(P&S-1) Roshan Mehta

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Assigment-2

Q1) Mean of distribution for home policy $= 800$

$\sigma = 100$

Mean of distribution for car policy $= 1200$

$\sigma = 300$

x is claim size on home policy

y is claim size on car policy

$$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$$

$$P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 800)$$

$$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3 \times 1200 - 4 \times 800,\ 300^2(3) + 100^2)$$

$$\sim N(400, 310000)$$

$$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$$

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$P(Z > 0.71842) \Rightarrow 1 - P(Z \leq 0.72)$

$= 1 - 0.76424$

$= 0.23576$

Q2) $N \sim$ -ve Binomial(K, P)

$P(N=n) = \binom{n+2}{n} 0.9^3 \, 0.1^n$

$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^2$

$= {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n$

So, $k = n$, $p = 0.1$

$K = 3$, $p = 0.9$

$M_y = M_N(\log M_x t)$

$X \sim$ Gamma$(6, 2)$

y = Total claim amount

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i) MGF of Y $\quad M_y(t) = E(E(e^{yt}/N=n))$

$E(e^{yt}/N=n) = E(e^{x_1 t + x_2 t + \cdots x t})$

$= E(e^{tx})\ldots$

$= \left(1 - \frac{t}{2}\right)^{-6n}$

So, $E(E(e^{yt}/N=n)) = E\left(\left(1-\frac{t}{2}\right)^{-6n}\right)$

$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$

$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k \Rightarrow \left(\frac{pe^{\log(1-\frac{t}{2})^{-6}}}{1-qe^{\log(1-\frac{t}{2})^{-6}}}\right)$

$\left[\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]^3$

$V(y) = E(N)V(X) + (E(X))^2 \cdot V(N)$

$\Rightarrow \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left(\frac{6}{2}\right)^2 \times \frac{(3)(0.1)}{(0.9)^2}$

$\sigma = 2.887.$

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Q3) i) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

$$MGF = E(e^{tx}) \Rightarrow \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} \, dx$$

$$M_x t = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda\alpha} \, dx \Rightarrow e \lambda \int_{\alpha}^{\infty} e^{-(\lambda-t)x} \, dx$$

$$\frac{e^{\alpha\lambda}\lambda}{t-\lambda}\left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty} \Rightarrow \frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[-e^{-(\lambda-t)\alpha}\right]$$

$$\frac{\cancel{e^{\lambda\alpha}}\lambda \, \cancel{e^{-\lambda\alpha}} e^{t\alpha}}{\lambda - t} \Rightarrow \frac{e^{t\alpha}\lambda}{\lambda-t}$$

ii) $M_x(t) = \dfrac{\lambda e^{t\alpha}}{\lambda - t} \Rightarrow \lambda e^{t\alpha}(\lambda-t)^{-1}$

$$M_x'(t) = \lambda[e^{t\alpha}(\lambda-t)^{-2}(+1) + (\lambda-t)^{-1} e^{t\alpha}(\alpha)]$$

$$\Rightarrow \lambda e^{t\alpha}(\lambda-t)^{-2}[\alpha(\lambda-t)+1]$$

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$t=0 \Rightarrow \lambda(\lambda)^{-2}[\alpha(\lambda)+1] \quad \alpha + \frac{1}{\lambda}$

$$\Rightarrow \frac{1}{\lambda}[\lambda\alpha+1] \Rightarrow \frac{\lambda\alpha+1}{\lambda}$$

$$M_\alpha''(t) \Rightarrow \lambda[\alpha e^{t\alpha}(\lambda-t)^{-2} + \alpha^2(\lambda-t)^{-1} e^{t\alpha} + e^{t\alpha}(+2)(\lambda-t)^{-3} + (\lambda-t)^{-2} e^{t\alpha}]$$

$t=0$

$$\lambda[\alpha(\lambda)^{-2} + \alpha^2(\lambda)^{-1} + (2)(\lambda)^{-3} + \alpha(\lambda)^{-2}]$$

$$\lambda\left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3}\right]$$

$$E(X^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(X) = E(X^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]^2$$

$$\frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \alpha^2 - \frac{1}{\lambda^2} - \frac{2\alpha}{\lambda} \Rightarrow \left[\frac{1}{\lambda^2}\right]$$

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Q4) $G_V(t) = \left(\frac{p}{1-qt}\right)^k$

$G_V t = \sum t^v \times P(V=v)$

$P(V=4) = \frac{\Gamma k+4}{\Gamma 5 \, \Gamma k} p^k \cdot q^p$

$= \frac{\Gamma k+4}{4! \, \Gamma k} p^k q^k$

$\Rightarrow \frac{(k+3)!}{(k-1)! \, 4!} p^k (1-p)^k$

Q5) $f(x,y) = ax(x+y)$

(i) $P\left(Y > \frac{1}{3} + 10\right)$

$a \int_{10}^{20} \int_0^5 (x^2 + xy) dx . dy = 1$

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$\int_{10}^{20} \frac{x^3}{3} + \frac{x^2}{2} y \Big|_0^5 dy = \frac{1}{a}$

$\int_{10}^{20} \left(\frac{125}{3} + \frac{25y}{2}\right) dy = \frac{1}{a}$

$\Rightarrow \left(\frac{125}{3} y + \frac{25}{4} y^2\right)_{10}^{20} = \frac{1}{a}$

$a\left(\frac{2500}{3} + 2500\right.$

$\left.\frac{1250}{3} - 625\right) = 1$

$a\left(\frac{1250}{3} + 1875\right) = 1$

$a = \frac{3}{6875}$

$833.33 + 2500 - (416.667 + 625) = \frac{1}{a}$

$a = 0.00043636$

$f_{y/x} = \frac{f(x,y)}{f(x)}$

$f(x) = \frac{3}{6875} \int_{10}^{20} (x^2 + xy) dy$

$\Rightarrow \frac{3}{6875} \left[x^2 y + \frac{xy^2}{2}\right]_{10}^{20}$

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$$= \frac{3}{6875}\left[10x^2 + 150x\right] \Rightarrow \frac{30x}{6875}\left[x+15\right]$$

$$f\, y/x = \frac{\frac{3}{6875}\, x(x+y)}{30x} \cancel{\times 6875(x+15)}\ \frac{\frac{3}{6875} \times (x+y)}{\frac{6875}{(x+15)}\ 10x}$$

$$E(y/x) = \int_{10}^{20} y \cdot \frac{(x+y)}{10(x+15)} \cdot dy \Rightarrow \frac{1}{10(x+15)}$$

$$\int_{10}^{20} (xy + y^2)\, dy$$

$$\Rightarrow \frac{1}{10(x+15)}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{10}^{20}$$

$$\Rightarrow \frac{1}{10(x+15)}\left[200x + \frac{8000}{3} - 50x - \frac{1000}{3}\right]$$

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$$\left[\frac{1}{x+15}\left(15x + \frac{700}{3}\right)\right]$$

ii) $P\left(y > \frac{1}{3}x + 10\right)$

$$f(y) = \frac{3}{6875}\int_0^5 x^2 + xy \cdot dx \Rightarrow \frac{3}{6875}\left[\frac{x^3}{3} + \frac{x^2 y}{2}\right]_0^5$$

$$f(y) \Rightarrow \frac{3}{6875}\left[\frac{125}{3} + \frac{25}{2}y\right]$$

$$\therefore \frac{3}{6875}\int_{\frac{1}{3}x+10}^{20}\left(\frac{125}{3} + \frac{25y}{2}\right)dy \Rightarrow \frac{3}{6875}\left[\frac{125y}{3} + \frac{25y^2}{4}\right]_{\frac{1}{3}x+10}^{20}$$

$$\frac{3}{6875}\left[\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3}\right.$$

$$\left. - \frac{25}{4}\left[\frac{1}{9}x^2 + 100 + \frac{2}{3}x\right]\right]$$

$$\frac{3}{6875}\left[\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3} - \frac{25x^2}{36} - \cancel{625} - \frac{25x}{6}\right] \Rightarrow 3\left[1250 + 18\right]$$

$$= 3804$$

Q6) $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$ $\quad M_Y(t) = e^{\mu(e^t-1)}$

So, $Z = X - Y$

$$M_Z(t) = E(e^{tZ}) = E(e^{t(x-y)})$$
$$= E(e^{tx} \cdot e^{-ty})$$
$$= E(e^{tx}) \times E(e^{-ty}) \Rightarrow \frac{E(e^{tx})}{E(e^{ty})}$$
$$\Rightarrow e^{(\lambda-\mu)(e^t-1)}$$

So, by uniqueness MGF of $Z =$
$$e^{(\lambda-\cancel{\mu}\mu)(e^t-1)}$$
$$\sim Poi(\lambda-\mu)$$

$Z = X + Y$

$$E(e^{tZ}) = e^{(\lambda+\mu)(e^t-1)}$$

$\therefore$ by uniqueness $Z \sim Poi(\lambda+\mu)$

Q7)

	X: 1	2	3	4
Y: 1	0.2	0	0.05	0.15
2	0	0.3	0.1	0.2

i) $V(X/Y=2) = E(X^2/Y=2) - [E(X/Y=2)]^2$

$$E(X^2/Y=2) \Rightarrow \sum_x x^2 \frac{P(X=x/Y=2)}{P(Y=2)}$$

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When $x = 1, 2, 3, 4$

$1^2 \frac{P(x=1, y=2)}{P(y=2)} + x=2 \ldots + x=4$

$\Rightarrow \frac{1(0)}{0.6} + \frac{(4)(0.3)}{0.6} + \frac{(9)(0.1)}{0.6} + \frac{16(0.2)}{0.6}$

$\Rightarrow 8.8333$

$E(X/y=2) \Rightarrow \sum_x x \frac{P(X=x, y=2)}{P(y=2)}$

$\frac{1(0)}{0.6} + \frac{2(0.3)}{0.6} + \frac{3(0.1)}{0.6} + \frac{4(0.2)}{0.6}$

$\Rightarrow \frac{0.6 + 0.3 + 0.8}{0.6} \Rightarrow 2.8333$

$\Rightarrow 0.807$

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ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$

$E(u/v=v) = ?$

$\frac{f(u,v)}{f(v)} \Rightarrow f\, u/v = v$

$f(v) \Rightarrow \frac{48}{67} \int_0^1 (2uv - u^2) \cdot du \Rightarrow \frac{48}{67}\left[u^2 v - \frac{u^3}{3}\right]_0^1$

$\frac{48}{67}\left[v - \frac{1}{3}\right] \Rightarrow \frac{16}{67}[3v - 1]$

$f(v) = \frac{16}{67}[3v - 1]$

$\frac{f(u,v)}{f(v)} = \frac{\overset{3}{\cancel{48}}}{\cancel{67}} \frac{(2uv^2 - u^2) \times \cancel{67}}{\cancel{16}(3v) - 1)}$

$\Rightarrow \frac{3}{3v-1}(2uv^2 - u^2)$

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$$E(U/V=v) = \frac{3}{3v-1}\int_0^1 u(2uv^2-u^2)\,du$$

$$\frac{3}{3v-1}\left[\frac{2u^3v^2}{3} - \frac{u^4}{4}\right]_0^1$$

$$\Rightarrow \frac{3}{3v-1}\left[\frac{2v^2}{3} - \frac{1}{4}\right]$$

$$\frac{\cancel{3}}{3v-1}\left[\frac{8v^2-3}{12\cancel{4}}\right] = \frac{8v^2-3}{4(3v-1)}$$

Q8) $X \sim$ Gamma$(3,2)$

$E(Y/X=x) = 3x+1 \quad var(Y/X=x) = 2x^2+5$

$$E(Y) = E(E(Y/X=x))$$

$$\Rightarrow 3E(X)+1 \Rightarrow 3\left(\frac{3}{2}\right)+1 = \frac{11}{2}$$

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$$V(Y) = E(V(X/Y)) + V(E(X/Y))$$

$$\Rightarrow E(2x^2+5) + V(3x+1)$$

$$\Rightarrow 2E(X^2)+5+9V(x)$$

$$V(X) = E(X^2) - [E(X)]^2$$

$$\frac{3}{4} + \frac{9}{4} = E(X^2)$$

$$\frac{12}{4} = E(X^2) \Rightarrow \boxed{E(X^2)=3}$$

$$V(Y) = 2(3)+5+9\left(\frac{3}{4}\right)$$

$$= \frac{44+21}{4} = \frac{71}{4}$$

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Q9) $E(x)=3$, $V(\cancel{x})=4$

$E(y)=4$, $V(y)=1$

$= -0.3\sqrt{4}$ $Cov(x,y)$

$= -0.6$

$Z = X+Y$

(i) $Cov(X, X+Y) = Cov(X,X) + Cov(X,Y)$
$= 4 + (-0.6)$
$= 3.4$

ii) $Var(Z) = Cov(X+Y, X+Y)$
$= Cov(X,X) + 2Cov(X,Y)$
$+ Cov(Y,Y)$

$= 4 + 2(-0.6) + 1$
$= 5 - 1.2$
$= 3.8$

$$f(x,y) = k x^{-\alpha} e^{-y/\beta} \qquad 1 < x < \infty, \quad 1 < y < \infty$$

$$k \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/\beta}\, dx\, dy = 1 \qquad \alpha > 1, \quad \beta > 0.$$

$$k \int e^{-y/\beta} \left[\frac{x^{-(\alpha - 1)}}{-\alpha + 1} \right]_1^\infty = 1$$

$$\frac{k}{1-\alpha} \int e^{-y/\beta} \left[-1 \right] \Rightarrow \frac{k}{\alpha - 1} \int e^{-y/\beta}\, dy = 1$$

$$\frac{k}{\alpha - 1} \left[\frac{e^{-y/\beta}}{\left(-\frac{1}{\beta}\right)} \right]_1^\infty = \frac{k}{\alpha - 1} (\beta) \left[+ e^{-1/\beta} \right] = 1$$

$$k = \frac{\alpha - 1}{\beta \left(e^{-1/\beta} \right)}$$