1) $X \sim Poi(\theta)$ $\quad \frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$ $\quad x = 200$

$\therefore \quad \theta = X \pm 1.96(\sqrt{\theta/n})$

2) $X_i \sim N(\mu, \sigma^2)$

$\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$Var(\overline{X}) = Var\left(\frac{\Sigma X_i}{n}\right) = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$

$E(\overline{X}) = E\left(\frac{\Sigma X_i}{n}\right) = \frac{n\mu}{n} = \mu$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ $\quad E(S^2) = \frac{(n-1)\sigma^2}{(n-1)} = \sigma^2$

$Var(S^2) = \frac{2(n-1)\sigma^4}{(n-1)^2} = \frac{2\sigma^4}{n-1}$

$(50 < S^2 < 150)\left(\frac{9 \times 50}{100} < \chi^2 < \frac{9 \times 150}{100}\right)$

4.5, 13.5 (Ans)

i)

$n = 4$ $\quad Bin(4, P)$ $\quad$ 1ßG 1fC policies

1)

no of claims	0	1	2	3	4
no. of policies	86	75	16	2	1

$$L(P) = \left((1-P)^4\right)^{86} \left(4p(1-P)^3\right)^{75} \left(6p^2(1-p)^2\right)^{16}$$
$$\left(4p^3(1-P)\right)^2 (p^4)^1$$

$$= (1-P)^{344} p^{75} (1-P)^{225} p^{32} (1-P)^{32} p^6 (1-P)^2 p^4$$

$$= (1-P)^{603} p^{117} \text{ Const}$$

$$\ln L(P) = 603 \ln(1-P) + 117 \ln P$$

$$\frac{d}{dP} \ln L(P) = \frac{-603}{1-P} + \frac{117}{P} = 0$$

$$117 - 117P = 603P$$

$q = 8375$ $\quad P = 0.1625$

ii) $P(X=0) = (1-P)^4 = 0.492$ $\quad P(X=2) = 6P^2(1-P)^2 = 0.111$

$P(X=1) = 4P(1-P)^3 = 0.388$ $\quad P(X=3) = 4P^3(1-P)$.
$= 0.0144$

$P(X=4) = 0.0007$

Q

	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.56	68.72	19.98	2.89	0.126

By clubbing :

	0	1	2
O_i	86	75	19
E_i	88.56	68.72	22.71
e_i			
$\frac{\Sigma(O-E_i)^2}{E_i}$	0.0757	0.5739	0.6061

$\Sigma e_i = 1.2557 \sim \chi^2_1$

$\therefore 1.2557 < 3.841$

$\therefore$ do not reject H

5) $u(a,b)$ $E(X) = \frac{a+b}{2}$ $Var(X) = \frac{(b-a)^2}{12}$

$a = \bar{X} - \sqrt{3}s$ $a = 2\bar{X} - b$ $a = b - 2\sqrt{3}s$

$2\bar{X} - b = b - 2\sqrt{3}s$

$b = \bar{X} + \sqrt{3}s$ $a = \bar{X} - \sqrt{3}s$

Sample = 2, 4, 6, 8, 10 $\bar{X} = 6$ $S = 3.16$

$a = 6 - \sqrt{3}(3.16)$

$b = 6 + \sqrt{3}(3.16)$

$2 \mid 11.47$

$= 0.527$

$U(0,b)$ $\quad f(x) = \frac{1}{b}$ $\quad L = \frac{1}{b^n}$ $\quad \log L = -n \log b$

$\frac{d}{dL} \log L = \frac{-n}{b}$

$\therefore b \to \alpha$

6) $H_0 P = 0.1$ $\quad H_1 P < 0.1$

P = Prob of hit of missiles

12 missiles fired $\quad H_0$ ∴ 3 or mor hits observed

12 missiles fired $\quad H_0 = 3$ or more hits observed

otherwise

24 missiles fired ~~total~~ total $\quad H_0 = 5$ or mor hits observed

7) $\bar{X} Pub = 30$ $\quad \bar{X} Poi = 35.056$

ii)

$S^2 Pub = 64.3125$ $\quad S^2 Poi = 62.403$

$Sp^2 = 58.123$

$$\frac{(\bar{X} Pub - \bar{X} Poi) - (\mu pub - \mu poi)}{Sp / (\sqrt{\frac{1}{n_1} + \frac{1}{n_2}})} \sim t_{n_1+n_2-2}$$

7) ii) $H_0 : \mu_{pub} = \mu_{pri}$ $H_1 : \mu_{pub} \neq \mu_{pri}$

$$\frac{(30 - 35.056)}{\sqrt{38.123}\sqrt{\frac{2}{9}}} = -1.407$$

$\therefore$ accept H

iii) Private hospital do not result in higher dem.

9) i) $t_K = \frac{N(0,1)}{\sqrt{\frac{\chi^2_K}{K}}}$ $\frac{\bar{x} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

ii) $\mu(t_K) = 0$ $Var(t_K) = \frac{K}{K-2}$

iii) $n = 10$ $\bar{x} = 50$

$s^2 = 48.667$

$CI = 99\%$

a) $t_{0.005, 9} = 3.25$

$\therefore \mu = \bar{x} \pm t_{0.5\%, 9} \frac{S}{\sqrt{n}} = 50 \pm 3.25\sqrt{\frac{48.667}{10}}$

$= 57.17, 42.83$

b) $t_{0.005}, 9 = 2.5758$ $\mu = 50 \pm 2.5758\left(\sqrt{\frac{48.667}{10}}\right)$

$55.68, 44.32$

10) $n = 1000$ $\quad X \sim Poi(3)$ $\quad X \sim N(3000, 3000)$

if $P(X < 3100)$ then $X \sim Poi(3)$

else X do not follow $Poi(3)$

i) Type 1 error = P(H₀ is reject / H₀ is true)

$= P(X > 3100, \lambda = 3) \quad \therefore X \sim N(3000, 3000)$

$= P\left(Z > \frac{3100 - 3000}{\sqrt{3000}}\right) = P(Z > 1.826) = 3.39\%$

ii) Typ 2 error $\Rightarrow$ P(H₀ accept / H₀ is false)

iii) Power = P(reject H₀ / H₀ false) $= P\left(Z < \frac{3100 - \mu_4}{\sqrt{\mu_4}}\right)$

11) $n = 12$ $\quad \Sigma x = 213200$ $\quad \Sigma x^2 = 4919860000$

$\bar{X} = 17767$ $\quad S = 10144$

$X_1 = 11000$ $\quad X_2 = 18000$ $\quad X_1' = 27500$

$X_2' = 31500$

$\Sigma x = 213200 = A + 11000 + 18000$

$A = 184200$

$\Sigma x' = A + x_1' + x_2' = 213200$ $\quad \bar{X}' = 17767$

$\Sigma x^2 = 4719860000 = B + 11000^2 + 48000^2$

$B = 2494860000$

$\Sigma' x^2 = B + x_i + x'^2_2 = 4243360000$

$S = \frac{1}{\sqrt{11}} \sqrt{4243360000 - 12(17767)^2}$

$= 6434.03$

8) i) unbias when $E(\hat{\theta}) = \theta$

ii) bias is when $E(\hat{\theta}) - \theta = 0$

iii) MSE is when $\theta = \text{var} + \text{bias}^2$