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Probablity & Statistics Assignment - 2

(1) Let x_i be claim size on home insurance.

Similary y_i is car insurance

$X \sim N(800, 100^2)$
$Y \sim N(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 800)$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4)$

$\sim N(3 \times 1200 - 4 \times 800,$

$300^2 (3) + 100^2 (4)$

$\sim N(400, 310000)$

$$P\left(z > \frac{800-400}{\sqrt{310000}}\right)$$

$$P\left(z > \frac{800-400}{\sqrt{310000}}\right).$$

$$P(z > 0.71842) \Rightarrow 1 - P(z < 0.71842)$$

$$1 - P(z < 0.72)$$

$$\Rightarrow \boxed{0.23576}$$

(2) $N \sim \text{Neg. Binomial}\ (K, P)$

$$P(N = n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$$

$$\Rightarrow {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n$$

So, $K = 3$, $P = 0.9$

$$M_y\ t = M_N(\log M_{x}t)$$

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$X \sim \text{Gamma}(6, 2)$

$Y \to$ Total claim amount

(i) MGF of y

$M_y(t) = \mathcal{E}(\mathcal{E}k^{4t}/N=n)$

$\mathcal{E}(e^{yt}/N=n) = \mathcal{E}(e^{x_1 t + x_2 t + \dots x_n t})$

$= \mathcal{E}(e^{tx_1}) \cdot \mathcal{E}(e^{tx_2}) \dots \mathcal{E}(e^{tx_n})$

$\Rightarrow \left(1 - t/2\right)^{-6n}$

so, $\mathcal{E}(\mathcal{E}(e^{yt}/N=n) = \mathcal{E}\left((1-t/2)^{-6n}\right)$

$= \mathcal{E}\left(e^{N\log(1-t/2)^{-6}}\right)$

$= M_N(\log(1-t/2)^{-6})$

$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k \Rightarrow \left(\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right)^k$

$\left[\frac{p(1-t/2)^{-6}}{(1-q)(1-t/2)^{-6}}\right]^3 \Rightarrow \left[\frac{0.9(1-t/2)}{6-0.1(1-t/2)^6}\right]^3$

$V(Y) = \mathcal{E}(N)V(X) + (\mathcal{E}(X))^2 \cdot V(N)$

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$$\Rightarrow \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \frac{3\left(6/2\right)^2 \times (3)(0.1)}{(0.9)^2}$$

Standard deviation $\rightarrow 2.887$

Q - 3 $\quad f(x) = \lambda e^{-\lambda(x-\alpha)}$

$$MGF = E(e^{tx}) \Rightarrow \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$$

$$M_x t = \lambda \int_{\alpha}^{\infty} e^{tx} \cdot e^{-\lambda x} \cdot e^{\lambda \alpha} dx$$

$$\Rightarrow e^{\lambda\alpha} \lambda \int_{\alpha}^{\infty} e^{-(\lambda - t)x} dx$$

$$\frac{e^{\alpha\lambda} \lambda}{t-\lambda} \left[e^{-(\lambda-t)x} \right]_{\alpha}^{\infty}$$

$$\Rightarrow \frac{e^{\lambda\alpha} \lambda}{t-\lambda} \left[- e^{-(\lambda-t)\alpha} \right]$$

$$\Rightarrow e^{t\alpha} \frac{\lambda}{\lambda - t}$$

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(ii). $M_x t = \dfrac{\lambda e^{t\alpha}}{\lambda - t} \Rightarrow \lambda e^{t\alpha}(\lambda - t)^{-1}$

$M_x'(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-2}(1)\right.$

$\left. + (\lambda-t)^{-1} e^{t\alpha}(\alpha)\right]$

$\Rightarrow \lambda\left[(\lambda-t)^{-1} e^{t\alpha}(\alpha) + e^{t\alpha}(\lambda-t)^{-2}\right]$

$\Rightarrow \lambda e^{t\alpha}(\lambda-t)^{-2}\left[\alpha(\lambda-t)+1\right]$

$\Rightarrow \lambda(\lambda)^{-2}\left[\alpha(\lambda)+1\right]$

$\Rightarrow \dfrac{1}{\lambda}\left[\lambda\alpha + 1\right] \Rightarrow \dfrac{\lambda\alpha + 1}{\lambda}$

$E(X)\ M_x''(t) \Rightarrow \lambda\left[\alpha e^{t\alpha}(\lambda-t)^{-2} + \right.$

$\alpha^2(\lambda-t)^{-1} e^{t\alpha} + e^{t\alpha}(+2)$

$\left.(\lambda-t)^{-3} + (\lambda-t)^{-2} e^{t\alpha}\right]$

$\lambda\left[\alpha(\lambda)^{-2} + \alpha^2(\lambda)^{-1} + (2)(\lambda)^{-3}\right.$

$\left. + \alpha(\lambda)^{-2}\right]$

$$\lambda \left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3} \right]$$

$$E(x)^2 = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(x) = E(x)^2 - [E(x)]^2 \Rightarrow$$

$$\frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left[\alpha + \frac{1}{\lambda}\right]$$

$$\frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left(\alpha^2 + \frac{1}{\lambda^2} + \frac{2\alpha}{\lambda}\right)$$

(4) $$G_v(t) = \left(\frac{P}{1-qt}\right)^K$$

$$G_v t = \sum t^v \times P(V=v)$$

$$P(V=4) \Rightarrow \frac{\Gamma K+4}{\Gamma 5 \, \Gamma K} \cdot p^K q^K$$

$$\frac{\Gamma K+4}{4! \, \Gamma K} \, p^K q^K$$

$$\Rightarrow \frac{(K+3)!}{(K-1)! \, 4!} \, p^K (1-p)^K$$

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Q-5 if $(x, y) = ax(x+y)$

(i) $P\left(Y > \frac{1}{3}X + 10\right)$

$$a\int_{10}^{20}\int_{0}^{5}(x^2+xy)\,dx\,dy = 1$$

$$\int_{10}^{20}\left(\frac{125}{3} + \frac{25}{2}y\right)dy$$

$$= \frac{1}{a} \Rightarrow \left(\frac{125}{3}y + \frac{25}{4}y^2\right)_{10}^{20}$$

$$833.33 + 2500 - (416.667 + 625) = \frac{1}{a}$$

$$a = 0.00043636$$

(iii) $E(Y/X)$

$$f\,Y/X = \frac{f(x,y)}{f(x)} \Rightarrow$$

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$$f(x) = \frac{3}{6875}\int_{10}^{20}(x^2+xy)\,dy$$

$$\Rightarrow \frac{3}{6875}\left[x^2y + \frac{xy^2}{2}\right]_{10}^{20}$$

$$\Rightarrow \frac{3}{6875}\left[20x^2 + x(200) - 10x^2 - 50x\right]$$

$$= \frac{3}{6875}\left[10x^2+150x\right] = \frac{30x}{6875}\left[x+15\right]$$

$$f_{y/x} = \frac{\cancel{3}}{\cancel{6875}}\,\frac{x(x+y)}{30\,x}\times\frac{6875}{(x+15)}$$

So, $$E(Y|X) = \int_{10}^{20} y\cdot\frac{(x+y)}{10(x+15)}\,dy$$

$$\Rightarrow \frac{1}{10(x+15)}\int_{10}^{20}(xy+y^2)\,dy$$

$$\Rightarrow \frac{1}{10(x+15)}\left[\frac{xy^2}{2}+\frac{y^3}{3}\right]_{10}^{20}$$

$$\Rightarrow \frac{1}{10(x+15)}\left[200x + \frac{8000}{3} - 50x - \frac{1000}{3}\right.$$

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$$\frac{1}{x+15}\left[15x + \frac{700}{3}\right]$$

(ii) $P\left(y > \frac{1}{3}x + 10\right)$

$$f(y) = \frac{3}{6875}\int_0^5 x^2 + xy \, dx$$

$$\Rightarrow \frac{3}{6875}\left[\frac{x^3}{3} + \frac{x^2 y}{2}\right]_0^5$$

$$f(y) = \frac{3}{6875}\left[\frac{125}{3} + \frac{25}{2}y\right]$$

$$\frac{3}{6875}\int_{\frac{1}{3}x+10}^{20}\left(\frac{125}{3} + \frac{25y}{2}\right)dy \Rightarrow \frac{3}{6875}\left[\frac{125y}{3} + \frac{25y^2}{4}\right]_{\frac{1}{3}x+10}^{20}$$

$$\Rightarrow \frac{3}{6875}\left[\frac{125}{3}(20) + \frac{25}{4}(400) - \frac{125}{3}\left(\frac{1}{3}x + 10\right)\right.$$

$$\left. - \frac{25}{4}\left(\frac{1}{3}x + 10\right)^2\right]$$

$$\frac{3}{6875}\left[\frac{2500}{3} + 2500 - \frac{125x}{9} - \frac{1250}{3}\right.$$

$$\left. - \frac{25}{4}\left[\frac{1}{9}x^2 + 100 + \frac{2}{3}x\right]\right]$$

$$\frac{2}{6875}\left[\frac{2500}{3}+2500-\frac{125}{9}\times-\frac{1250}{3}-625-\frac{25}{4}\times\right]$$

(6) $X \sim Poi(\lambda)$ $Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$ $M_Y(t) = e^{\mu(e^t-1)}$

So $Z = X - Y$

$M_Z(t) = E(e^{tZ}) = E(e^{t(X-Y)})$

$= E(e^{tX} \cdot e^{-tY})$

$= E(e^{tX}) \times E(e^{-tY}) = \frac{E(e^{tX})}{E\, e^{tY}}$

$\Rightarrow \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$

$\Rightarrow e^{(\lambda-\mu)(e^t-1)}$

So. By uniqueness MGF of Z

$= e^{(\lambda-\mu)(e^t-1)}$

$\sim Pois(\lambda-\mu)$

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Now let $z = X + Y$

$E(e^{tz}) = e^{\lambda(e^t-1)} \times e^{\mu(e^t-1)}$

$\Rightarrow e^{(\lambda+\mu)(e^t-1)}$

so, By uniqueness $z \sim Poi(\lambda+\mu)$

(7)

		X			
		1	2	3	4
Y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

(i) $V(X/Y=2) = E(X^2/Y=2) - [E(X/Y=2)]^2$

Now

$E(X^2/Y=2) \Rightarrow \dfrac{\sum x^2 P(X=x, Y=2)}{P(Y=2)}$

when $x=1$ when $x=2$
when $x=3$ when $x=4$

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$$\Rightarrow (1)^2 \frac{P(x=1, y=2)}{P(y=2}$$

$$+ \frac{(2)^2 (P(x=2, y=2)}{P(y=2)}$$

$$+ (3)^2 \frac{P(x=3, y=2)}{P(y=2}$$

$$+ (4)^2 \frac{P(x=4, y=3)}{P(y=2)}$$

$$= \frac{(1)(0)}{0.6} + \frac{(4)(0.3)}{0.6} + \frac{(9)(0)}{0.6}$$

$$+ \frac{16(0.2)}{0.6}$$

$$= 8.8333$$

Now: $E(X/y=2) \Rightarrow \sum_x x \frac{(Px=x, y=2)}{P(y=2)}$

$$\Rightarrow \frac{1 \cdot 0}{0.6} + \frac{2 \cdot (0.3)}{0.6)} + \frac{(3)(0.1)}{0.6} + \frac{4(0.2)}{0.6}$$

$$\Rightarrow \frac{0.6 + 0.3 + 0.8}{0.6} \Rightarrow 2.8333$$

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$\Rightarrow 0.807$

ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$

$0 < u < 1 ; \frac{u}{2} < v < 2$

find $E(v/u=v)$

$\frac{f(u,v)}{f(v)} \Rightarrow f_{u/v} = v$

$f(v) = \frac{48}{67}\int_0^1 (2uv - u^2)\,du$

$\Rightarrow \frac{48}{67}\left[u^2 v - \frac{u^3}{3}\right]_0^1$

$\frac{48}{67}\left[v - \frac{1}{3}\right] \Rightarrow \frac{16}{67}\left[3v-1\right]$

$f(v) = \frac{16}{67}\left[3v-1\right]$

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$$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}}{67 \times 16} \frac{(2uv^2 - u^2)}{(3v-1)} \times \cancel{67}$$

$$\Rightarrow \frac{3}{3v-1} (2uv^2 - u^2) \quad 2u^2v^2 - u^3$$

Now $E(U/V=v) = \frac{3}{3v-1} \int_0^1 u(2uv^2 - u^2)\, du$

$$\frac{3}{3v-1} \left[\frac{2u^3v^3}{3} - \frac{u^4}{4} \right]_0^1 \Rightarrow \frac{3}{3v-1} \left[\frac{2v^3}{3} - \frac{1}{4} \right]$$

$$\frac{\cancel{3}}{3v-1} \left[\frac{8v^2 - 3}{\cancel{12}\, 4} \right] \Rightarrow \boxed{\frac{8v^2 - 3}{4(3v-1)}}$$

8. $X \sim \text{Gamma}(3,2)$ $\quad E X \; 3/2 \quad V(x) \; \frac{3}{4}$

$E(Y/X=x) = 3x+1$

$Var(Y/X=x) = 2x^2 + 5$

$E(Y) = E(E(Y/X=x)) \Rightarrow E(3x+1)$

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$\Rightarrow 3(E(X)) + 1$

$\Rightarrow 3(3/2) + 1$

$\Rightarrow \frac{9}{2} + \frac{1}{1} = \frac{9+2}{2}$

$V(4) = E(V(X/4) + V(E(X/4)$

$\Rightarrow E(2x^2+5) + V(3x+11)$

$\Rightarrow 2E(x^2) + 5 + 9V(x)$

$V(X) = E(x^2) - [E(X)]^2$

$\frac{3}{4} + \frac{9}{4} = E(x^2)$

$\frac{12}{4} = E(x^2) \Rightarrow \boxed{E(x^2) = 3}$

$V(4) = 2(3) + 5 + 9(3/4)$

$= 11 + \frac{27}{4} = \frac{44+27}{4}$

$\Rightarrow \boxed{\frac{71}{4}}$

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(9.) $E(X) = 3$, $V(X) = 4$

$E(Y) = 4$, $V(Y) = 1$

$= 0.3\sqrt{4} \Rightarrow \mathrm{cov}(X, Y)$
$= -0.6$

$Z = X + Y$

(i) $\mathrm{cov}(X, X+Y)$

$= \mathrm{cov}(X, X) + \mathrm{cov}(X, Y)$

$= V(X) + \mathrm{cov}(X, Y)$

$= 4 + (-0.6)$

(ii) $\mathrm{Var}(Z) = \mathrm{cov}(X+Y, X+Y)$

$= \mathrm{cov}(X, X) + \mathrm{cov}(X, Y) + \mathrm{cov}(Y, Y)$

$= V(X) + 2\,\mathrm{cov}(X, Y) + V(Y)$

$= 4 + 2(-0.6) + 1$

$\Rightarrow 5 - 1.2$

$\Rightarrow 3.8$

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(7) $f(x, y) = Kx^{-\alpha} e^{-y/\beta}$

$$K \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$$

$$K \int e^{-y/B} \left[\frac{x^{-(\alpha-1)}}{-\alpha+1} \right]_1^{\infty} = 1$$

$$\frac{K}{1-\alpha} \int e^{-y/B} [-1]$$

$$\Rightarrow \frac{K}{\alpha-1} \int e^{-y/B} \, dy = 1$$

$$\frac{K}{\alpha-1} \frac{\left[e^{-y/B} \right]_1^{\infty}}{(-1/\beta)} , \quad \frac{-K}{\alpha-1} (\beta) \left(+e^{-1/\beta}\right) = 1$$

$$\boxed{K = \frac{\alpha-1}{(\beta)(e^{-1/\beta})}}$$