# Financial Mathematics
## Assignment - 1

given => $d^{(4)} = 8\% = 0.08$

① i.

$$\left(1-\frac{d^{(4)}}{4}\right)^4 = e^{-\delta}$$

$$-\delta = \ln\left(1-\frac{0.08}{4}\right)^4$$

$$\delta = \ln\left(1-0.02\right)^{-4}$$

$$\underline{\delta = 8.081\ \%}$$

ii.

$$e^{\delta} = 1+i$$
$$i = e^{\delta} - 1$$
$$i = e^{0.08081} - 1$$
$$\underline{i = 8.41648\ \%}$$

iii.

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$$

$$1-\frac{d^{(12)}}{12} = e^{-\delta/12}$$

$$d^{(12)} = \left[1-e^{-\delta/12}\right] \times 12$$

$$\underline{d^{(12)} = 8.0539\ \%}$$

② $\delta(t) = 0.004t + 0.0002t^2$

i. $PV_0 = 1000\left[e^{-\int_0^{10} 0.004t + 0.0002t^2\, dt\ \cancel{(10)}}\right]$

$$PV_0 = 1000\, e^{-\cancel{10}\left[0.00\cancel{2}t^2 + \frac{0.0002}{3}t^3\right]_0^{10}}$$

$$PV_0 = 1000\, e^{-\cancel{10}\left[0.2 + \frac{0.2}{3}\right]}$$

$$= 1000\, e^{-\left[0.2 + \frac{0.2}{3}\right]}$$

$$\underline{PV_0 = 765.9283}$$

ii. $\frac{1000}{765.9283} = (1+i)^{10}$

$$i = \cancel{1-} \left(\frac{1000}{765.9283}\right)^{1/10} - 1$$

$$\underline{i = 2.70254\ \%}$$

3 i. Since we know that, $v = \frac{1}{1+i}$

$\Rightarrow d = 1 - v \quad \ldots\ (v = 1-d)$

$\Rightarrow d = 1 - \frac{1}{1+i}$

$\Rightarrow d = \frac{\cancel{1} + i - \cancel{1}}{1+i}$

$\Rightarrow d = i\left(\frac{1}{1+i}\right) \quad \ldots \left(\frac{1}{1+i} = v\right)$

$\Rightarrow \boxed{d = iv}$

ii. a. $d^{(2)} = ?$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = (1-d)^4 \quad \ldots (d = 0.0225)$$

$$1 - \frac{d^{(2)}}{2} = (1-d)^2$$

$$d^{(2)} = \left[1 - (1-d)^2\right] \times 2$$

$$\underline{d^{(2)} = 8.89875\ \%}$$

b. $d^{(2)} = ?$

$$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} \quad \ldots \left(d^{(1/2)} = 0.05\right)$$

$$1 - \frac{d^{(2)}}{2} = \left(1 - (0.05)2\right)^{1/4}$$

$$d^{(2)} = \left[1 - (1 - 0.1)^{1/4}\right] \times 2$$

$$\underline{d^{(2)} = 5.19925\ \%}$$

iii. $$(1+i)^{91/365} = \left(1 - \frac{91}{365} d\right)^{-1}$$

$$\left(1 - \frac{91}{365} d\right) = (1+i)^{\cancel{365}\,-91/365}$$

$$d = \left[1 - (1+i)^{-91/365}\right] \times \frac{365}{91}$$

$$\underline{d = 4.84946\ \%}$$

④ i. Same as Q. ③ $\rightarrow$ i.

ii. $i = 0.08$

a. $\left(1 - \frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$

$d^{(12)} = \left[1 - (1+i)^{-1/12}\right] \times 12$

$d^{(12)} = \left[1 - (1.08)^{-1/12}\right] \times 12$

$\underline{d^{(12)} = 7.67147\%}$

b. $\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1+i$

$i^{(365)} = \left[(1+i)^{1/365} - 1\right] \times 365$

$\underline{i^{(365)} = 7.6969\%}$

c $e^{\delta} = 1+i$

$\delta = \ln(1+i) = \ln(1.08)$

$\underline{\delta = 7.6961\%}$

d. $\left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5} = 1+i$

$i^{(0.5)} = \left[(1+i)^{1/0.5} - 1\right] \times 0.5$

$i^{(0.5)} = 8.32\%$

(5) i. $d^{(4)} = 6\% = 0.06$

a. $\left(1 + \frac{i^{(2)}}{2}\right)^{2} = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$

$$1 + \frac{i^{(2)}}{2} = \left(1 - \frac{d^{4}}{4}\right)^{-2}$$

$$i^{(2)} = \left[\left(1 - \frac{d^{(4)}}{4}\right)^{-2} - 1\right] \times 2$$

$$\underline{i^{(2)} = 6.13775\ \%}$$

b. $\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{4}$

$$d^{(12)} = \left[1 - \left(1 - \frac{d^{(4)}}{4}\right)^{1/3}\right] \times 12$$

$$d^{(12)} = 6.03025\ \%$$

ii. $\frac{220\,000}{200\,000} = 1 + i$

$$i = 0.1 = 10\%$$

Time period between 15th April, 2005 to 17th July, 2005 $\cong$ 3 months

Sum paid on 17th July, 2005 $= \left[(1+i)^{3/12}\right] \times 200\,000$

$$= [1.1]^{3/12} \times 200\,000$$

$$= \underline{204822.7378}$$

(6) Doubt i. $X = \cancel{100} \; R(1-0.3)$ Retail Price = R

... (Cash Payment)

$Y = R(1-0.25)^2$

.... ( Six months credit)

$$\frac{X}{Y} = \frac{\cancel{R}(1-0.3)}{\cancel{R}(1-0.25)^{\cancel{2}}} = 1-d$$

$$\frac{0.7}{(0.75)^{\cancel{2}}} = 1-d$$

$$d = \frac{0.75-0.7}{0.75}$$

$$d = \frac{0.05}{0.75} = \frac{5}{75} = \underline{6.6667\,\%}$$

ii. $d^{(12)} = \left[1-(1-d)^{1/12}\right] \times 12$

$\underline{d^{(12)} = 6.8798\,\%}$

$i^{(2)} = \left[(1-d)^{-1/2} - 1\right] \times 2$

$\underline{i^{(2)} = 7.02003\,\%}$

$\delta = -\ln(1-d)$

$\underline{\delta = 6.8998\,\%}$

⑦ $PV = 20\,000$

$AV_{17/12} = 26\,000$

i. $$\left(1+\frac{i^{(4)}}{4}\right)^{4(17/12)} = \frac{26\cancel{000}}{20\cancel{000}} = 1.3$$

$$\left(1+\frac{i^{(4)}}{4}\right) = (1.3)^{3/17}$$

$$i^{(4)} = \left[(1.3)^{3/17} - 1\right] \times 4$$

$$i^{(4)} = 18.95525\,\%$$

ii. $$\left(1-\frac{d^{(2)}}{2}\right)^{2} = \left(1+\frac{i^{(4)}}{4}\right)^{-4}$$

$$\left(1-\frac{d^{(2)}}{2}\right) = \left(1+\frac{i^{(4)}}{4}\right)^{-2}$$

$$d^{(2)} = \left[1-\left(1+\frac{i^{(4)}}{4}\right)^{-2}\right] \times 2$$

$$d^{(2)} = 17.68823\,\%$$

iii. $$\frac{26000}{2000} = \left(1+\frac{17}{12}\,i\right)$$

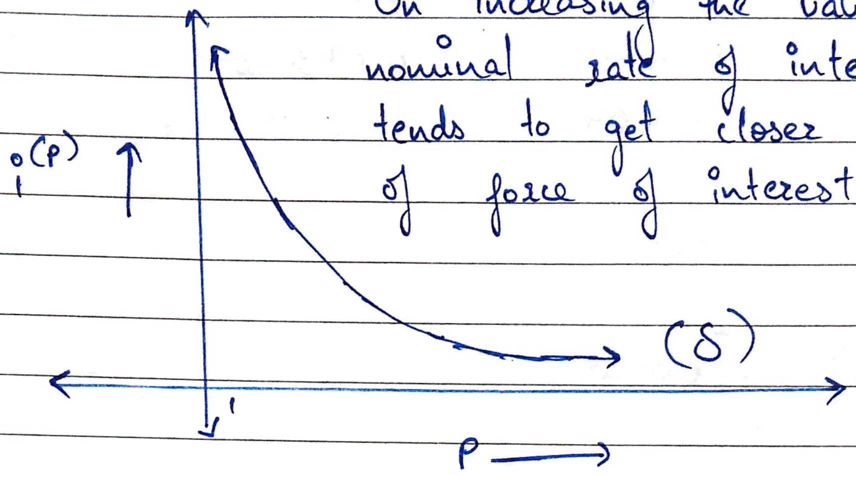
$$\frac{17}{12}\,i = 1.3 - 1$$

$$i = \frac{(0.3)(12)}{17}$$

$$i = 21.17647\,\%$$

(8)

On increasing the value of p, the nominal rate of interest ($i^{(p)}$) tends to get closer to the value of force of interest ($\delta$).




(9) i. Force of interest refers to a nominal interest rate or a discount rate compounded infinite number of times per time period.

ii. $i^{(2)} = 0.0775$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i$$

$$i = \left(1+\frac{0.0775}{2}\right)^2 - 1$$

$$i = 7.90015\%$$

$$\delta = \ln(1+i)$$

$$\delta = \ln(1.0790015)$$

$$\delta = 7.60361\%$$

$$\left(1+2i^{(1/2)}\right)^{1/2} = (1+i)$$

$$1+2i^{(1/2)} = (1+i)^2$$

$$2i^{(1/2)} = (1+i)^2 - 1$$

$$i^{(1/2)} = \frac{\left[(1+i)^2 - 1\right]}{2}$$

$$i^{(1/2)} = 8.21221\%$$

$$\left(1-\frac{d^{(12)}}{12}\right)^{12} = (1+i)^{-1}$$

$$d^{(12)} = \left[1-(1+i)^{-1/12}\right] \times 12$$

$$d^{(12)} = 7.579568\%$$

(10) $\delta(t) = \begin{cases} 0.08, & 0 \le t \le 5 \\ 0.06, & 5 \le t \le 10 \\ 0.04, & t \geq 10 \end{cases}$

- when $0 \le t \le 5$

$$v(t) = e^{-\int_0^t 0.08\, ds}$$

$$= e^{-[0.08s]_0^t} = e^{-0.08t} = v(t)$$

- when $5 \le t \le 10$

$$v(t) = e^{-0.08(5)} \, e^{-\int_5^t 0.06\, ds}$$

$$= e^{-0.4}\left(e^{-[0.06s]_5^t}\right)$$

$$= e^{-0.4}\left(e^{-0.06t+0.3}\right)$$

$$v(t) = e^{-0.1-0.06t}$$

- when $t \geq 10$

$$v(t) = e^{-0.1-0.06(10)} \, e^{-\int_{10}^t 0.04\, ds}$$

$$= e^{-0.1-0.6} \, e^{-[0.04s]_{10}^t}$$

$$= e^{-0.7} \, e^{-0.04t+0.4}$$

$$v(t) = e^{-0.3-0.04t}$$

(11) a. $T = 9$ month loan

$C = 50000$

$d = 0.18$ p.a

$$PV_0 = C(1-d)^{T/12}$$

$$PV_0 = 50000(1-0.18)^{9/12}$$

$$PV_0 = 43085.42623$$

b. $\delta = 6\%$

i.

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$$

$$d^{(12)} = \left[1 - e^{-\delta/12}\right] \times 12$$

$$d^{(12)} = 5.98502\%$$

ii. $d^{(4)} = 6\%$

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{-4}$$

$$1 + \frac{i^{(12)}}{12} = \left(1 - \frac{d^{(4)}}{4}\right)^{-1/3}$$

$$i^{(12)} = \left[\left(1 - \frac{d^{(4)}}{4}\right)^{-1/3} - 1\right] \times 12$$

$$i^{(12)} = 6.060708\%$$

c. $d < \delta < i^{(4)} < i$

(12) a. $AV_6 = 7049\left(1+\frac{i^{(12)}}{12}\right)^{24}\left(1+ \cancel{\ } 10(0.015)\right)\left(1-\frac{d^{(12)}}{12}\right)^{-18}$

... $(i^{(12)} = 0.06)$, $(d^{(12)} = 0.06)$

$AV_6 = 7049(1.127159)(1.15)(1.094421)$

$\underline{AV_6 = 9999.883762}$

(13) ~~PV = 2Y~~ ~~C=~~

i. $AV = 2X \quad C = X$

$AV = C(1+0.1)^t$

$2X = X(1.1)^t$

$\ln(2) = t\ln(1.1)$

$t = \frac{\ln(2)}{\ln(1.1)}$

$\underline{t = 7.2725}$ years

ii. $\left(1-\frac{0.1}{12}\right)^{-12t} = 2$

$\ln 2 = -12t\ln\left(1-\frac{0.1}{12}\right)$

$t = \frac{\ln(2)}{-12\ln\left(1-\frac{0.1}{12}\right)} = \underline{6.90255}$