NMA Assignment 1

Q1) $r_t = 12.65 cm$

$r = 12.5 cm$

$A.E = r_t - r$

$= 12.65 - 12.5$

$= 0.15 cm$

$R.E = \frac{Error}{r} = \frac{0.15}{12.5} = 0.012$

$P.E = R.E \times 100 = 0.012 \times 100 = 1.2\%$

Q2)

x	$f(x) = \log x$
4	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.7781513

1) $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$

$\frac{5 - 4}{4 - 6} = \frac{y - 0.60206}{0.60206 - 0.7781513}$

$y = 0.69010565$

2) $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$

$\therefore \frac{5 - 4.5}{4.5 - 5.5} = \frac{y - 0.6532125}{0.6532125 - 0.7403627}$

$\therefore y = 0.6967876$

Q3) $x^4 - x - 10 = 0$

$a = 1.5$, $b = 2$

x	y
1.5	-6.4375
2	4

① $x = \frac{x_1 + x_2}{2} = \frac{1.5 + 2}{2} = 1.75$

$\therefore y = -2.37109$

x	y
1.75	-2.37109
2	4

② $\therefore x = \frac{x_1 + x_2}{2} = \frac{1.75 + 2}{2} = 1.875$

$\therefore y = 0.484619141$

$\therefore$

x	y
1.75	-2.37109
1.875	0.484619

③ $x = \frac{x_1 + x_2}{2} = 1.8125$

$y = -1.020248$

x	y
1.8125	-1.020248
1.875	0.484619

(4) $x = \frac{x_1 + x_2}{2}$

$x = \frac{1.8125 + 1.875}{2} = 1.84375$

$y = -0.287734032$

x	y
1.84375	-0.287734
1.875	0.484619

(5) $x = \frac{x_1 + x_2}{2}$

$= \frac{1.875 + 1.84375}{2}$

$= 1.859375$

$\therefore y = 0.093378$

Q4) $f(x) = x^3 - x - 1$ $\quad f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
1	-1
2	5

$$x_1 = 1 - \frac{(-1)}{2}$$

$$= 1 + \frac{1}{2}$$

$$= 1.5$$

$$f(x_1) = 0.875$$
$$f'(x_1) = 5.75$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \frac{0.875}{5.75}$$

$$= \cancel{1.652173913}$$

$$= 1.347826087$$

$$f(x_2) = 0.100682173$$
$$f'(x_2) = 4.449905482$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.325200399$$

$$f(x_3) = 0.002058362$$
$$f'(x_3) = 4.268468293$$

$$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.324718174$$

$$f(x_4) = 0.000000924$$

$= 7A - (I + A)^3$

$= \cancel{7A - (I + 3A^2 + 9A + A^3)}$ $\quad 7A - (I^3 + 3I^2A + 3IA^2 + A^3)$

$= \cancel{7A}$ $\quad 7A - I^3 - 3A - 3A^2 - A^2 \cdot A$

$= 7A - I - 3A - 3A - A \cdot A$

$= 7A - I - 3A - A^2$

$= 7A - I - 7A$

$= -I$

Q6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} = 1(-6) + 1(-4-0) + 2(4)$
$= -2$

$A_{Minor} = \begin{bmatrix} -6 & -4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix}$

$A_{C.F} = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$

$adj \cdot A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3 & -0.5 & 3 \\ -2 & 0.5 & -1 \\ -2 & 0.5 & -2 \end{bmatrix}$

Q7)

$A = 8\hat{i} - 9\hat{j}$

$B = 7\hat{i} - 9\hat{j}$

$|A| = \sqrt{8^2 + 9^2}$
$= \sqrt{145}$

$|B| = \sqrt{7^2 + 9^2}$
$= \sqrt{130}$

$\therefore A \cdot B = (8\hat{i} - 9\hat{j}) \cdot (7\hat{i} - 9\hat{j})$
$= (8 \times 7) + (-9 \times -9)$
$= 56 + 81$
$= 137$

$\therefore A \cdot B = |A||B| \cdot \cos\theta$
$\therefore 137 = \sqrt{145} \times \sqrt{130} \times \cos\theta$
$\therefore 137 = 5\sqrt{754} \times \cos\theta$
$\therefore \cos\theta = \dfrac{137}{5\sqrt{754}}$
$\therefore \cos\theta = 0.997849$
$\therefore \theta = \cos^{-1}(\cancel{-}0.998)$
$= 3.6243$

Q8) Mag. $= \sqrt{(75)^2 + (25)^2}$
$= \sqrt{5625 + 625}$
$= \sqrt{6250}$
$= 79.057$

Q9) $a = 101$
$a_n = 998$
$d = 3$

$\therefore a_n = a + (n-1)d$
$\therefore 998 = 101 + (n-1)3$
$\therefore 897 = (n-1)3$
$\therefore \frac{897}{3} = (n-1)$
$\therefore 299 = n-1$
$\therefore n = 300$

$\therefore S_{300} = \frac{n}{2}(2a + (n-1)d)$
$= \frac{300}{2}(2(101) + (299)(3))$
$= 1,64,850$

Q10) $a_6 = 32 = a \cdot r^5$
$a_8 = 128 = a \cdot r^7$

$\therefore \frac{a_8}{a_6} \Rightarrow \frac{128}{32} = \frac{a \cdot r^7}{a \cdot r^5}$

$\Rightarrow 4 = r^2$
$\Rightarrow r = 2$

Q11) $(4+3x)^5$

$= (3x)^5 + 5(3x)^4(4) + 10(3x)^3(4)^2 + 10(3x)^2(4)^3$
$+ 5(3x)(4)^4 + (4)^5$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

Q12) $A = \begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix}$

$|A - \lambda I|$

$\begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$

$\begin{vmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0$

$(2-\lambda)[(-5-\lambda)(3-\lambda)-0] + 3(6-2\lambda-0) - 0(\quad) = 0$

$(2-\lambda)(-15+2\lambda+\lambda^2) + 3(6-2\lambda) = 0$

$-30+4\lambda+2\lambda^2+15\lambda-2\lambda^2-\lambda^3+18-6\lambda = 0$
$-\lambda^3-13\lambda-12=0$
$-\lambda^3-12\lambda-\lambda-12=0$
$-\lambda(\lambda^2+12)-1(\lambda+12)=0$

$(2-\lambda)[(-5-\lambda)(3-\lambda)] + 3[(2-\lambda)(3-\lambda)] + 0[0] = 0$

$(2-\lambda)(-5-\lambda)(3-\lambda) + 3(2-\lambda)(3-\lambda) = 0$
$(2-\lambda)(3-\lambda)(-5-\lambda+3) = 0$

$\therefore \quad (2-\lambda)(3-\lambda)(-\lambda-2)=0$

$\therefore \quad (2-\lambda)(3-\lambda)(\lambda+2)=0$

$\therefore \quad \lambda = -2, 2, 3$

$$\begin{vmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$2x_1 - 3x_2 = -2x_1 \quad \therefore \; 4x_1 - 3x_2 = 0$

$2x_1 - 5x_2 = -2x_2 \quad \therefore \; 4x_1 - 5x_2 = 0$

$3x_3 = -2x_3 \qquad 5x_3 = 0 \quad \therefore \; x_3 = 0$

Q13) $f(x) = x^3 - 7x^2 + 8x - 3$ $\quad f'(x) = 3x^2 - 14x + 8$

$\therefore x_0 = 5$

$\therefore f(x_0) = -13$

$\therefore f'(x_0) = 13$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 5 - \frac{(-13)}{13}$

$= 5 + 1$

$x_1 = 6$

$\therefore f(x_1) = 9$

$\therefore f'(x_1) = 32$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 6 - \frac{9}{32}$

$\therefore x_2 = 5.71875$

Q14) $(1 - x + x^2)^4$

$= [x^2 + (1-x)]^4$

$= x^8 + 4(x^2)^3(1-x) + 6(x^2)^2(1-x)^2 + 4(x^2)(1-x)^3 + (1-x)^4$

$= x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$

$= x^8 + 4x^6 - 4x^7 + 6x^4 - 12x^5 + 6x^6 + 4x^2(1 - 3x + 3x^2 - x^3)$
$+ (1 - 4x + 6x^2 - 4x^3$
$+ x^4)$

$= x^8 + 4x^6 - 4x^7 + 6x^4 - 12x^5 + 6x^6 + 4x^2 - 12x^3 + 12x^4 - 4x^5 +$
$1 - 4x + 6x^2 - 4x^3 + x^4$

$= x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

Q15) $e^{-x} = 3 \cdot \log x$

$e^{-x} - 3 \cdot \log x = 0$

x	$f(x)$
2	-1.944106258
1	0.367879441

① $\therefore x_0 = \frac{x_1 + x_2}{2} = \frac{2+1}{2} = 1.5$

$f(x) = -0.993265164$

x	y
1.5	-0.993265164
1	0.367879441

② $\therefore x = \frac{x_1 + x_2}{2} = \frac{1.5 + 1}{2} = 1.25$

$f(x) = -0.382925857$

x	y
1.25	-0.3829
1	0.3678

③ $x = \frac{x_1 + x_2}{2} = \frac{1.25 + 1}{2} = 1.125$

$f(x) = -0.02869$

x	y
1.125	-0.02869
1	0.3678

(i) $x = \frac{x_1 + x_2}{2} = \frac{1.125 + 1}{2} = 1.0625$

$f(x) = 0.1637168 87$

$x = 1.0625$