underline{I}$

6 Find out inverse of $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & 1 \end{bmatrix}$

$|A| = 1(0-6) + 1(4-0) + 2(4-0)$

$= -6 + 4 + 8$

$= -6 + 12 = 6.$

$A A^{-1} = I$

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 + 2R_3 \qquad R_1 \to R_1 \mp 2R_3$

$$\cancel{\begin{bmatrix} 1 & & 0 \\ 0 & 6 & 0 \\ 0 & 1 & 1 \end{bmatrix}}$$

$R_3 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 4 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 4 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -4 & 1 & 0 \end{bmatrix}$$

$R_1 \longrightarrow R_1 + R_2$ $\quad$ $R_3 \longrightarrow R_3 - \cancel{8}4R_2$.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ -4 & 1 & -4 \end{bmatrix}$$

$\div R_3$ by -6.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 6 & 0 & 6 \\ 0 & 0 & 6 \\ 4 & -1 & 4 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_3$ $\quad$ $R_1 \longrightarrow R_1 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -6 & 3 & -6 \\ \cancel{0}-4 & 1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$I A^{-1} = \begin{bmatrix} -6 & 3 & -6 \\ -4 & 1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -6 & 3 & -6 \\ -4 & 1 & 2 \\ 4 & 1 & 4 \end{bmatrix}}$$

7 Using the definition of scalar product, find the angle between the following vector. (Find the smallest nonnegative ∠)

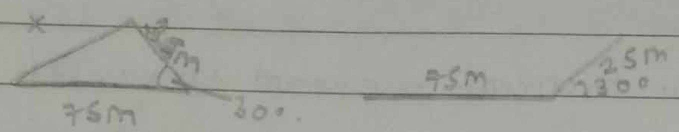
a) $A = 8i - 9j$ and $B = 7i - 9j$

$\bar{A} \times \bar{B} = 8 \times 7 + (-9 \times -9)$

$= 56 + 81$

$= 137$

8 Sabrina walked 75 meters to the east. Then she turned 30° to the ~~f~~ left and walked 25 meters. Determine the maginitude of Sabrina's displacement vector.



$$\text{Magnitude} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{75^2 + 25^2 + 2(75)(25)\cos(30)}$$

$$= \sqrt{9497.595264}$$

$$= \underline{97.4556}$$

9 What is the sum of all 3 digit number that leave a remainder of 2 when divided by 3.

Lowest 3 digit number ~~is~~ when ÷ by 3 leaves a remainder of 2 is 101
& highest = 998

∴ AP = 101, 104, 107 ... 998 ; first term $a = 101$

$d = 104 - 101 = 3$ $\quad$ n^{th} term / last term = 998.

$t_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$\frac{998 - 101}{3} = n - 1$

$299 = n - 1$

$\boxed{n = 300}$

$S_n = \frac{n}{2}(a + l)$

$= \frac{300}{2}(101 + 998)$

$= 150(1099)$

$\boxed{S_n = 164850}$

10 The 6th term of GP is 32 and 8th term is 128, find 'r' of the GP.

$t_6 = 32$ $\qquad$ $t_8 = 128$

$ar^{n-1} = 32$ $\qquad$ $ar^{n-1} = 128$

$ar^{6-1} = 32$ $\qquad$ $ar^{8-1} = 128$

$ar^5 = 32$ $\qquad$ $ar^7 = 128$

$a = \frac{32}{r^5}$ $\qquad$ $\frac{32}{\cancel{r^5}} \times r^{\cancel{7}\,2} = 128$

$$r^2 = \frac{128}{32}$$

$$r = \sqrt{4}$$

$$\boxed{r = \pm 2}$$

12) Use the Binomial Theorem to expand $(4+3x)^5$.

$(4+3x)^5 = (3x+4)^5$

$= (3x)^5 + 5(3x)^4(4) + 10(3x)^3(4)^2 +$

$10(3x)^2(4)^3 + 5(3x)(4)^4 +$

$(4)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= 243x^5 + 1620x^4 + 4320x^3$

$+ 1920x^2 + 3840x + 1024$

12 Find all eigen values and corresponding eigen matrix.

$$\begin{pmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} \right| = 0$$

$$\lambda^3 - (a+q+w)\lambda^2 + (M_{11} + M_{22} + M_{33})\lambda - |A| = 0.$$

$$(2-\lambda)\left[(-5-\lambda)(3-\lambda)\right] + 3\left[2(3-\lambda)\right] = 0$$

$$2-\lambda\left[(-15-3\lambda+5\lambda+\lambda^2)\right] + 3\left[6-2\lambda\right] = 0$$

$$2-\lambda\left[-15+2\lambda+\lambda^2\right) + \left[18-\cancel{3}^{6}\lambda\right] = 0$$

$$-30+4\lambda+\cancel{2\lambda^2}+15\lambda-\cancel{2\lambda^2}-\lambda^3+18-\cancel{3}^{6}\lambda = 0.$$

$$-\lambda^3 + \cancel{16}^{13}\lambda - 12 = 0.$$

$$\lambda^3 - \cancel{18}^{13}\lambda + 12 = 0.$$

let $\lambda = 1$

$(1)^3 - 13(1) + 12$

$= 0$ $\quad\therefore \lambda - 1$ satisfies the equation one factor is $\lambda = 1$

1	1	0	−13	12
	↓	1	+1	−12
	1	1	−12	0

$$\lambda^2 + \lambda - 12 = 0$$

$$+4 \quad -3$$

$$(\lambda+4)(\lambda-3) = 0$$

$$\lambda = -4 \qquad \lambda = 3$$

Eigen values $= 3, 1, -4.$

$(A - \lambda I)x = 0$ $\qquad$ for $\lambda = 3$

$$\left|\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right| \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \sim 0$$

$R_2 \longrightarrow R_2 + 2R_1$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$R_2/-14$

$$\begin{bmatrix} -1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$R_1 \to R_1 + 3R_2$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x = 0$ $\quad y = 0$ $\quad z =$ Dependent variable (Indeterminant) α.

$\lambda = 3$, Eigen vector $\begin{bmatrix} 0 \\ 0 \\ \alpha \end{bmatrix}$

$(A - \lambda I)(x) = 0$ for $\lambda = 1$

$$\left(\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\lambda = 1$ Eigen ~~values~~ vectors $\begin{bmatrix} 3y \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} y$

$$\begin{bmatrix} x - 3y \\ 0 \\ 2z \end{bmatrix} = 0$$

$x - 3y = 0$ $\qquad 2z = 0$

$x = 3y$ $\qquad z = 0$

(Independent)

$$(A - \lambda I)x = 0 \qquad \lambda = -4$$

$$\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_1/3$

$$\begin{bmatrix} 2 & -1 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 2x - y \\ 0 \\ 7z \end{bmatrix} = 0$$

$$2x - y = 0 \qquad 7z = 0$$

$$2x = y$$

$\lambda = -4$ Eigen vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 2x \\ 0 \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

13 Use Newton's Method to determine x_2 for $f(x) = x^3 - 7x^2 + 8x - 3$ if $x_0 = 5$

$f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5 - \frac{(-13)}{13} = 5 + 1 = 6$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 6 - \frac{f(6)}{f'(6)} = 6 - \frac{9}{32} = 5.71875.$$

$\boxed{x_2 = 5.71875}$

14 Expand the following $(1 - x + x^2)^4$

$(1 - x + x^2)^4$

Let $1 - x = y$

$(y + x^2)^4$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$= y^4 + 4(y^3)(x^2) + 6(y^2)(x^2)^2$
$+ 4(y)(x^2)^3 + 1(x^2)^4$

$= (1-x)^4 + 4(1-x^3)(x^2) + 6(1-x)^2 x^4 + 4(1-x)x^6 + x^8.$

$= (1 - 4x + 6x^2 - 4x^3 + x^2) + 4x^2(1 - 3x + 3x^2 - x^3) + 6x^4(1 - 2x + x^2)$
$+ 4x^6(1-x) + x^8$

$= (1 - 4x + 6x^2 - 4x^3 + x^2) + (4x^2 - 12x^3 + 12x^4 - 4x^5) + (6x^4 - 12x^5 + 6x^6)$
$+ (4x^6 - 4x^7) + x^8$

$= \boxed{x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1}$

15 Find the approximated value of x the 4 iterations for $e^{-x} = 3\log(x)$ using Bisection method.

$f(x) = 3\log(x) - e^{-x}$.

$3\log(x) - e^{-x} = 0$.

x	$f(x)$
1	-0.3678
2	0.767754

$x_3 = \frac{1+2}{2} = 1.5$.

$y_3 = 0.30514$

x	y
1	-0.3678
1.5	0.305143.

$x_4 = \frac{1+1.5}{2} = \frac{2.5}{2} = 1.25$

$y_4 = 0.0042252$

x	y
1	-0.3678
1.25	0.~~305143~~ 0042252

$x_5 = \frac{1+1.25}{2} = \frac{2.25}{2} = 1.125$

$y_5 = -0.1711949$.

x	y
1.125	-0.171194
1.25	0.~~305148~~ 0042252

$x_6 = \frac{1.125+1.25}{2} = \frac{2.375}{2} = 1.1875$

$y_6 = -0.0810819138.2$

x	y
1.1875	-0.081081
1.25	0.~~305148~~ 004252

$x_7 = \frac{1.1875+1.25}{2} = \frac{\cancel{3.125}}{\cancel{2}}$

$= \cancel{1.} \frac{\cancel{2.4375}}{2}$

$= 1.21875$

$y_7 = -0.037855$

x	y
1.21875	-0.037855
1.25	0.004252

$\boxed{x \approx 1.21875}$