Numerical Methods
and
Algebra

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Ans 1. Given :

radius (wrong) = 12.65 cm
radius (actual) = 12.5 cm

(i) Absolute error

$\Rightarrow$ | Actual Value - Measured Value |

$= |12.5 - 12.65|$

$= 0.15$

(ii) ~~Measured~~ Relative error

$\Rightarrow \dfrac{|\text{Measured} - \text{Actual}|}{\text{Actual}}$

$\Rightarrow \dfrac{12.65 - 12.5}{12.5}$

$= 0.012$

(iii) Percentage Error

$\Rightarrow$ 1.2 %

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Ans 2. Given

x	$f(x) = \log(x)$
4.0	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

(a) $\log(5) \Rightarrow$

Let $\log(5)$ be y

$$y = 0.60206 + \frac{1}{2}(0.7781513 - 0.60206)$$

$\log(5) \Rightarrow 0.690105$

(b) $\log(5) \Rightarrow$

Let $\log(5)$ be z

$$z = 0.6532125 + \frac{1}{2}(0.7403627 - 0.6532125)$$

$\log(5) = 0.6967876$

Ans 3. root of the eq $\Rightarrow x^4 - x - 10 = 0$

upto 5 iterations using Bisection

$a = 1.5$

$b = 2$

x	y
1.5	-6.4375
2	4

(I)

$$x_1 = \frac{1.5 + 2}{2} = 7/4$$

$$y_1 = -2.37109375$$

(II)

$$x_2 = \frac{1.75 + 2}{2} = 1.875$$

$$y_2 = 0.4846191$$

(III)

$$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$$

$$y_3 = -1.00202484 13$$

(IV)

$$x_4 = \frac{1.8125 + 1.875}{2} = 1.84375$$

$$y_4 = -0.2877340$$

(V)

$$x_5 = \frac{1.84375 + 1.875}{2} = 1.859375$$

$$y_5 = 0.0933781 2662$$

Therefore the root of the eq $\simeq 1.859375$

Ans 4. $f(x) = x^3 - x - 1$

using Newton Raphson Method

x	$f(x)$
0	-1
1	-1
2	5

$f'(x) = 3x^2 - 1$

$x_0 = 1$

$f(x_0) = -1$

$f'(x_0) = 2$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 1 - (-1/2)$

$= 1.5$

$y_1 = 0.875$

$f'(x_1) = 5.75$

$x_2 = x_1 - f x_1 / f' x_1$

$= 1.5 - 0.875/5.75$

$= 1.347826$

$y_2 = 0.100681$

$f'(x_2) = 4.449904$

$x_3 = x_2 - f(x_2)/(f'(x_2))$

$= 1.325200$

$y_3 = 0.0020566$

$f'(x_3) = 4.268465$

$x_4 = x_3 - f(x_3)/(f'(x_3))$

$= 1.3247181$

$y_4 = 0.0000006\,0879$

Ans 5. Given $A^2 = A$

$$7A - (I+A)^3 \Rightarrow 7A - (I^3 + A^3 + 3IA + 3A^2)$$

$$= 7A - (I + A \cdot A^2 + 3A + 3A^2)$$

$$= 7A - (I + A \cdot A + 3A + 3A)$$

$$= 7A - (I + A + 6A)$$

$$= 7A - (I + 7A)$$

$$= 7A - I - 7A$$

$$= -I$$

$$= -1$$

Ans 6.

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$, To find ⇒ Inverse of A

$$|A| = 1(-1(0) - 6(1)) + 1(-4 - 0) + 2(4)$$

$$= -6 - 4 + 8$$

$$= -2$$

Since $|A| \neq 0$

A^{-1} exists.

Adjoint of A

$$\Rightarrow \begin{bmatrix} -6 & +4 & 4 \\ 1 & -1 & -1 \\ -6 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 1 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$$

Ans 7 Given $A = 8i - 9j$
$B = 7i - 9j$

$|\vec{a}| = \sqrt{64+72} = 2\sqrt{34}$

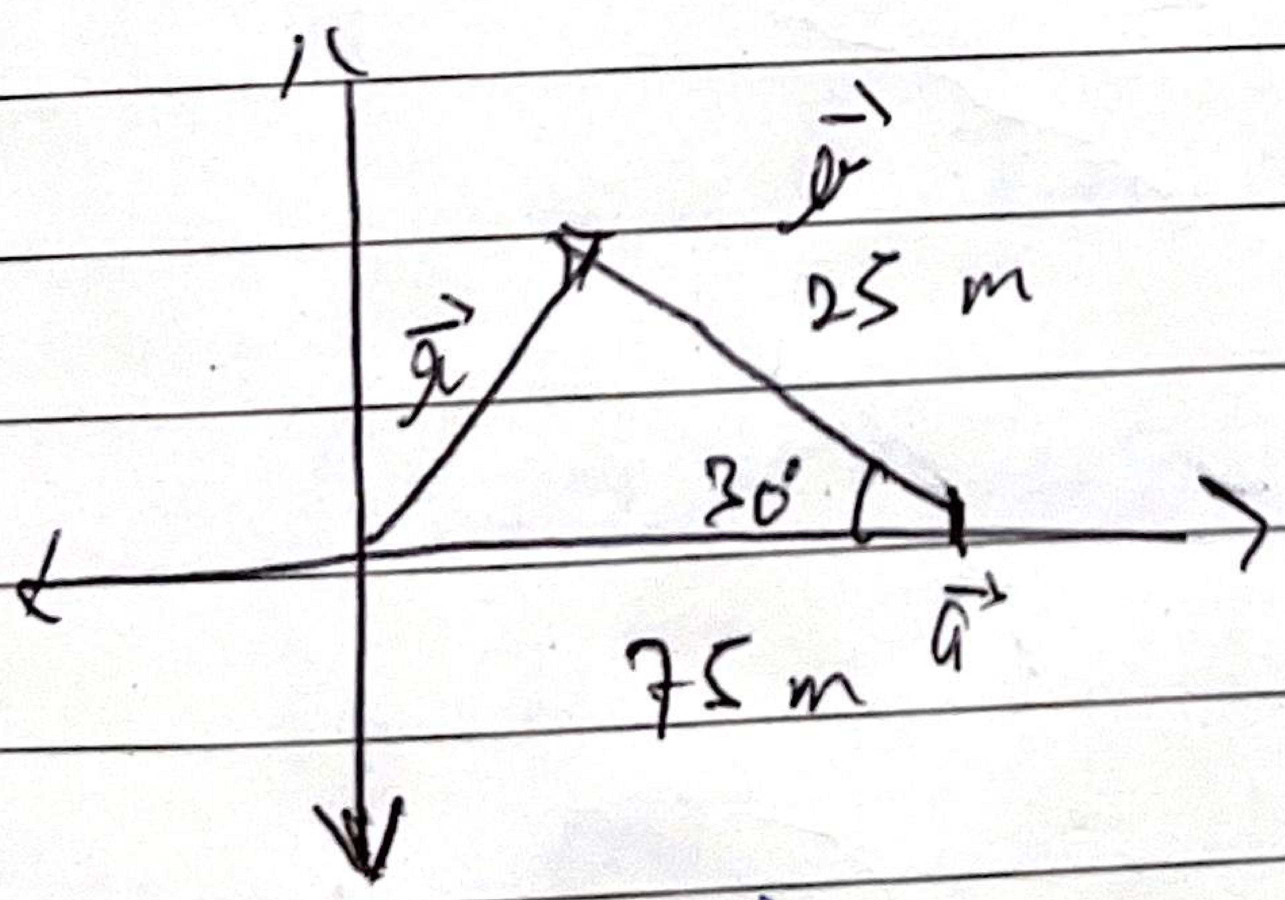
$|\vec{b}| = \sqrt{49+81} = \sqrt{130}$

$$\cos\theta = \cancel{84-93}\ \frac{(8i - 9j)\cdot(7i - 9j)}{2\sqrt{34} \times \sqrt{130}}$$

$$= \frac{56+81}{2\sqrt{34 \times 130}} = \frac{137}{2\sqrt{34 \times 130}}$$

$$\theta = \cos^{-1}\frac{137}{2\sqrt{4420}}$$

Ans 8.



$\vec{R} = \bar{a} + \bar{b}$

$R^2 = |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a}\,\bar{b}$

$= |\bar{a}|^2 + |\bar{b}|^2 + 2|\bar{a}||\bar{b}| \cos 30°$

$= 75^2 + 25^2 + 2(75)(25)\frac{\sqrt{3}}{2}$

$\vec{R}^2 = 9497.5952\ m$

$\bar{R} = 97.46\ m$

Ans 9.

$a = 101$

$a_n = 998$

$d = 3$

$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$300 = n$

$S_n = \frac{n}{2}(a + a_n)$

$= \frac{300}{2}(101 + 998)$

$= 164850$

Ans 10.

$ar^5 = 32$

$ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$

Ans 11. $f(x) = (4+3x)^5$

Using the binomial theorem

$\Rightarrow {}^5C_0 (3x)^5 + {}^5C_1 (3x)^4 (4) + {}^5C_2 (3x)^3 (4)^2 + {}^5C_3 (3x)^2 (4)^3$

$+ {}^5C_4 (3x)(4)^4 + {}^5C_5 (4)^5$

$\Rightarrow 1 \times 243x^5 + 5(81)x^4(4) + 10(27)x^3(16) + 10(9)x^2(64)$

$+ 5(3)x(256) + 1(1024)$

$\Rightarrow 243x^5 + \cancel{20}\ 1620x^4 + 4320x^3 + 7560x^2 + 3840x$

$+ 1024$

Ans 12.

$$A = \begin{Bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{Bmatrix}$$

We know that $|A - \lambda I| = 0$

$$\begin{Bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{Bmatrix} = 0$$

$\Rightarrow \cancel{2(}\ -(2-\lambda)(5+\lambda)(3-\lambda) - (-3)[(3-\lambda)(2) - 0] + 0 = 0$

$\Rightarrow (-2+\lambda)(15 - \lambda^2 - 2\lambda) + 3(6 - 2\lambda) = 0$

$\Rightarrow (-30 + 2\lambda^2 + 4\lambda + 15\lambda - \lambda^3 - 2\lambda^2) + (18 - 6\lambda) = 0$

$\Rightarrow -\lambda^3 + 19\lambda - 30 + 18 - 6\lambda = 0$

$\Rightarrow +\lambda^3 - 13\lambda + 12 = 0$

if $\lambda = 1$

$\Rightarrow 1^3 - 13 + 12$

$\Rightarrow 0$ Hence $\lambda = 1$ is one of the roots.

Using synthetic division

1	1	0	-13	12
	1	1	1	-12
	1	1	-12	0

$\lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda+4) - 3(\lambda+4) = 0$

$(\lambda+4)(\lambda-3) = 0$

Eigen Values $\Rightarrow -4, 1, 3$

Ans 13. $f(x) = x^3 - 7x^2 + 8x - 3$

if $x_0 = 5$

$x_0 = 5$ $\qquad f'(x) = 3x^2 - 14x + 8$

$f(x_0) = -13$

$f'(x_0) = 13$

$x_1 = 5 - \frac{(-13)}{13}$

$= 6$

$f(x_1) = 9$

$f'(x_1) = 32$

$x_2 = 6 - \frac{9}{32}$

$= 5.71875$

Ans 14. Expand $\Rightarrow (1 - x + x^2)^4$

Let $1 - x = y$

the $(y + x^2)^4$

Using binomial theorem

$\Rightarrow {}^4C_0\, y^4 + {}^4C_1\, y^3 (x^2) + {}^4C_2\, y^2 (x^2)^2 + {}^4C_3\, y^1 (x^2)^3 + {}^4C_4\, (x^2)^4$

$\Rightarrow y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$

$\Rightarrow (1-x)^4 + 4(1-x)^3 x^2 + 6x^4(1-x)^2 + 4x^6(1-x) + x^8$

$\Rightarrow (1-x)^2(1-x)^2 + 4x^2(1 - x^3 - 3x + 3x) + 6x^4(1 + x^2 - 2x) + 4x^6 - 4x^7 + x^8$

$\Rightarrow 1 - 4x + 10x^4 - 16x^3 + 14x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$

Ans 15. $e^{-x} = 3\log(x)$

for value of x upto 4 iterations.

$e^{-x} - 3\log(x) = 0$

x	y
0.5	1.50962
1	0.36787
1.5	-0.305143

(i) $x_1 = \frac{1+1.5}{2} = 1.25$

$y_1 = -0.0042252$

(ii) $x_2 = \frac{(1+1.25)}{2} = 1.125$

$y_2 = 0.1711944$

(iii) $x_3 = \frac{1.125 + 1.25}{2} = 1.1875$

$y_3 = 0.0810819$

(iv) $x_4 = \frac{1.1875+1.25}{2} = 1.21875$

$y_4 = 0.0378555$

Therefore after 4 iterations the value of $x = 1.21875$