22.12.2021 unit 3 practise question

2. $N \sim Poi(\lambda)$ $E(x) = \mu$, $v(x) = \sigma^2$

$M_N(t) = e^{\lambda(e^t - 1)}$

$S = X_1 + X_2 + \dots + X_n$

$M_S = M_N(\ln M_X(t)) = e^{\lambda(e^{\ln M_X(t)} - 1)}$

$= e^{\lambda(M_X(t) - 1)}$

$S = X_1 + X_2 + X_3 \dots + X_N$

$S_{/N=n} = X_1 + X_2 + X_3 \dots + X_n$

$M_S(t) = E(e^{tS}) = E(E(e^{tS}/N=n))$

$= E(E(e^{t(X_1 + X_2 \dots X_n)}))$

$= E(E(e^{tX_1}) . E(e^{tX_2}) \dots E(e^{tX_n}))$

$= E(M_X(t) . M_X(t) \dots M_X(t))$

$= E([M_X(t)]^n)$

$= E(M_X(t)^N)$

$= E(e^{\ln(M_X(t)^N)}) = M_N(\ln(M_X(t)))$

7.

ii) $X_1 \sim Gamma(\alpha, \lambda)$ $X_2 \sim Gamma(\alpha_2, \lambda)$

$M_{X_1}(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha_1}$ $M_{X_2}(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha_2}$

$Y = X_1 + X_2$

$M_Y(t) = E(e^{tY}) = E(e^{t(X_1 + X_2)})$

$= E(e^{tX_1 + tX_2})$

$= E(e^{tX_1}) E(e^{tX_2})$

$= M_{X_1}(t) . M_{X_2}(t)$

$= \left(1 - \frac{t}{\lambda}\right)^{-\alpha_1} \left(1 - \frac{t}{\lambda}\right)^{-\alpha_2}$

$= \left(1 - \frac{t}{\lambda}\right)^{-(\alpha_1 + \alpha_2)}$

$Y \sim Gramma(\alpha_1 + \alpha_2, \lambda)$

9. $M_X(t) = (1-10t)^{-2}$ $\lambda = \frac{1}{10}$ $\alpha = 2$

$X \sim Gramma(2, \frac{1}{10})$

$V(X) = E(X^2) - [E(X)]^2$ $E(X) = \frac{\alpha}{\lambda} = \frac{2}{\frac{1}{10}} = 20$

$E(X^2) = V(X) + [E(X)]^2$ $V(X) = \frac{\alpha}{\lambda^2} = \frac{2}{\frac{1}{100}} = 200$

$= 200 + 400$

$= 600$

$E(X^3) = M'''_X(t)$ a $(t=0) = M_X'''(0)$

8. $M_N(t) = e^{\lambda(e^t - 1)}$ $N \sim Poi(\lambda)$

$M_S(t) = M_N(\ln(M_X(t)))$

$= e^{\lambda(M_X(t) - 1)}$

$C_X(t) = \ln[M_X(t)]$ $C_S(t) = \ln[M_S(t)]$

$= \ln(e^{\lambda(M_X(t)-1)})$

$= \lambda(M_X(t) - 1)$

Assignment - 2

2. $P(N=n) = \binom{n+2}{n} \underset{p}{0.9^3} \underset{q}{0.1^n}$

i) $\left(\frac{pe^t}{1-qe^t}\right)^r$

$\frac{0.9 e^t}{1-0.1.e^t}$

$\left(1-\frac{t}{\lambda}\right)^{-\alpha}$, $t<\lambda$

$\left(1-\frac{t}{6}\right)^{-2}$

Y = Total claim = claim amount × no of claim

$= \frac{0.9e^t}{1-0.1e^t} \times \left(1-\frac{t}{6}\right)^{-2}$

3.

Assignment of unit 4

1. i) $E(U) = E(E[Y/X)] = E(Y) = 5$

ii) $Var(E) = Var(E(Y/X)) = Var(Y) - E[Var(Y/X)]$
$= 4 - 2 = 2$

2. i) $f(x,y) = Ke^{-(x+2y)} = Ke^{-x}e^{-2y}, \; x>0, y>0$

ii) $\iint_0^\infty f(x,y)\,dx\,dy = K\iint_0^\infty e^{-(x+2y)}\,dx\,dy$

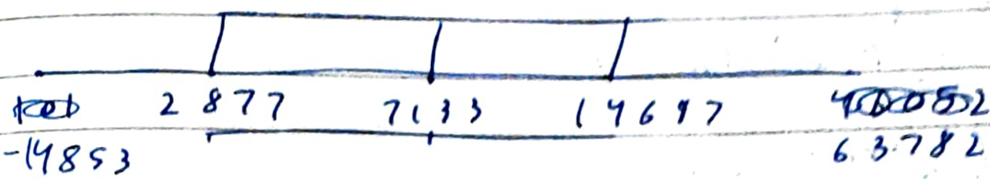
$= K\int_0^\infty e^{-x}\,dx \int_0^\infty e^{-2y}\,dy$

$= \cancel{A[-e^{x}]_0^\infty \{-\frac{1}{2}e^{-2y}\}_0^\infty =}$

$[\,[-e^{x}]_0^\infty = 1, \; [-\frac{1}{2}e^{-2y}]_0^\infty = \frac{1}{2}$

$= K[1][\frac{1}{2}] \quad K = 2 \quad f(x,y)$ is 1

) $f_Y(y) = 2\int_0^\infty e^{-x}e^{-2y}\,dx = 2e^{-2y}\int_0^\infty e^{-x}\,dx$
$= 2e^{-2y}$



Inter ~~qu~~ quantile range = $Q_1 - Q_1$ = 11820

minimum = $Q_1 - 1.5\, IQR = 2877 - 1.5 \times 11820$

$= -14853$

maximum = 63782

4. i) even no = 2, 4, 6 $\quad \frac{3}{6} = \frac{1}{2}$

ii) more than avg = 4, 5, 6 $\quad \frac{3}{6} = \frac{1}{2}$

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$

correction of mock paper

1.a i) Exponential if we assume memoryless p

ii) Poisson distribution as it is countin
random variable

iii) uniform distribution with n possibi
n being the number of distinct numbers

iv) Depending on the skewness in data,
model can be normal

v) Beta distribution as value is between
0 and 1.

b. For exponential distribution :
PDF = $\lambda \cdot \exp(-\lambda x)$
CDF $F(x) = 1 - \exp(-\lambda x)$

mean = $\int x f(x)\,dx = \int_0^{\infty} x \lambda e^{-\lambda x}\,dx$

c) $Q_1 = \frac{(50+2)^{th}}{4} = 13^{th}$ value $= 28,77,000$

$Q_2 = \frac{2(50+2)}{4} = 25.5^{th}$ value $= 69,33,000$

$Q_3 = \frac{3(50+2)}{4} = 38^{th}$ value $= 1,38,63,000$

2 a) A = even number B = more than average

Addition rule : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

multiplication : $P(A \cap B) = P(A) \cdot P(B/A)$