Assignment.

① i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$n = 10$

$\text{Mean}(\bar{x}) = \frac{\Sigma x_n}{n} = \frac{134}{10} = 13.4$

$\text{Median} = \left(\frac{n+1}{2}\right)^{th} = 5.5^{th}$

$= \frac{11+15}{2} = 13$

Mode = 18.

$SD = \sqrt{\frac{\Sigma(x_i - x)^2}{n}}$

x_i	$(x_i - \bar{x})^2$
4	88.36
7	40.96
9	19.36
9	19.36
11	5.76
15	2.56
18	21.16
18	21.16
18	21.16
25	134.56

$\sigma^2 = \frac{374.4}{10} = 37.44$

$\boxed{\sigma = 6.12}$

$\Rightarrow Q_1 = \left(\frac{n}{4}\right)^{th} = 2.5^{th}$ term

last term of $Q_1 = \frac{16}{2} = 8$

$Q_3 = 7.5^{th}$

$Q_3 = 18$

Interquartile Range $\Rightarrow Q_3 - Q_1$

$= 10$

(ii)

x_i	f_i	cf
0	5	5
1	11	16
2	26	42
3	15	57
4	3	60

$n = 60$

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{n}$

$= \frac{120}{60} = 2.$

Median $= \left(\frac{n+1}{2}\right)^{th} = (30.5^{th})$

Median $= 2$

Mode $= 2$

x_i	f_i	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
0	5	4	20
1	11	1	11
2	26	0	0
3	15	1	15
4	3	4	12

$\Sigma f_i(x_i-\bar{x})^2 = 58$

$\sigma^2 = \frac{58}{60} = 0.97$

$\sigma = \sqrt{0.97} = 0.98$

Interquartile Range $= Q_3 - Q_1$

$= 3 - 1$

$= 2.$

Q2) $\lambda = 0.5$

Poi(0.5)

P(no. claim arrising) = P(X=0)

$$P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \frac{(0.5)^0 e^{-0.5}}{0!}$$

$$= e^{-0.5}$$

$$= 0.60653066$$

(ii) $P(a) = P(X \geq 0) = 1 - P(X=0)$

$= 1 - 0.60653066$

$= 0.39346934$

$P(b) = P(X=0) = 0.60653066$

Thus, $X = (3, 0.39346934)$

$P(X=1) = {}^3C_1 (0.39346934)^1 (0.60653066)^2$
$= 0.434247843$
$= 0.434$

III) $\alpha = 1$
$P(X>2) = 1 - P(X \leq 2)$
$= 1 - F(2)$
$= 1 - (1 - e^{-\lambda\alpha})$
$= e^{-\lambda x}$
$= e^{-0.5 \times 2}$
$= e^{-1}$

$P(X>2) = 0.367879441$
$= 0.368.$

Q3) $\alpha = 2 \qquad \lambda = 1/2$

$E(X) = \frac{\alpha}{\lambda}$
$= \frac{2}{1/2}$
$= 4$

$V(X) = \frac{\alpha}{\lambda^2}$
$= \frac{2}{1/4}$
$= 8$

$\sigma = \sqrt{8} = 2\sqrt{2} = 2.83$

Page

Date

(ii) coefficient of skewness $= \frac{2}{\sqrt{\alpha}}$

$= \frac{2}{\sqrt{2}}$

$= 1.414$

iii) $f_x(x) = \lambda^{\alpha} x^{\alpha-1} e^{-\lambda\alpha}$

$= (1/2)^2 x^1 e^{-x/2}$

$= (2-1)!$

$= \frac{1}{4}(xe^{-x/2})$

$F_x(x) = \frac{1}{4}\int_0^x xe^{-x/2}$

$= \frac{1}{4}\left[2e^{-x/2} - 2xe^{-x/2}\right]_0^x$

$= \frac{1}{4}\left[2e^{-x/2} - 2xe^{-x/2} - 2\right]_0^x$

$= \frac{1}{2}\left(e^{-x/2} - xe^{-x/2} - 1\right)$

Q4) P(choosing 2 nos. from the group) $= {}^{18}C_2 = P(A)$

P(atleast 1 female) $= 1 - P$

$P(F) = 1 - \frac{{}^{10}C_2}{{}^{18}C_2}$

$= 0.705882353$

P(2 qualified females) $= \frac{{}^{7}C_2}{{}^{18}C_2}$

$P(F \cap Q) = 0.137254902$

$$P(Q|F) = \frac{P(F \cap Q)}{P(F)}$$

$$= \frac{0.137254902}{0.705882353}$$

$$= 0.1945$$

Q5) $P(L_1) = 0.4$

$P(L_2) = 0.5$

$P(L_3) = 0.1$

$P(L|L_1) = 0.2$

$P(L/L_2) = 0.4$

$P(L/L_3) = 0.1$

$$P(L_2/L) = \frac{P(L_2)P(L_1/L_2)}{P(L_1)P(L/L_1) + P(L_2)P(L/L_2) + P(L_3)P(L/L_3)}$$

$$= 0.69$$

Q6) $x \sim v(1,2)$

$$f_x x = \frac{1}{\beta - \alpha} = 1$$

$$y = \frac{3}{6-x}$$

$$y(6-x) = 3$$

$$6 - x = \frac{3}{y}$$

$$x = -\frac{3}{x} + 6$$

$$g^{-1}(y) = 6 - \frac{3}{x}$$

$$\frac{d}{dy} g^{-1}(y) = -3\left(\frac{-1}{y^2}\right)$$

$$= \frac{3}{y^2}$$

Page :
Date :

$$f_Y(y) = f_X\, g^{-1}(y) \left| \frac{d}{dy} g^{-1}(y) \right|$$

$$= \frac{1 \times 3}{y^2}$$

$$= 3y^{-2}$$

Q7) i) $F(3) = P(X \leq 3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

(ii) $E(X) = x_1p_1 + x_2p_2 + x_3p_3 + x_4p_4 + x_5p_5$

$= 3.25$

iii) $V(X) = 11.45$

coefficent of skewness $= \frac{\mu_3}{\sigma^3}$

$$\frac{E(X-\mu)^3}{\sigma^3}$$

$= \mu E(X) = [E(X)]^2$

$$= \frac{E(x^3) - 3E(x)\sigma^2 - (E(x))^2}{\sigma^3}$$

$= 42.55$

coefficent of skewness $= -2.67$

Camlin

Q8) Let success be defined as winning the prize.

$P(A) = 0.01$

$P(F) = 0.99$

$P(X=3) = {}^{7}C_{3}(0.01)^{3}(0.99)^{4}$

$= 0.000033621$

Q9) $P(S) = 0.2$

$P(F) = 0.8$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= 0.398$

Q10) $\lambda = \frac{2}{5} = 0.4$

$P(X>2) = 1 - P(X \leq 2)$

$P(X=0) = \frac{\lambda^{x} e^{-\lambda}}{x!}$

$= \frac{(0.4)^{0} e^{-0.4}}{0.1}$

$= e^{-0.4}$

$= 0.670320046$

$P(X=1) = \frac{(0.4)^{1} e^{-0.4}}{1!}$

$= 0.268128018$

$P(X=2) = \frac{(0.4)^{2} e^{-0.4}}{2!}$

$= 0.053625604$

$P(X \leq 2) = 0.992073668$

Q11) $P(b) = P(g)$

$P(b) + P(g) = 1$

$2P(b) = 1$

$P(b) = 1/2$

$\therefore P(g) = 1/2$

Let E be the event of having 3 boys.

$P(E) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Q12) $\lambda = 10$

$\alpha = 1$

$t = 0.1$ days

$P(T < 0.1) = F(0.1)$

$= 1 - e^{-\lambda t}$

$= 1 - e^{-1}$

$= 0.632120559.$

Q13) $X \sim U(75, 120)$

$$f_X(x) = \frac{1}{120-75}$$

$$= \frac{1}{45}$$

$$P(80 < x < 100) = \frac{100-80}{45}$$

$$= \frac{20}{45}$$

$$= 0.444$$

Q14) $X \sim \text{gamma}(5, 0.4)$

$2\lambda x \sim \chi^2_{2\alpha}$

$0.8X = \chi^2_{10}$

$P(X < 22.89) = P(0.8X < 18.312)$

$= P(\chi^2_{10} < 18.312)$

$P(\chi^2_{10} < 18) = 0.9450$

Page :

Date :

$$P\left(\chi^2_{10} < 18.5\right) = 0.9529$$

$$0.5 = 0.0079$$

$$0.312 = \frac{0.0079}{0.5} \times 0.312$$

$$= 0.0049296$$

$$\therefore P\left[x^2_{10} < 18.312\right) = 0.9499296$$

Thus, $P(X < 22.89) = P\left(x^2_{10} < 18.312\right)$

$$= 0.9499296.$$