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# Numerical Methods & Algebra.

Q.1) Original error radius of circle = 12.5 cm = $r_1$

Area of circle = $\pi r_1^2$
$= \pi (12.5)^2$
$= 491.0714286$

radius of circle with error ~~ad~~ radius = 12.65 = $r_2$

Area of circle = $\pi r_2^2$
$= \pi (12.65)^2$
$= 502.9278571$

↦ Absolute error = Area of circle with errored radius - Area of circle with og radius.
$= 502.9278571 - 491.0714286$
$= 11.85642850$

↦ Relative error = $\frac{\text{Absolute error}}{\text{Original area}}$
$= 0.024144000$

↦ Percentage error = Relative error $\times 100$
$= 2.414399985$

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24 a)

| x | f(x) = log x |
|---|---|
| 4 | 0.60206 |
| 6 | 0.7781513 |

Interpolation

$(6-4) = (0.7781513 - 0.60206)$

$2 = 0.17609300$

1 causes change of $= \frac{0.17609300}{2}$

$1 = 0.088045650$

Thus $x = 5$

$\log(5) = \log(4) + \log(1)$

$= 0.60206 + 0.088045650$

$= 0.690105650$.

b)

| x | log(x) |
|---|---|
| 4.5 | 0.6532125 |
| 5.5 | 0.7403627 |

$5.5 - 4.5 = 0.7403627 - 0.6532125$

$1 = 0.087150200$.

0.5 causes change of $= \frac{0.087150200}{2}$.

$= 0.043575100$.

$x = 5$

$\log(5) = \log(4.5) + 0.043575100$

$= 0.6532125 + 0.043575100$

$= 0.65362650 \quad 0.6967\ 87600$

Q.34 $x^4 - x - 10 = 0$

| | $x$ | $y$ |
|---|---|---|
| $x_0 =$ | 1.5 | $-6.437$ |
| $x_1 =$ | 2. | 4. |

$\rightarrow x_2 = \frac{x_0 + x_1}{2}$ $\quad y_2 = -2.371693750$

$= 1.75$

$\rightarrow x_3 = \frac{x_1 + x_2}{2}$ $\quad y_3 = 0.4846\ 1941$

$= \frac{2 + 1.75}{2} = 1.875$

$\rightarrow x_4 = \frac{x_2 + x_3}{2}$ $\quad y_4 = -1.020248413$

$= 1.8125$

$\rightarrow x_5 = \frac{x_3 + x_4}{2}$ $\quad y_5 = -0.2877\ 34032$

$= 1.84375$

$\rightarrow x_6 = \frac{x_4 + x_5}{2} = 1.8235$

$y_6 = -0.7668625$

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8.44 $f(x) = x^3 - x - 1$ roots using Newton Raphson method.

$f'(x) = 3x^2 - 1$

| x | y |
|---|---|
| 0 | -1 |
| 1 | -1 |
| 2 | 5 |

$x_0 = 1$

$f(x_0) = -1.$

$f'(x_0) = 2.$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$1 - \left(\frac{-1}{2}\right)$$

$= 1.5.$

$y = 0.875$

$f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \left(\frac{0.875}{5.75}\right)$$

$x_2 = 1.34783$

$f(x_2) = 0.100699$

$f'(x_2) = 4.449945$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$= 1.3252$

$y_3 = 0.00205$

$f'(x_3) = 4.26846.$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$= 1.32472.$

$y_4 = 0.000048712.$

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Q.5) $A^2 = A$

$7A - 7A(1+A)^3 = 7A - [I^3 + A^3 + 3A^2I + 3AI^2]$

$= 7A - (I + A^2 + 6A)$

$= 7A - I + 7A$

$= -I$

Q.6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 0 - 1(6-8) + (-1)(0+4)$

$= 2 - 4$

$= -2$

$\text{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \times \text{adj}[A]$

$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$

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8.74 $\vec{a} = 8\uparrow - 9\uparrow$.

$\vec{b} = 7\uparrow - 9\uparrow$.

$\vec{a} \cdot \vec{b} = ab\cos\theta$.

$= (8\uparrow - 9\uparrow)(7\uparrow - 9\uparrow)$

$= 56 + 81$

$= 137$.

$a = \sqrt{64+81}$

$= \sqrt{145}$

$b = \sqrt{49+81}$

$= \sqrt{130}$.

$\vec{a} \cdot \vec{b} = ab\cos\theta$.

$\cos\theta = \frac{137}{\sqrt{130} \times \sqrt{145}}$

$\cos\theta = 0.997849146$

$\theta = 3.758\,56^\circ$.

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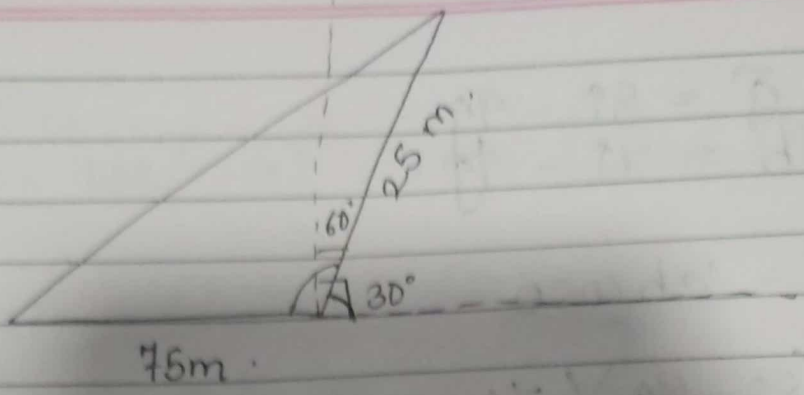
Q.8)




Displacement Vector = $\vec{R}$

$\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 30°$

$R^2 = 75^2 + 25^2 + 2(75)(25)\cos\theta$

$R^2 = 9497.595264$

$R = 97.4556$ metres.

Thus displacement = 97.4556 metres.

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Q.9) Common difference, $d = 3$

A.P = 101, 104, 107.

$a = 101$

$a_n = a + (n-1)d$.

$998 = 101 + (n-1)3$.

$n = 300$

$S_n = \frac{n}{2}(2a + (n-1)d) \quad \Big| \quad \frac{n}{2}(2a + a_n)$

$= \frac{300}{2}(2(101) + 998 + 101)$

$= 150 \times 1099$

$= 1,64,850$

Q.10) $a_6 = 32$.

$ar^5 = 32$

$a_8 = 128$

$ar^7 = 128$

$\frac{ar^7}{ar^5} = \frac{128}{32} \qquad r^2 = 4$

$r = 2$ or $-2$.

$r = 2$.

Q.11) $(3x+4)^5$.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | 1 | | | $a$ |
| | | 1 | | 1 | | $(a+b)$ |
| | | 1 | 2 | 1 | | $(a+b)^2$ |
| | 1 | 3 | | 3 | 1 | $(a+b)^3$ |
| | 1 | 4 | 6 | 4 | 1 | $(a+b)^4$ |
| 1 | 5 | 10 | | 10 | 5 | 1 $(a+b)^5$ |

$$(3x+4)^4 = (3x)^5 + (5(3x)^4 \times 4) + (10 \times (3x)^3 \times 4^2) + (10 \times (3x^2) \times 4^3) + 5(3x \times 4^4) + 4^5.$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024.$$

Q.12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}.$$

$$|A-\lambda I| = 0 = 0 + (3-\lambda)[(2-\lambda)(-5-\lambda)+6]$$

$$= (3-\lambda)[-10-2\lambda+5\lambda+\lambda^2+6]$$

$$= [\lambda^2+3\lambda-4](3-\lambda)$$

$$= 9\lambda - 12 - \lambda^3$$

$$|A-\lambda I) = -\lambda^3+13\lambda-12$$

Comparing Coefficents

$$a+b+c+d = 1-13+12$$
$$= 0$$

Thus one root = 1.

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | -13 | 12 |
| | | 1 | 1 | -12 |
| | 1 | -12 | 0 | |

$$\lambda^2+\lambda-12=0$$

$$\lambda = 3 \text{ or } -4$$

Eigen Values = -4, 1, 3.

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Q.13) $f(x) = x^3 - 7x^2 + 8x - 3$.

$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$

$f(5) = -13$.

$f'(5) = 13$.

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 5 - \left(\frac{-13}{13}\right)$

$= 6$.

$f(x_1) = 9$.

$f'(x_1) = 32$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$x_2 = 6 - \frac{9}{32}$$

$x_2 = 5.71875$

$f(x_2) = 0.84787$.

8.14 $(x^2 - x + 1)^4 = ((x-1)^2 + x)^4$.

Using Pascal's Triangle.

| | | | | | |
|---|---|---|---|---|---|
| | | 1 | | | $a$ |
| | 1 | | 1 | | $a+b$ |
| 1 | | 2 | | 1 | $(a+b)^2$ |
| 1 | 3 | | 3 | 1 | $(a+b)^3$ |
| 1 | 4 | 6 | 4 | 1 | $(a+b)^4$. |

$(x^2 - x + 1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$.

8.15 $e^{-x} = 3\log(x)$.

$3\log(x) - e^{-x} = 0$

| $x$ | $y$ |
|---|---|
| 0.1 | -2.07516 |
| 0.2 | -1.27818 |
| 0.3 | -0.82782 |
| 0.4 | -0.5235 |
| 0.5 | -0.29656 |
| 0.6 | -0.11673 |
| 0.7 | 0.03188 |

$x_0 = 0.6$ , $y_0 = -0.11673$

$x_1 = 0.7$ , $y_1 = 0.03188$

$$x_2 = \frac{x_0 + x_1}{2}$$

$$= 0.65$$

$$y_2 = -0.03921$$

$$x_3 = \frac{x_2 + x_1}{2}$$

$$= 0.675$$

$$y_3 = -0.00293$$

$$x_4 = \frac{x_3 + x_1}{2}$$

$$= 0.6875$$

$$y_4 = 0.01465$$

$$x_5 = \frac{x_4 + x_3}{2}$$

$$= 0.68125$$

$$y_5 = 0.0059$$