

FINANCIAL MATHS ASSIGNMENT

ASSMATE
Date _____
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1) loan = 20000.
 Payment = 427.90, (monthly, arrears)
 t = 5 y

2) $20000 = 427.90 a_{\overline{5}|i}^{(12)}$
 $= \frac{427.90 (1 - (1+i)^{-5})}{12((1+i)^{1/12} - 1)}$
 $520.87 ((1+i)^{1/12} - 1) = 1 - (1+i)^{-5}$

APR is double flat rate

Flat rate = $\frac{\text{total interest}}{\text{loan} \times \text{total years}}$
 $= \frac{5 \times 12 \times 427.90 - 20k}{20k \times 5} = 5.67\%$

APR is likely 11%.

At $i = 11\% \Rightarrow 3.8787 = a_{\overline{5}|i}^{(12)}$
 At $i = 10\% \Rightarrow 3.96154 = a_{\overline{5}|i}^{(12)}$

Interpolation:

APR $\Rightarrow \frac{11\% - 10\%}{3.89499 - 3.96154} \Rightarrow 10.8\%$

2) After 1 year, capital left to pay: $12 \times 427.90 a_{\overline{1} \mid i}^{(12)} @ i$.

$$= 12 \times 427.90 \times \frac{(1 - (1 + 10.8\%)^{-1})}{i^{(12)}}$$

$$= 16775.98.$$

let t denote term of new loan:

$$16775.98 = 12 \times 274.49 a_{\overline{t} \mid i}^{(12)}$$

$$16775.98 = 3293.88 \frac{(1 - 1.08^{-t})}{12(1.108^{1/12} - 1)}$$

$$1.108^{-t} = 0.4754$$

$$t = 7.257.$$

3) The original loan had a repayment of 4 Y.

$$\text{Total Interest} = 4 \times 12 \times 427.90 - 16775.98$$

$$= 3763.22$$

new loan has a term of 7.25 Y

$$\text{Total Interest} = 7.25 \times 12 \times 274.49 - 16775.98$$

$$= 7104.65$$

Mr & Mrs Jones will pay 3341.43

(7104.65 - 3763.22) more interest under restructured loan

Q.2)

(i)

a) The discounted payback period is the smallest t for which the present or accumulated value of the returns up to time t exceeds the P.V or A.V of costs up to the time

b) The payback period is same as discounted payback period, except the present value calculations is carried out using an interest rate 0. In other words, it is the earliest time for which the monetary value of returns exceeds the monetary value of costs

(ii) Unlike NPV, neither DPP or PP give any indication of how profitable a project is, they ignore cash flows after the acc value of zero is reached

There may not be one unique time when the balance in the investor's account changes from negative to positive. However, the NPV can always be calculated

the PP can give misleading results, as it does not take into account the time value of money

Thus NPV is superior measure as compared to DPP or PP

a) Internal rate of return

The IRR for Project A is the rate of Interest that satisfies the equation of value:

$$-170k - 20kv - 10kv^2 + 20v + 20v^2 + 200v^3 = 0$$

$$10v^2 + 200v^3 = 170$$

considering SS. using table Method

$$10(1.074)^2 + 200(1.074)^3 = 170 \quad ? = 0.074 = 7.4\%$$

For Project B: (Income paid at the end of each of the first 6 Y) represent 7% of the initial capital which is returned at the end of 6Y

so IRR for Project B is 7%.

b) NPV:

Project A:

$$NPV = -170k + 10kv^2 + 200kv^3 = 6823.82$$

Project B:

$$NPV = -200k + 14k \frac{2}{6} + 200kv^6 = 9834.65$$

c) Minimum interest rate beyond which ~~are~~ are unprofitable
Each project will be profitable if the rate of return at which the funds can be borrowed is less than IRR.
So to be profitable, Project A requires borrowing lower than 7.4% and Project B requires a rate lower than 7%.

Other factors:

Project A provides no net income during F.Y.

Lower IRR for P.B. app for longer period

Risk associated with risk of return

Q. 3

→

Value at 1st Nov 1985 :

$$\text{Annuity 1} = 200 \times (v)^{1/4} \times a_{\overline{22}|} \text{ at } 8\%$$

$$= 200 \times 0.980944 \times 11.016767$$

$$= 2161.362$$

$$\text{Annuity 2} = 320 \times (1+i)^{1/2} \times a_{\overline{16.25}|}^{(4)} @ 8\%$$

$$= 320 \times 1.006434 \times 9.184257$$

$$= 2957.872$$

$$\text{Annuity 3} = 180 \times a_{\overline{18.75}|}^{(12)}$$

$$= 180 \times 9.892583$$

$$= 1780.665$$

$$\text{Total value of all 3 annuities} = 6889.90$$

Let Revised Annuity be A

$$\text{value of revised annuity be} = A(1+i)^{1/4} \times a_{\overline{21.5}|}^{(2)}$$

$$A = \frac{6889.90}{1.019427 \times 10.30886}$$

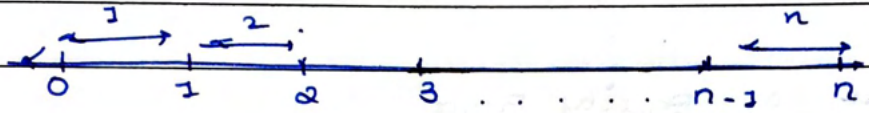
$$= 656.56$$

Q.4)

→

$\int \bar{a} \, \bar{n}$ for $i > 0$

From first principles



$$\begin{aligned} \int \bar{a} \, \bar{n} &= \int_0^1 1 \cdot v^t \, dt + \int_1^2 2 \cdot v^t \, dt + \dots + \int_{n-1}^n n v^t \, dt \\ &= \bar{a} \, \bar{n} + 2v \bar{a} \, \bar{n} + 3v^2 \bar{a} \, \bar{n} + \dots + nv^n \bar{a} \, \bar{n} \\ &= \bar{a} \, \bar{n} (1 + 2v + 3v^2 + \dots + nv^n) \end{aligned}$$

We know :

$$1 + 2v + 3v^2 + 4v^3 + \dots + nv^n$$

$$(1+i)(1 + 2v + 3v^2 + \dots + nv^n) = 1 + 2v + 3v^2 + \dots + nv^{n-1}$$

Subtracting 2 from 1.

$$\int \bar{a} \, \bar{n} = \frac{\bar{a} \, \bar{n} - nv^n}{i}$$

$$= \bar{a} \, \bar{n} (1 + 2v + 3v^2 + \dots + nv^n)$$

$$= \bar{a} \, \bar{n} (\bar{a} \, \bar{n} - nv^n)$$

$$= \frac{(1-v)(\bar{a} \, \bar{n} - nv^n)}{i}$$

$$= \frac{\bar{a} \, \bar{n} - nv^n}{i}$$

Q.5.

(i) The TWR of Property fund.

$$\Rightarrow \frac{16.4}{12.4} - 1 = 0.3226$$

The TWR of Equity Fund

$$\frac{15.5}{12.1} - 1 = 0.2810$$

(ii) Assuming that the investor bought 100 units per quarter in property fund the equation of value

$$1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1580(1+i)^{1/4} = 4 \times 1640$$

where i is yield per annum = 0.2932

Assuming that the investor purchases INR 100 worth of units each quarter

$$400 s_{\overline{4}|i}^{(+)} = 100 \times 16.4 \left(\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{15.8} \right)$$

$$\therefore s_{\overline{4}|i}^{(+)} = 3.7801, i = 0.2979$$

(iii)

- a) Assuming investor bought 100 units per quarter in the Equity fund the equation of value is given by:

$$1210(1+i) + 920(1+i)^{3/4} + 1030(1+i)^{1/2} + 1310(1+i)^{1/4} = 4 \times 1550$$

Where i is yield per annum = 0.8431.

- b) Assume that the investor purchase 100 worth of units each quarter

$$400 \ddot{s}_{\overline{4}|i} = 100 \times 15.5 \left(\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1} \right)$$

$$\ddot{s}_{\overline{4}|i} = 1.14135, \text{ this gives } i = 7.88\%$$

Q.6

$$\text{loan} = ₹ 160k$$

$$t = 10 \text{ y (annuities)}$$

$$i = 8\%$$

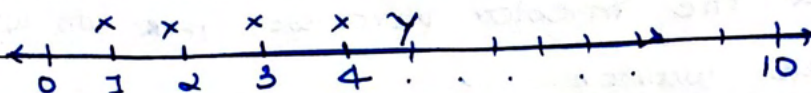
(i) Initial amt of annual repayment

$$160000 = x a_{\overline{10}|8\%}$$

$$x = \frac{1600000 \times 8\%}{6.7101}$$

$$= 1907157 \cdot 23844.65$$

(ii)



$$\Rightarrow \frac{23844.65}{1907157} a_{\overline{4}|8\%} @ 8\% = 6318.88 \text{ } \sqrt{4}$$

$$\text{Remaining loan} = 1600000 - 6318.88 \sqrt{4}$$

$$= 160k - 58049.63$$

$$= 101950.36, 110231.442$$

$$101950.36 = Y a_{\overline{7}|10\%}$$

$$Y = \frac{101950.36}{4.3553} = 25309.72$$

iii

The loan outstanding after 7 payments paid:

$$25309.72 a_{\overline{3}|10\%}$$

$$= 25309.72 \times 2.4869 = 62942.74$$

Let Z be revised payments:

$$Z a_{\overline{3}|9\%} = 62942.74 @ 9\%$$

$$Z = \frac{62942.743}{2.48692.5313} = 24865.77$$

The yield:

$$160k = 23844.65 a_{\overline{4}|i} + 25309.72 a_{\overline{3}|i} \sqrt{4} + \sqrt{4} \times 24865.77$$

$$i = 8.4\%$$

Q.2

(i) Reasons:

- Return on property may be lower, especially in short term
- There is less flexibility because of the larger unit sizes.
- It may be difficult & expensive to value the property
- There are large buying & selling expenses
- The maintenance cost associated with property investments are also high.
- There may be periods when the property investments are also high.
- Marketability is poor

(ii) a) Govt can raise money by floating loan on stock exchange.

The terms of issues are set out by tender. The investors are invited to nominate the price that they are prepared to pay & loan is issued to the highest bidder, subject to certain rules of allocation.

b) Advantages:

Govt will know price investor will pay at outset so

It will know cost of borrowing

It will be administered less difficult by tender

Disadvantages

The investor may be willing to pay more for the bond than the set price & so the money could be borrowed more cheaply if tender is used. Many investors may be prepared to pay set price calling only part of the offer to be sold & govt may not meet financing requirement

c) Securities with uncertain income

→ Equities

→ Property

→ Bonds

Q.8 Workup in units of Rs 81 crore & assume i as the rate of interest:

(i) The P.V of outlay is:

$$2.7 + 0.2 a_{\overline{3}|} @ i$$

The expected present value:

$$0.1v^3 + a_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3) @ i$$

Equating the P.V of the outlay to the P.V of the Income & solve for i

$$2.7 + 0.2 a_{\overline{3}|} = 0.1v^3 + a_{\overline{10}|} (0.25 + 0.2v^2 + 0.1v^3) @ i$$

$$i = 9.3\%$$

(ii) Based on information, the P.V of the company Income is:

$$\rightarrow 0.1v^3 + 0.85 a_{\overline{10}|} v^3$$

$$\rightarrow 0.1v^3 + 0.85 a_{\overline{10}|} v^3 - 2.7 - 0.2 a_{\overline{3}|} @ i = 10\%$$

$$\rightarrow 0.1 \times 0.7513 + 0.85 \times 0.7513 \times 6.4459 - 2.7 - 0.2 \times 2.4668$$

$$\rightarrow 4.79 - 3.79$$

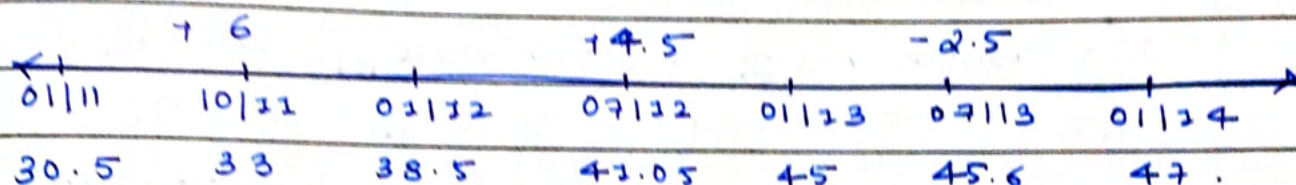
$$\rightarrow 1.0089$$

Since NPV is positive it is worth to invest in the projects.

Q.9

Q.9(i)

TWRR. In crore



TWRR

$$(1+i)^3 = \frac{33}{30.5} \times \frac{(41.05 - 4.5)}{33 + 6} \times \frac{(45.6)}{41.05} \times \frac{47}{45.6 - 2.5}$$

$$= 1.08196 \times 0.937179 \times 1.12084 \times 1.090487$$

$$(1+i)^3 = 1.228313$$

$$= 1.070957$$

$$\text{TWRR} = 7.0957\%$$

MWRR

$$30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$$

$$i = 7.206\%$$

(ii) LRR:

Year 2011 - Equation of value:

$$30.5(1+i) + 6(1+i)^{0.25} = 38.5$$

$$i = 6.266\%$$

Year 2012 - EOY

$$38.5(1+i) + 4.5(1+i)^{0.5} = 45$$

$$i = 4.92\%$$

Year 2013 - EOY

$$45(1+i) - 2.5(1+i)^{-0.5} = 47$$

$$i = 10.276\%$$

$$\text{LRR} \Rightarrow (1+i)^3 = (1+6.266\%)(1+4.92\%)(1+10.276\%)$$

$$i = 7.13\%$$

(iii)

TWRR eliminates effect of C.F. amt & timing & gives a fairer basis for assessing investment performance.

Q. 10 .

(i) Let the loan amt be Rs L

$$L = 180000 \times a_{\overline{15}|} + 20000 (3a_{\overline{15}|} @ 12\%$$

$$L = 180000 \times 6.8109 + 20000 (\ddot{a}_{\overline{15}|} - 15v^{15}) / i$$

$$L = 1225962 + 20000 (7.6282 - 2.74044) / 0.12$$

$$L = 1225962 + 20000 \times 40.7313$$

$$= 2040588$$

Amt of loan from the bank = Rs 2040588

Total cost of house = 2040588 / 0.8

$$= 2550735$$

(ii) Loan outstanding at the beg of the 9th Y

$$L = (180kv + 180kv^2 + \dots + 180kv^7) + (9 \times 20kv + 10 \times 20kv^2 + \dots + 15 \times 20kv^7)$$

$$L(9) = 340k a_{\overline{7}|} + 20k 3a_{\overline{7}|} @ 12\%$$

$$L(9) = 340k \times 4.5638 + 20k \times (5.114 - 7 \times 0.45237) / 0.12$$

$$= 1557672 + 324158.33$$

$$= 1875850.33$$

Loan schedule

Year	Loan o/s	Loan installment	Interest	Capital Rep.
9	1875850.33	360k	225702.04	134897.96
10	1740952.37	380k	208914.28	171085.72
11	1569866.65			

(iii) Let the new installment be Rs P.

$$1569866.65 = P a_{\overline{10}|10\%}$$

$$1569866.65 = P \times 3.790786$$

$$P = 414126.87$$

Extra payment made by Mr. loan continued with first bank

Total installments less outstanding at end of 10th year

$$= (40k + 42k + 44k + 46k + 48k) - 1569866.65$$

$$= 630133.35$$

Extra payments made by Sam if loan is transferred to second bank:

Total installments + 1% processing fees - loan outstanding at end of Year 10

$$= 5 \times 414126.90 \times 0.1 + 1569866.65 - 1569866.65$$

$$= 2070634.537 - 1569866.65$$

$$= 516466.35$$

Since extra payment (Interest + processing fee) to second bank is lower than the payments to the first bank, it is advisable to Mr. Sam to transfer his loan to the second bank.

Q.11

		+ 40	- 50		
	01/15	04/15	01/16	07/16	01/17
(*)	460	500	550	600	x

(i)

TWRR :

$$(1+i)^2 = \frac{500}{460} \times \frac{(550+50)}{500+40} \times \frac{x}{550}$$

$$(1+i)^2 = 1.44$$

$$x = (1.44 \times 550)$$

$$1.086957 \times 1.1111$$

$$x = 655.78$$

(ii)

MWRR :

$$460(1+i)^2 + 40(1+i)^{3/4} - 50(1+i) = 655.78$$

$$\text{MWRR} = 19.85\%$$

(iii)

The effective rate of return for the 7 years given by solving equation of value

$$460(1+i) + 40(1+i)^{3/4} = 600$$

$$i = 20.44\%$$

The effective rate of return for the year 2015 is given by solving equation of value:

$$(1+i) = \frac{655.78}{550} = 1.1923$$

LRR :

$$(1+i)^2 = 1.1923 \times 1.2044$$

$$= 19.83\%$$

(iv) When the sub interval chosen for calculating IRR coincide with interval b/w net cash flows b/w received then the linked internal rate of return will be identical to the time weighted rate of return.

(v) → The TWRR is a better measure of fund manager's performance.

→ The TWRR is sensitive to the amounts & timings of the net cash flows but fund managers does not have any control on these.

→ TWRR is calculated using the "growth factors" reflecting the change in fund value b/w the times of consecutive cash flows.

Thus TWRR is independent of amt & timing of cash flow.

→ Hence TWRR is a better measure of a fund managers performance.

Q.12

(i) The DPP of an investment project is the first time when NPV of cash flows from project is positive.

(ii) a) Let us work in units of 1000 to find the DPP
 $-800(1+i)^t - 50(1+i)^{t-1} + 100s_{t-2} @ 7\% = 0$

$$\text{At } t=17, \text{ NPV} = 75$$

$$\text{At } t=18, \text{ NPV} = 22.85$$

$$\text{By Interpolation } t = 17.767$$

b) The accumulated profit after 22y is found by accumulating the income after DPP has elapsed at 6% p.a.

$$100s_{4.233} \text{ at } 6\% = 480$$

The accumulated amt is INR 480k

(iii) The businessman may accumulate his rental income for each year is 6%.

$$100s_1 = 102.971 \text{ at the end of the year}$$

The DPP is thus the smallest integer t such that

$$-800(1+i)^t - 50(1+i)^{t-1} + 102.97s_{t-2} \text{ at } 7\% \geq 0$$

By trial & error $t = 18$ y

The balance in the businessman's A/c just after the transaction at $t=18$ is

$$-800(1+i)^{18} - 50(1+i)^{17} + 102.97s_{16} @ 7\% = 9.7714$$

$$\text{Acc Amt after 22y} = 9.7714(1+i)^4 + 100s_4 @ 6\%$$

*

Q.13

$$\text{loan} = 988000 \quad (10/07/15)$$

$$t = 25 \quad (\text{monthly arrears})$$

$$i = 7\%$$

$$(1) \quad 988000 = x a_{\overline{25}|}^{(12)} @ 7\%$$

$$x = \frac{988000}{a_{\overline{25}|}^{(12)} \times \frac{i}{12}}$$

$$x = \frac{988000 \times 0.067875}{7\% \times 11.6592}$$

$$= 6848.82176$$

$$\text{Monthly repayment} = 6848$$

ii) Loan outstanding after 10th Mar 2029

$$= x a_{\overline{34/3}|}^{(12)} @ 7\% = 648600$$

iii) Loan outstanding 10/09/26 :

$$x a_{\overline{83/6}|}^{(12)} @ 7\%$$

Loan outstanding 10/10/26 :

$$x a_{\overline{13.75}|}^{(12)} @ 7\%$$

Capital repaid on 10th Oct :

$$= x \left[a_{\overline{83/6}|}^{(12)} - a_{\overline{13.75}|}^{(12)} \right] @ 7\%$$

$$= x \left[\frac{v^{\overline{83/6}} - v^{\overline{13.75}}}{i^{(12)}} \right] @ 7\%$$

$$= 26860$$

iv) Capital repayment continued in 12 Instalments

$$x \left[a_{\overline{22/3}|}^{(12)} - a_{\overline{19/3}|}^{(12)} \right] @ 7\% = 576.20$$

Total Interest

$$x - 576.20 = 385.5630576$$